Fermat Contest  (Grade 11)
Wednesday, February 19, 2003

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C. There is no penalty for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have sixty minutes of working time.
Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of \(3^3 - 3^2 + 3^1 - 3^0\) is
   (A) 18     (B) 6     (C) 9     (D) 40     (E) 20

2. If \(a = 5\) and \(a^2 + ab = 60\), the value of \(b\) is
   (A) 7     (B) 4     (C) 2     (D) 10     (E) 30

3. In the diagram, the value of \(x\) is
   (A) 22.5     (B) 25     (C) 20     (D) 36     (E) 18

4. In the diagram, the numbers 1 to 10 are placed around a circle. Sandy crosses out 1, then 4, and then 7. Continuing in a clockwise direction, she crosses out every third number of those remaining, until only two numbers are left. The sum of these two numbers is
   (A) 13     (B) 10     (C) 8     (D) 14     (E) 17

5. During hibernation, a bear loses 20% of its original mass. After hibernation, its mass is 220 kg. What was its mass, in kilograms, just before hibernation?
   (A) 176     (B) 264     (C) 240     (D) 275     (E) 1100

6. There are 2 girls and 6 boys playing a game. How many additional girls must join the game so that \(\frac{5}{8}\) of the players are girls?
   (A) 6     (B) 3     (C) 5     (D) 8     (E) 7

7. A fish tank, in the shape of a rectangular prism, has a base measuring 20 cm by 40 cm with a height of 30 cm. The tank sits on a horizontal table and is half full of water. If 4000 cm\(^3\) of water is added, what is the new depth of the water?
   (A) 5 cm     (B) 15 cm     (C) 20 cm     (D) 25 cm     (E) 10 cm
8. In the diagram, \( D \) is the point on \( BC \) so that \( AD \) is perpendicular to \( BC \). The slope of \( AD \) is
   
   \[
   \begin{align*}
   (A) & \quad \frac{3}{11} \\
   (B) & \quad 1 \\
   (C) & \quad \frac{15}{11} \\
   (D) & \quad \frac{2}{7} \\
   (E) & \quad \frac{2}{5}
   \end{align*}
   \]

9. The average (mean) of \( \frac{1}{3} \) and \( \frac{1}{10} \) is \( \frac{1}{x} \). The value of \( x \) is
   
   \[
   \begin{align*}
   (A) & \quad \frac{20}{3} \\
   (B) & \quad \frac{3}{20} \\
   (C) & \quad 30 \\
   (D) & \quad \frac{10}{3} \\
   (E) & \quad \frac{2}{15}
   \end{align*}
   \]

10. Carly takes three steps to walk the same distance as Jim takes in four steps. Each of Carly’s steps covers 0.5 metres. How many metres does Jim travel in 24 steps?
   
   \[
   \begin{align*}
   (A) & \quad 16 \\
   (B) & \quad 9 \\
   (C) & \quad 36 \\
   (D) & \quad 12 \\
   (E) & \quad 18
   \end{align*}
   \]

Part B: Each correct answer is worth 6.

11. In the diagram, it is only possible to travel along an edge in the direction indicated by the arrow. Hazel studied the figure, and determined all the possible routes from \( A \) to \( B \). She selected one of these routes at random. What is the probability that she selected a route which passes through \( X \)?
   
   \[
   \begin{align*}
   (A) & \quad \frac{8}{11} \\
   (B) & \quad \frac{3}{11} \\
   (C) & \quad 1 \\
   (D) & \quad \frac{9}{11} \\
   (E) & \quad \frac{6}{11}
   \end{align*}
   \]

12. In the diagram, \( \angle ABC = 90^\circ \) and \( AB = BC = CD = 10 \). The length of \( AD \) is closest to
   
   \[
   \begin{align*}
   (A) & \quad 14 \\
   (B) & \quad 5 \\
   (C) & \quad 9 \\
   (D) & \quad 10 \\
   (E) & \quad 4
   \end{align*}
   \]

13. If \( x + y = 1 \) and \( x - y = 3 \), what is the value of \( 2x^2 - y^2 \)?
   
   \[
   \begin{align*}
   (A) & \quad 4 \\
   (B) & \quad 8 \\
   (C) & \quad 2 \\
   (D) & \quad 16 \\
   (E) & \quad 32
   \end{align*}
   \]
14. In the diagram, AMN, APQ, QRM, and PRN are all straight lines. The value of \(a + b\) is
   (A) 70       (B) 55       (C) 80
   (D) 90       (E) 75

