Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

2003 Solutions
Fermat Contest (Grade 11)

for

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING
Awards

2003 Fermat Contest Solutions

1. Evaluating,
$3^3 - 3^2 + 3^1 - 3^0 = 27 - 9 + 3 - 1 = 20.$

ANSWER: (E)

2. Substituting the given value of a, we obtain

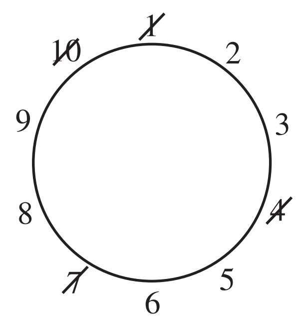
$$\begin{aligned} a^2 + ab &= 60 \\ 25 + 5b &= 60 \\ 5b &= 35 \\ b &= 7 \end{aligned}$$

ANSWER: (A)

3. Since the two angles below the line sum to 180°, then the given line must be a straight line. This tells us that $4x^\circ + x^\circ = 90^\circ$ or $5x^\circ = 90^\circ$ or $x = 18$.

ANSWER: (E)

4. On her first pass around the circle, Sandy crosses out 1, 4, 7, and 10. This leaves her with the numbers 2, 3, 5, 6, 8, and 9 remaining.
Starting from the 10, the third of the remaining numbers is 5, and the third of the remaining numbers after the 5 is 9.
Thus Sandy crosses off 5, 9, 6, and 3, leaving 2 and 8 as the last two remaining numbers.



ANSWER: (B)

5. *Solution 1*
Since the bear has lost 20% of its original mass, then the 220 kg represents 80% of its original mass. Therefore, 20% of its original mass is 55 kg (one quarter of 220 kg), and so its mass before hibernation was $220 + 55 = 275$ kg. (Notice that this just amounts to $\frac{5}{4} \times 220 = 275$.)

Solution 2
Let x be the bear's mass just prior to hibernation.
Since the bear loses 20% of its mass during hibernation, then

$$\begin{aligned} \frac{80}{100}x &= 220 \\ x &= \frac{100}{80}(220) \\ x &= 275 \end{aligned}$$

Therefore, the bear's mass just before hibernation was 275 kg.

ANSWER: (D)

6. *Solution 1*

When $\frac{5}{8}$ of the players are girls then $\frac{3}{8}$ of the players will be boys. Since the number of boys playing is 6 (and does not change), then after the additional girls join, there must be 16 players in total for $\frac{3}{8}$ of the players to be boys. Since there were 8 players initially, then 8 additional girls must have joined the game.

Solution 2

Let the number of additional girls be g.
Then

$$\begin{aligned} \frac{2+g}{8+g} &= \frac{5}{8} \\ 16+8g &= 40+5g \\ 3g &= 24 \\ g &= 8 \end{aligned}$$

ANSWER: (D)

7. Since the height of the fish tank is 30 cm, and it is half full of water, then the depth of the water is 15 cm.
Now the area of the base of the tank is $(20 \text{ cm})(40 \text{ cm}) = 800 \text{ cm}^2$, and so 4000 cm^3 of water will cover this base to a depth of $\frac{4000 \text{ cm}^3}{800 \text{ cm}^2} = 5 \text{ cm}$.
Therefore, the new depth of water is $15 \text{ cm} + 5 \text{ cm} = 20 \text{ cm}$.

ANSWER: (C)

8. From the diagram, we see that the line segment AD is perpendicular to the line segment BC, and so the product of the slopes of these two line segments is -1.
The slope of segment BC is $\frac{7-(-4)}{6-9} = -\frac{11}{3}$, and so the slope of AD is $\frac{3}{11}$.
(We notice that we did not use the coordinates of A!)

ANSWER: (A)

9. *Solution 1*

The average of two numbers is half-way between the two numbers.
So what number is half-way between $\frac{1}{5}$ and $\frac{1}{10}$?
We can write $\frac{1}{5} = \frac{4}{20}$ and $\frac{1}{10} = \frac{2}{20}$, so the number half-way in between is $\frac{3}{20}$ or $\frac{1}{\left(\frac{20}{3}\right)}$

Therefore, $x = \frac{20}{3}$.

Solution 2

The average of two numbers X and Y is $\frac{1}{2}(X+Y)$, so

$$\frac{\frac{1}{5}+\frac{1}{10}}{2}=\frac{1}{x}$$

$$\frac{\frac{3}{10}}{2}=\frac{1}{x}$$

$$x=\frac{2}{\left(\frac{3}{10}\right)}$$

$$x=\frac{20}{3}$$

ANSWER: (A)

10. Since the distance covered by Jim in 4 steps is the same as the distance covered by Carly in 3 steps, then the distance covered by Jim in 24 steps is the same as the distance covered by Carly in 18 steps.
Since each of Carly's steps covers 0.5 m, then she covers 9 m in 18 steps, ie. Jim covers 9 m in 24 steps.

