Cayley Contest (Grade 10)
Wednesday, February 19, 2003

Instructions
1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   - There is no penalty for an incorrect answer.
   - Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have sixty minutes of working time.

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Time: 1 hour
Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\frac{3-(-3)}{2-1}$ is
   (A) 2  (B) 0  (C) 3  (D) 6  (E) -3

2. $17^2 - 15^2$ equals
   (A) $8^2$  (B) $2^2$  (C) $4^2$  (D) $7^2$  (E) $6^2$

3. The integer 42 is
   (A) an odd number  (B) a prime number  (C) a perfect cube
   (D) divisible by 7  (E) a perfect square

4. If 5% of a number is 8, what is 25% of the same number?
   (A) 40  (B) 0.1  (C) 320  (D) 10  (E) 200

5. The integer closest to the value of $\frac{3 \times 4 \times 9 + 7}{2}$ is
   (A) 3  (B) 4  (C) 5  (D) 6  (E) 7

6. In the diagram, $ABC$ is a straight line. The value of $x$ is
   (A) 27  (B) 33  (C) 24
   (D) 87  (E) 81

7. In the diagram, the sum of the numbers in each quarter circle is the same. The value of $x + y + z$ is
   (A) 75  (B) 64  (C) 54
   (D) 171  (E) 300

8. An equilateral triangle has a side length of 20. If a square has the same perimeter as this triangle, the area of the square is
   (A) 25  (B) 400  (C) 225  (D) 60  (E) 100
9. If \( \frac{1}{x+1} = \frac{5}{3} \), then \( x \) equals

(A) \( \frac{2}{3} \)  (B) \( \frac{4}{3} \)  (C) \( \frac{1}{3} \)  (D) \( -\frac{2}{3} \)  (E) \( -\frac{22}{3} \)

10. There are 2 girls and 6 boys playing a game. How many additional girls must join the game so that \( \frac{5}{8} \) of the players are girls?

(A) 6  (B) 3  (C) 5  (D) 8  (E) 7

Part B: Each correct answer is worth 6.

11. Let \( N = 10^3 + 10^4 + 10^5 + 10^6 + 10^7 + 10^8 + 10^9 \). The sum of the digits of \( N \) is

(A) 12  (B) 1  (C) 6  (D) 9  (E) 7

12. The points \( A(a, 1) \), \( B(9, 0) \) and \( C(-3, 4) \) lie on a straight line. The value of \( a \) is

(A) 3  (B) \( \frac{8}{3} \)  (C) \( \frac{7}{2} \)  (D) 6  (E) \( \frac{5}{2} \)

13. In the diagram, \( ABCD \) is a square with a side length of 10. If \( AX = CX = 8 \), the area of the shaded region is

(A) 16  (B) 20  (C) 40  (D) 48  (E) 24

14. Carly takes three steps to walk the same distance as Jim walks in four steps. Each of Carly’s steps covers 0.5 metres. How many metres does Jim travel in 24 steps?

(A) 16  (B) 9  (C) 36  (D) 12  (E) 18

15. In the diagram, line \( L_1 \) is parallel to line \( L_2 \) and \( BA = BC \). The value of \( x \) is

(A) 35  (B) 30  (C) 37.5  (D) 45  (E) 40

16. The value of \( \left( \begin{array}{c} 4^{2003} \\ 3^{2002} \\ 6^{2002} \\ 2^{2003} \end{array} \right) \) is

(A) 1  (B) 2  (C) 12  (D) 4  (E) \( \frac{1}{2} \)
17. In the diagram, the four circles have a common centre, and have radii of 1, 2, 3, and 4. The ratio of the area of the shaded regions to the area of the largest circle is

(A) 5 : 8  (B) 1 : 4  (C) 7 : 16  
(D) 1 : 2  (E) 3 : 8

18. If $496 = 2^m - 2^n$, where $m$ and $n$ are integers, then $m + n$ is equal to

(A) 13  (B) 9  (C) 4  (D) 14  (E) 5

19. The product of the digits of a four-digit number is 810. If none of the digits is repeated, the sum of the digits is

(A) 18  (B) 19  (C) 23  (D) 25  (E) 22

20. A car uses 8.4 litres of gas for every 100 km it is driven. A mechanic is able to modify the car’s engine at a cost of $400 so that it will only use 6.3 litres of gas per 100 km. The owner determines the minimum distance that she would have to drive to recover the cost of the modifications. If gas costs $0.80 per litre, this distance, in kilometres, is between

(A) 10 000 and 14 000  (B) 14 000 and 18 000  (C) 18 000 and 22 000  
(D) 22 000 and 26 000  (E) 26 000 and 30 000

Part C: Each correct answer is worth 8.

21. Troye and Daniella are running at constant speeds in opposite directions around a circular track. Troye completes one lap every 56 seconds and meets Daniella every 24 seconds. How many seconds does it take Daniella to complete one lap?

(A) 32  (B) 36  (C) 40  (D) 48  (E) 42

22. In the diagram, $\triangle ABC$ is isosceles with $AB = AC$ and $BC = 30$ cm. Square $EFGH$, which has a side length of 12 cm, is inscribed in $\triangle ABC$, as shown. The area of $\triangle AEF$, in cm$^2$, is

(A) 27  (B) 54  (C) 51  
(D) 48  (E) 60

23. A pyramid has a square base which has an area of 1440 cm$^2$. Each of the pyramid’s triangular faces is identical and each has an area of 840 cm$^2$. The height of the pyramid, in cm, is

(A) $30\sqrt{2}$  (B) 40  (C) $20\sqrt{6}$  (D) $20\sqrt{3}$  (E) 30

24. In how many ways can $a$, $b$, $c$, and $d$ be chosen from the set $\{0, 1, 2, ..., 9\}$ so that $a < b < c < d$ and $a + b + c + d$ is a multiple of three?

(A) 54  (B) 64  (C) 63  (D) 90  (E) 72

continued ...
25. \( \angle BAC \) is said to be “laceable” if distinct points \( X_1, X_2, \ldots, X_{2n} \) can be found so that
- \( X_{2k-1} \) is on \( AC \) for each value of \( k \),
- \( X_{2k} \) is on \( AB \) for each value of \( k \), and
- \( AX_1 = X_1X_2 = X_2X_3 = \cdots = X_{2n-1}X_{2n} = X_{2n}A \).

For example, the angle 20° is laceable, as shown. The number of laceable acute angles, whose sizes in degrees are integers, is

(A) 3  (B) 4  (C) 5  
(D) 6  (E) 7
Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

Copies of Previous Canadian Mathematics Competitions
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