Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

1999 Solutions
Pascal Contest (Grade 9)

for the
NATIONAL BANK OF CANADA
Awards

Part A

1. The value of $\dfrac{4\times 4+4}{2\times 2-2}$ is

(**A**) 2 (**B**) 6 (**C**) 10 (**D**) 12 (**E**) 18

Solution

$\dfrac{4\times 4+4}{2\times 2-2}=\dfrac{16+4}{4-2}=\dfrac{20}{2}=10$ ANSWER: (C)

2. If $k=2$, then $\left(k^3-8\right)(k+1)$ equals

(**A**) 0 (**B**) 3 (**C**) 6 (**D**) 8 (**E**) -6

Solution

For $k=2$, $\left(k^3-8\right)(k+1)$

$=\left(2^3-8\right)(2+1)$

$=0(3)$

$=0$ ANSWER: (A)

3. If $4(\heartsuit)^2=144$, then a value of $\heartsuit$ is

(**A**) 3 (**B**) 6 (**C**) 9 (**D**) 12 (**E**) 18

Solution

$4(\heartsuit)^2=144$

$\heartsuit^2=36$

$\heartsuit=\pm 6$ ANSWER: (B)

4. Which of the following numbers divide exactly into $\left(15+\sqrt{49}\right)$?

(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 7 (**E**) 11

Solution

$15+\sqrt{49}=15+7=22$

The only integer listed that divides 22 evenly is 11. ANSWER: (E)

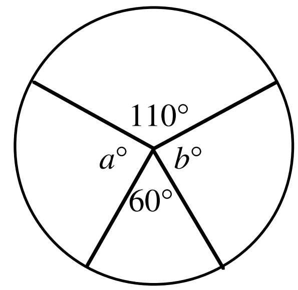
5. If 10% of 400 is decreased by 25, the result is

(**A**) 15 (**B**) 37.5 (**C**) 65 (**D**) 260 (**E**) 3975

Solution

(10% of 400) $-25=40-25=15$. ANSWER: (A)

6. In the diagram, $a+b$ equals



(**A**) 10 (**B**) 85 (**C**) 110
(**D**) 170 (**E**) 190

Solution
The number of degrees at the centre of a circle is 360.
Thus, $a+b+110+60=360$ (measured in degrees).
Therefore $a+b=190$. ANSWER: (E)

7. If $2x-1=5$ and $3y+2=17$, then the value of $2x+3y$ is

(**A**) 8 (**B**) 19 (**C**) 21 (**D**) 23 (**E**) 25

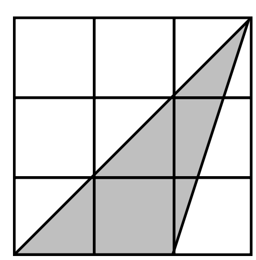
Solution
$2x-1=5$, $3y+2=17$
$2x=6$ $3y=15$
Thus, $2x+3y=6+15=21$. ANSWER: (C)
Note: It is not necessary to solve the equations to find actual values for x and y although this would of course lead to the correct answer. It is, however, a little more efficient to solve for $2x$ and $3y$.

8. The average of four test marks was 60. The first three marks were 30, 55 and 65. What was the fourth mark?

(**A**) 40 (**B**) 55 (**C**) 60 (**D**) 70 (**E**) 90

Solution
The total number of marks scored on the four tests was 4×60 or 240. The total number of marks scored on the first three tests was 150. The fourth mark was $240-150=90$. ANSWER: (E)

9. In the diagram, each small square is 1 cm by 1 cm. The area of the shaded region, in square centimetres, is



(**A**) 2.75 (**B**) 3 (**C**) 3.25
(**D**) 4.5 (**E**) 6

Solution
The shaded triangle has a base of 2 cm and a height of 3 cm.

Its area is $\frac{2 \times 3}{2} = 3$ (sq. cm). ANSWER: (B)

10. $10 + 10^3$ equals

(**A**) 2.0×10^3 (**B**) 8.0×10^3 (**C**) 4.0×10^1 (**D**) 1.0×10^4 (**E**) 1.01×10^3

Solution
$10 + 10^3 = 10 + 1000 = 1010 = 1.01 \times 10^3$ ANSWER: (E)

Part B

11. Today is Wednesday. What day of the week will it be 100 days from now?

(**A**) Monday (**B**) Tuesday (**C**) Thursday (**D**) Friday (**E**) Saturday

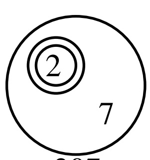
Solution
Since there are 7 days in a week it will be Wednesday in 98 days.
In 100 days it will thus be Friday. ANSWER: (D)

12. The time on a digital clock is 5:55. How many minutes will pass before the clock next shows a time with all digits identical?