ANSWER: (B)

11. We determine all of the possible routes:
Travelling A to X to B, there are 2 routes, since there are 2 paths A to X.
Travelling A to X to Y to B, there are 6 routes since there are two paths from A to X and 3 paths from Y to B.
Travelling A to Y to B, there are 3 routes, since there are 3 paths Y to B.
So there are 11 routes from A to B, of which 8 pass through X.
Therefore, the probability that Hazel chooses a route that passes through X is $\frac{8}{11}$.

ANSWER: (A)

12. Since ΔABC is right-angled, then we can use Pythagoras' Theorem to say $AC^2=10^2+10^2=200$, or $AC=\sqrt{200}=10\sqrt{2}$. Therefore, $AD=AC-DC=10\sqrt{2}-10\approx 14.1-10=4.1$.
Thus, the length of AD is closest to 4.

ANSWER: (E)

13. *Solution 1*
Since $x+y=1$ and $x-y=3$, then $x^2-y^2=(x-y)(x+y)=3$, and so $2^{x^2-y^2}=2^3=8$.

Solution 2
Since $x+y=1$ and $x-y=3$, then adding these two equations, we obtain $2x=4$ or $x=2$. Substituting this value for x back into the first equation we see that $y=1$.
Therefore, $2^{x^2-y^2}=2^{2^2-1^2}=2^3=8$.

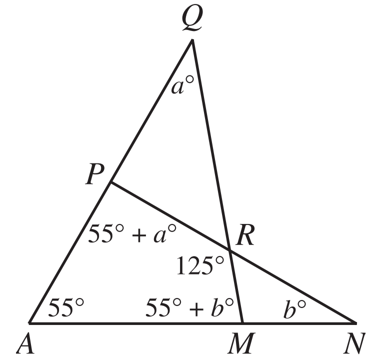
ANSWER: (B)

14. Since $\angle PRM = 125^\circ$, then $\angle QRP = \angle NRM = 55^\circ$.
Then $\angle APR = 180^\circ - \angle QPR = 180^\circ - \left[180^\circ - a^\circ - 55^\circ\right] = 55^\circ + a^\circ$.
Similarly, $\angle QPR = 55^\circ + b^\circ$. (This is an external angle in ΔQPR.)
Since AQ is a straight line,

$$\left(55^\circ + a^\circ\right) + \left(55^\circ + b^\circ\right) = 180^\circ$$

$$a^\circ + b^\circ + 110^\circ = 180^\circ$$

$$a + b = 70$$



ANSWER: (A)

15. Suppose that T is the side length of the equilateral triangle and S is the side length of the square. (Both S and T are integers.) Then, since the perimeters of the triangle and the square are equal, we have $3T = 4S$.
Since $3T = 4S$ and each side of the equation is an integer, then T must be divisible by 4 because 4 must divide into $3T$ evenly and it does not divide into 3.
The only one of the five possibilities which is divisible by 4 is 20.
(We should check that $T = 20$ does indeed yield an integer for S, which it does ($S = 15$).)

ANSWER: (D)

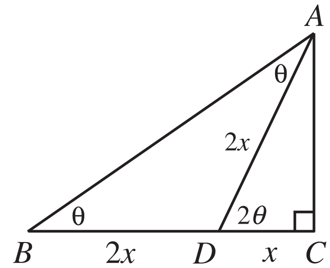
16. Suppose that the four digit number has digits a, b, c, and d, ie. the product $abcd = 810$.
We must determine how to write 810 as the product of 4 *different* digits, none of which can be 0. So we must start by factoring 810, as $810 = 81 \times 10 = 3^4 \times 2 \times 5$.
So one of the digits must have a factor of 5. But the only non-zero digit having a factor of 5 is 5 itself, so 5 is one of the required digits.
Now we need to find 3 different digits whose product is $3^4 \times 2$.
The only digits with a factor of 3 are 3, 6, and 9, and since we need 4 factors of 3, we must use each of these digits (the 9 contributes 2 factors of 3; the others contribute 1 each). In fact, $3 \times 6 \times 9 = 3^4 \times 2 = 162$.
Therefore, the digits of the number are 3, 5, 6, and 9, and so the sum of the digits is 23.