15. The side lengths of an equilateral triangle and a square are integers. If the triangle and the square have the same perimeter, which of the following is a possible side length of the triangle?
   (A) 1       (B) 10       (C) 18       (D) 20       (E) 25

16. The product of the digits of a four-digit number is 810. If none of the digits is repeated, the sum of the digits is
   (A) 18       (B) 19       (C) 23       (D) 25       (E) 22

17. In the diagram, \(\triangle ABC\) is right-angled at \(C\). If \(BD = 2x\), \(DC = x\), and \(\angle ADC = 2(\angle ABC)\), then the length of \(AB\) is
   (A) \(2\sqrt{2}x\)   (B) \(\sqrt{6}x\)   (C) \(2\sqrt{3}x\)
   (D) \(3x\)       (E) \(4x\)

18. A car uses 8.4 litres of gas for every 100 km it is driven. A mechanic is able to modify the car’s engine at a cost of $400 so that it will only use 6.3 litres of gas per 100 km. The owner determines the minimum distance that she would have to drive to recover the cost of the modifications. If gas costs $0.80 per litre, this distance, in kilometres, is between
   (A) 10 000 and 14 000       (B) 14 000 and 18 000       (C) 18 000 and 22 000
   (D) 22 000 and 26 000       (E) 26 000 and 30 000

19. In an art gallery, a 2 m high painting, \(BT\), is mounted on a wall with its bottom edge 1 m above the floor. A spotlight is mounted at \(S\), 3 m out from the wall and 4 m above the floor. The size of \(\angle TSB\) is closest to
   (A) 27°       (B) 63°       (C) 34°
   (D) 45°       (E) 18°

20. If \(a\), \(b\) and \(c\) are positive, consecutive terms of a geometric sequence (that is, \(\frac{c}{b} = \frac{b}{a}\)), then the graph of \(y = ax^2 + bx + c\) is
   (A) a curve that intersects the \(x\)-axis at two distinct points
   (B) entirely below the \(x\)-axis
   (C) entirely above the \(x\)-axis
   (D) a straight line
   (E) tangent to the \(x\)-axis
21. A sequence of numbers has 6 as its first term, and every term after the first is defined as follows: If a term, \( t \), is even, the next term in the sequence is \( \frac{1}{2} t \). If a term, \( s \), is odd, the next term is \( 3s + 1 \). Thus, the first four terms in the sequence are 6, 3, 10, 5. The 100th term is
(A) 1  (B) 2  (C) 3  (D) 4  (E) 6

22. Pentagon \( ABCDE \) is such that all five diagonals \( AC, BD, CE, DA, \) and \( EB \) lie entirely within the pentagon. If the area of each of the triangles \( ABC, BCD, CDE, DEA, \) and \( EAB \) is equal to 1, the area of the pentagon \( ABCDE \) is closest to
(A) 3.62  (B) 3.64  (C) 3.66  (D) 3.68  (E) 3.70

23. Three faces of a rectangular box meet at a corner of the box. The centres of these faces form the vertices of a triangle having side lengths of 4 cm, 5 cm and 6 cm. The volume of the box, in cm\(^3\), is
(A) \( 45 \sqrt{3} \)  (B) \( 45 \sqrt{6} \)  (C) \( 90 \sqrt{6} \)  (D) 125  (E) \( 120 \sqrt{2} \)

24. When the expression \( \left[ (1 + x)(1 + 2x^3)(1 + 4x^9)(1 + 8x^{27})(1 + 16x^{81})(1 + 32x^{243})(1 + 64x^{729}) \right]^2 \) is expanded and simplified, the coefficient of \( x^{2003} \) is
(A) 0  (B) \( 2^{28} \)  (C) \( 2^{30} \)  (D) \( 2^{29} \)  (E) \( 2^{31} \)

25. The set \( \{1, 4, n\} \) has the property that when any two distinct elements are chosen and 2112 is added to their product, the result is a perfect square. If \( n \) is a positive integer, the number of possible values for \( n \) is
(A) 8  (B) 7  (C) 6  (D) 5  (E) 4
Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

**Copies of Previous Canadian Mathematics Competitions**
Copies of previous contests and solutions are available at no cost in both English and French at http://www.cemc.uwaterloo.ca

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Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is $15. *(Available in English only.)*

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