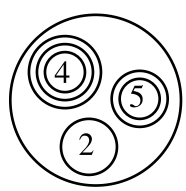
(**A**) 71 (**B**) 72 (**C**) 255 (**D**) 316 (**E**) 436

Solution
The digits on the clock will next be identical at 11:11. This represents a time difference of 316 minutes. (Notice that times like 6:66, 7:77 etc. are not possible.) ANSWER: (D)

13. In *Circle Land*, the numbers 207 and 4520 are shown in the following way:

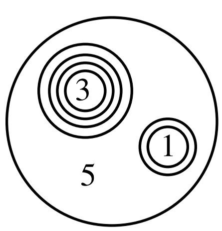


207



4520

In *Circle Land*, what number does the following diagram represent?



(**A**) 30 105 (**B**) 30 150 (**C**) 3105 (**D**) 3015 (**E**) 315

Solution 1

$= 3 \times 10^4 = 30\,000$

$= 1 \times 10^2 = 100$

5 $= 5 \times 10^0 = 5$

The required number is $30\,000 + 100 + 5 = 30\,105$.

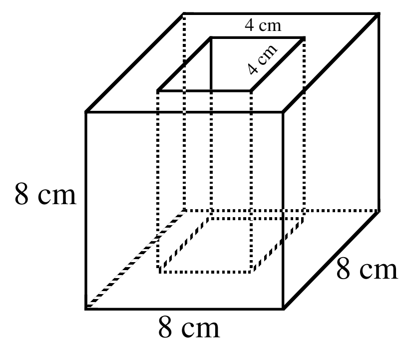
Solution 2

Since there are four circles around the '3' this corresponds to $3 \times 10^4 = 30\,000$.
The '5' corresponds to a 5 in the units digit which leads to $30\,105$ as the only correct possibility.

ANSWER: (A)

14. An 8 cm cube has a 4 cm square hole cut through its centre, as shown. What is the remaining volume, in cm^3?

(**A**) 64 (**B**) 128 (**C**) 256
(**D**) 384 (**E**) 448



Solution

Remaining volume $= 8 \times 8 \times 8 - 8 \times 4 \times 4$ (in cm^3)
$= 8(64 - 16)$
$= 8 \times 48$
$= 384$

ANSWER: (D)

15. For how many different values of k is the 4-digit number $7k52$ divisible by 12?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

Solution

Since $12 = 4 \times 3$ the number $7k52$ must be divisible by both 4 and 3. Since 52 is the number formed by the last two digits divisible by 4 then we need only ask, 'for what values of k is $7k52$ divisible by 3?' If a number is divisible by 3 the sum of its digits must be a multiple of 3. Thus $7 + k + 5 + 2$ or $14 + k$ must be a multiple of 3. The only acceptable values for k are 1, 4 or 7.
Thus, are three values.

ANSWER: (D)

16. In an election, Harold received 60% of the votes and Jacquie received all the rest. If Harold won by 24 votes, how many people voted?

(**A**) 40 (**B**) 60 (**C**) 72 (**D**) 100 (**E**) 120

Solution

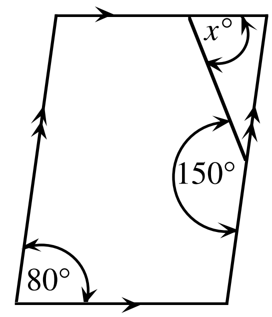
If Harold received 60% of the votes this implies that Jacquie received 40% of the total number of votes. The difference between them, 20%, represents 24 votes.

Therefore, the total number of votes cast was $5 \times 24 = 120$. ANSWER: (E)

17. In the parallelogram, the value of x is

(**A**) 30 (**B**) 50 (**C**) 70

(**D**) 80 (**E**) 150



Solution

The angle in the parallelogram opposite the angle measuring $80°$ is also $80°$. The angle supplementary to $150°$ is $30°$.

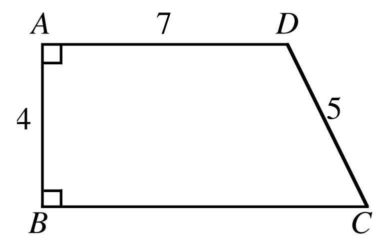
In the given triangle we now have, $x° + 80° + 30° = 180°$.

Therefore $x = 70$. ANSWER: (C)

18. In the diagram, $AD < BC$. What is the perimeter of $ABCD$?

(**A**) 23 (**B**) 26 (**C**) 27

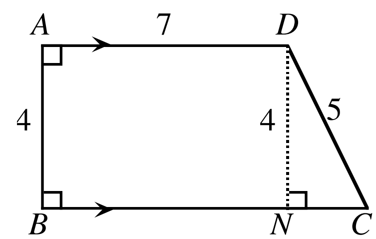
(**D**) 28 (**E**) 30



Solution

From D we draw a line perpendicular to BC that meets BC at N. Since $ADNB$ is a rectangle and $AD \| BC$, $DN = 4$. We use Pythagoras to find $NC = 3$. We now know that $BC = BN + NC = 7 + 3 = 10$.