ANSWER: (C)

17. *Solution 1*

Let $\angle ABC = \theta$. Then $\angle ADC = 2\theta$, and so $\angle ADB = 180^\circ - 2\theta$ and $\angle BAD = \theta$.
Thus ΔADB is isosceles with $BD = DA$, and so $DA = 2x$.

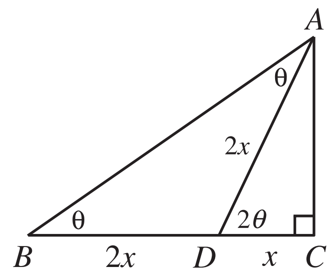
Since AD is twice the length of DC and ΔADC is right-angled, then ΔADC is a 30-60-90 triangle, that is, $\angle ADC = 60^\circ$ and so $\angle ABC = 30^\circ$.
Therefore, ΔABC is also a 30-60-90 triangle, and so $\angle BAC = 60^\circ$ (which is opposite side BC of length $3x$).
Thus, $AB = \frac{2}{\sqrt{3}} BC = \frac{2}{\sqrt{3}}(3x) = 2\sqrt{3}x$.



Solution 2

Let $\angle ABC = \theta$. Then $\angle ADC = 2\theta$, and so $\angle ADB = 180^\circ - 2\theta$ and $\angle BAD = \theta$.
Thus ΔADB is isosceles with $BD = DA$, and so $DA = 2x$.
Since ΔADC and ΔABC are both right-angled, then

$$\begin{aligned} AB^2 &= BC^2 + AC^2 \\ &= BC^2 + \left(AD^2 - DC^2\right) \\ &= (3x)^2 + \left((2x)^2 - x^2\right) \\ &= 9x^2 + 3x^2 \\ &= 12x^2 \end{aligned}$$



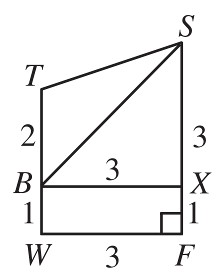
and so $AB = \sqrt{12}x = 2\sqrt{3}x$.

ANSWER: (C)

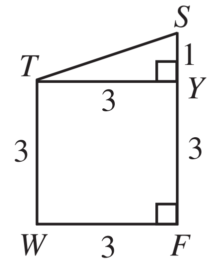
18. The cost to modify the car's engine (\$400) is the equivalent of the cost of $\dfrac{400}{0.80} = 500$ litres of gas. So the car would have to be driven a distance that would save 500 L of gas in order to make up the cost of the modifications.
Originally, the car consumes 8.4 L of gas per 100 km, and after the modifications the car consumes 6.3 L of gas per 100 km, a savings of 2.1 L per 100 km.
Thus, in order to save 500 L of gas, the car would have to be driven $\dfrac{500}{2.1} \times 100 = 23809.52$ km.

ANSWER: (D)

19. Let X be the point on SF so that BX is perpendicular to SF.
Then $BX = 3$, $XF = 1$ and $XS = 3$.
Therefore, ΔBXS is an isosceles right-angled triangle, and so $\angle BSX = 45^\circ$.
Let Y be the point on SF so that TY is perpendicular to SF.
Then $TY = 3$ and $SY = 1$, and ΔSTY is right-angled.



Therefore, $\tan(\angle TSY) = \frac{3}{1} = 3$ and so $\angle TSY \approx 71.6^\circ$.
Thus, $\angle TSB = \angle TSY - \angle BSF \approx 71.6^\circ - 45^\circ = 26.6^\circ$, which is closest to 27°.



ANSWER: (A)

20. Since a, b and c are the consecutive terms of a geometric sequence, then $\frac{c}{b} = \frac{b}{a}$ or $b^2 = ac$.
Therefore, the discriminant of the quadratic equation $ax^2 + bx + c = 0$ is
$\Delta = b^2 - 4ac = ac - 4ac = -3ac < 0$ since a and c are both positive.
Since the discriminant is negative, the parabola does not intersect the x-axis. Since the leading coefficient, a, is positive, then the parabola is entirely above the x-axis.