The required perimeter is $7 + 5 + 10 + 4 = 26$.



ANSWER: (B)

19. The numbers 49, 29, 9, 40, 22, 15, 53, 33, 13, 47 are grouped in pairs so that the sum of each pair is the same. Which number is paired with 15?

(**A**) 33 (**B**) 40 (**C**) 47 (**D**) 49 (**E**) 53

Solution

If we arrange the numbers in ascending order we would have: 9, 13, 15, 22, 29, 33, 40, 47, 49, 53. If the sum of each pair is equal they would be paired as: $9 \leftrightarrow 53$, $13 \leftrightarrow 49$, $15 \leftrightarrow 47$, $22 \leftrightarrow 40$, $29 \leftrightarrow 33$.

ANSWER: (C)

20. The units (ones) digit in the product $(5+1)\left(5^3+1\right)\left(5^6+1\right)\left(5^{12}+1\right)$ is

(**A**) 6 (**B**) 5 (**C**) 2 (**D**) 1 (**E**) 0

Solution

We start by observing that each of 5^3, 5^6 and 5^{12} have a units digit of 5. This implies that each of $5+1$, 5^3+1, 5^6+1 and $5^{12}+1$ will then have a units digit of 6.

If we multiply any two numbers having a units digit of 6, their product will also have a units digit of 6. Applying this to the product of four numbers, we see that the final units digit must be a 6.

ANSWER: (A)

Part C

21. A number is *Beprisque* if it is the only natural number between a prime number and a perfect square (e.g. 10 is Beprisque but 12 is not). The number of *two-digit* Beprisque numbers (including 10) is

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Solution

We start with the observation that it is necessary to consider only the odd perfect squares and the integers adjacent to them. It is not necessary to consider the even perfect squares because if we add 2 or subtract 2 from an even number the result is even and it is required by the conditions set out in the question that this number be prime. Considering then the odd perfect squares we have: $\{9, 10, 11\}$, $\{23, 24, 25, 26, 27\}$, $\{47, 48, 49, 50, 51\}$, $\{79, 80, 81, 82, 83\}$.

The Beprisque numbers are those that are circled.

ANSWER: (E)

22. If $w = 2^{129} \times 3^{81} \times 5^{128}$, $x = 2^{127} \times 3^{81} \times 5^{128}$, $y = 2^{126} \times 3^{82} \times 5^{128}$, and $z = 2^{125} \times 3^{82} \times 5^{129}$, then the order from smallest to largest is

(**A**) w, x, y, z (**B**) x, w, y, z (**C**) x, y, z, w (**D**) z, y, x, w (**E**) x, w, z, y

Solution

We start with the observation that $2^{125} \times 3^{81} \times 5^{128}$ is a common factor to each of the given numbers.

For the basis of comparison, we remove the common factor and write the numbers as follows:

$$w = 2^4 \cdot 5^3\left(2^{125} \times 3^{81} \times 5^{128}\right) = 2000k$$

$$x = 2^2 \cdot 5^3\left(2^{125} \times 3^{81} \times 5^{128}\right) = 500k$$

$$y = 2 \cdot 3 \cdot 5^3\left(2^{125} \times 3^{81} \times 5^{128}\right) = 750k$$

$$z = 3 \cdot 5^4\left(2^{125} \times 3^{81} \times 5^{128}\right) = 1875k, \text{ where } k = 2^{125} \times 3^{81} \times 5^{128}.$$

Thus, $x < y < z < w$. ANSWER: (C)

23. Al and Bert must arrive at a town 22.5 km away. They have one bicycle between them and must arrive at the same time. Bert sets out riding at 8 km/h, leaves the bicycle and then walks at 5 km/h. Al walks at 4 km/h, reaches the bicycle and rides at 10 km/h. For how many minutes was the bicycle not in motion?

(**A**) 60 (**B**) 75 (**C**) 84 (**D**) 94 (**E**) 109

Solution
Let x represent the distance that Bert rides his bicycle.
Therefore, he walks for $(22.5 - x)$ km.
Bert's total time for the trip is $\left(\frac{x}{8} + \frac{22.5 - x}{5}\right)$ hours and Al's is $\left(\frac{x}{4} + \frac{22.5 - x}{10}\right)$ hours.
Since their times are equal,

$$\frac{x}{8} + \frac{22.5 - x}{5} = \frac{x}{4} + \frac{22.5 - x}{10}$$

$$\frac{22.5 - x}{5} - \frac{22.5 - x}{10} = \frac{x}{4} - \frac{x}{8}$$

$$\frac{2(22.5 - x)}{10} - \frac{22.5 - x}{10} = \frac{2x}{8} - \frac{x}{8}$$

$$\frac{22.5 - x}{10} = \frac{x}{8}$$

$$10x = 180 - 8x$$

$$18x = 180$$

$$x = 10.$$

This means that Bert rode for 1.25 h before he left the bicycle and Al walked for 2.5 h before he picked it up. The bicycle was thus not in motion for 1.25 h or 75 minutes. ANSWER: (B)

24. A number is formed using the digits 1, 2, ..., 9. Any digit can be used more than once, but adjacent digits cannot be the same. Once a pair of adjacent digits has occurred, that pair, in that order, cannot be used again. How many digits are in the largest such number?

(**A**) 72 (**B**) 73 (**C**) 144 (**D**) 145 (**E**) 91

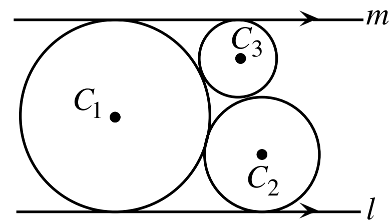
Solution

Since there are $9(8)=72$ ordered pairs of consecutive digits, and since the final digit has no successor, we can construct a 73 digit number by adding a 9. The question is, of course, can we actually construct this number? The answer is 'yes' and the largest such number is,

98 97 96 95 94 93 92 91 87 86 85 84 83 82 81 76 75 74 73 72 71 65 64 63 62 61
54 53 52 51 43 42 41 32 31 21 9.

If we count the numbers in the string we can see that there are actually 73 numbers contained within it. ANSWER: (B)

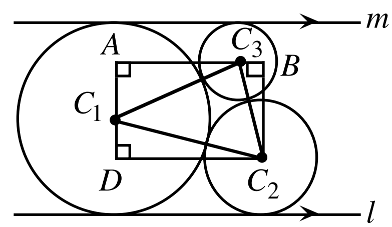
25. Two circles C_1 and C_2 touch each other externally and the line l is a common tangent. The line m is parallel to l and touches the two circles C_1 and C_3. The three circles are mutually tangent. If the radius of C_2 is 9 and the radius of C_3 is 4, what is the radius of C_1?



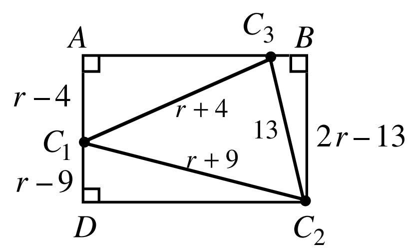
(**A**) 10.4 (**B**) 11 (**C**) $8\sqrt{2}$
(**D**) 12 (**E**) $7\sqrt{3}$

Solution

We start by joining the centres of the circles to form $\Delta C_1C_2C_3$. (The lines joining the centres pass through the corresponding points of tangency.)
Secondly, we construct the rectangle ABC_2D as shown in the diagram. If the radius of the circle with centre C_1 is r we see that: $C_1C_2=r+9$, $C_1C_3=r+4$ and $C_2C_3=13$.



We now label lengths on the rectangle in the way noted in the diagram.



To understand this labelling, look for example at C_1D. The radius of the large circle is r and the radius of the circle with centre C_2 is 9. The length C_1D is then $r-9$.
This same kind of reasoning can be applied to both C_1A and BC_2.

Using Pythagoras we can now derive the following:

In ΔAC_3C_1, $C_3A^2=(r+4)^2-(r-4)^2$
$=16r$.

Therefore $C_3A=4\sqrt{r}$.

In ΔDC_1C_2, $\quad (DC_2)^2 = (r+9)^2 - (r-9)^2$
$\quad = 36r.$

Therefore $DC_2 = 6\sqrt{r}$.

In ΔBC_3C_2, $\quad (C_3B)^2 = 13^2 - (2r-13)^2$
$\quad = -4r^2 + 52r.$

Therefore $C_3B = \sqrt{-4r^2 + 52r}$.

In a rectangle opposite sides are equal, so:

$DC_2 = C_3A + C_3B$

or, $6\sqrt{r} = 4\sqrt{r} + \sqrt{-4r^2 + 52r}$

$2\sqrt{r} = \sqrt{-4r^2 + 52r}$.

Squaring gives, $4r = -4r^2 + 52r$

$4r^2 - 48r = 0$

$4r(r-12) = 0$

Therefore $r = 0$ or $r = 12$.

Since $r > 0$, $r = 12$. ANSWER: (D)