ANSWER: (C)

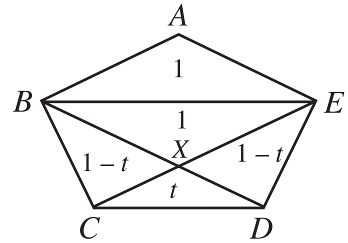
21. In order to get a better feel for this sequence we should write out the first few terms (and hope we spot a pattern!). We'll use the notation t_n for the nth term in the sequence; that is, t_1 is the first term, t_2 is the second term, and so on.
So we have:

$$t_1 = 6,\ t_2 = \tfrac{1}{2}t_1 = 3,\ t_3 = 3t_2 + 1 = 10,\ t_4 = \tfrac{1}{2}t_3 = 5,\ t_5 = 16,\ t_6 = 8,\ t_7 = 4,\ t_8 = 2,$$
$$t_9 = 1,\ t_{10} = 4,\ t_{11} = 2,\ t_{12} = 1,\ t_{13} = 4,\ \ldots$$

Since each term in the sequence depends only on the previous term, then if a term repeats (as it does here), the sequence will then cycle. Here, the terms (starting with the 7th) will cycle 4, 2, 1, 4, 2, 1 etc.
We notice that the cycle has length 3, and $t_9 = 1$, so $1 = t_9 = t_{12} = t_{15} = t_{18} = \cdots$, and so every term whose subscript is a multiple of 3 will be equal to 1.
Thus, $t_{99} = 1$ and so $t_{100} = 4$.

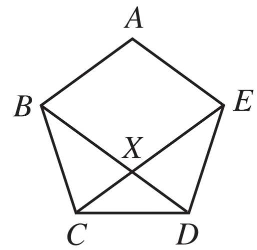
ANSWER: (D)

22. *Solution 1*
We will consider an arbitrary pentagon satisfying the given condition.
First, we will show that each of the diagonals is parallel to the corresponding side.
Since Area ΔBCD = Area ΔCDE and these triangles have a common base, then their heights must be equal, that is, the points B and E must be the same perpendicular distance from the line CD, that is, the line BE is parallel to the line CD. The same can be said for all of the other diagonals.



Now join BD, CE and BE. Since BD is parallel to AE and CE is parallel to AB, then $BAEX$ is a parallelogram, and so the area of ΔBEX is equal to the area of ΔEAB, which is 1.

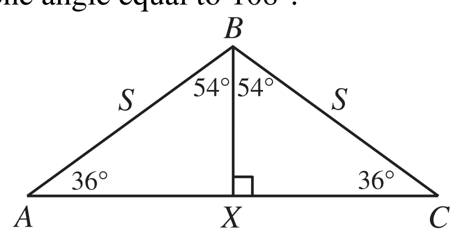
Suppose the area of ΔCXD is t. Then the areas of ΔCXB and ΔEXD are both $1-t$.
Since ΔCXD and ΔCXB have a common height, then the ratio of their bases is the ratio of their areas, ie.
$\frac{DX}{BX}=\frac{t}{1-t}$. Similarly, $\frac{CX}{EX}=\frac{t}{1-t}$.



But since BE is parallel to CD, then ΔCXD is similar to ΔEXB, and so the ratio of their areas is the square of the ratio of their side lengths, ie. $\frac{\text{Area }\Delta EXB}{\text{Area }\Delta CXD}=\frac{1}{t}=\left(\frac{1-t}{t}\right)^2$.
Simplifying, $t^2-3t+1=0$ or $t=\frac{3-\sqrt{5}}{2}$ (since t is smaller than 1).
Therefore, the area of the entire pentagon is

$$1+1+t+(1-t)+(1-t)=4-t=\tfrac{5+\sqrt{5}}{2}\approx 3.62.$$

Solution 2
Since the problem implies that the area of the pentagon is independent of the configuration (assuming the appropriate area conditions hold), let us assume that the pentagon is a regular pentagon, with side length s.
We first determine the value of s. Consider ΔABC. Since a regular pentagon has each interior angle equal to 108°, then ΔABC is isosceles with one angle equal to 108°.
Let X be the midpoint of AC, and join B to X. Then BX is perpendicular to AC since ΔABC is isosceles, and
$\angle ABX=\angle CBX=54^\circ$ (BX bisects $\angle ABC$).
Therefore, $BX=s(\cos 54^\circ)$ and $AX=CX=s(\sin 54^\circ)$.



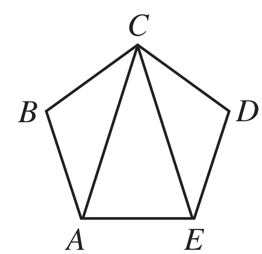
Therefore, the area of ΔABC is

$$1=\tfrac{1}{2}\left[s(\cos 54^\circ)\right]\left[2s(\sin 54^\circ)\right]=s^2\sin 54^\circ\cos 54^\circ$$

which enables us to determine that $s^2=\frac{1}{\sin 54^\circ\cos 54^\circ}$.

$$\begin{aligned}\text{Area of pentagon}&=\text{Area }\Delta ABC+\text{Area }\Delta ACE+\text{Area }\Delta CDE\\&=2+\text{Area }\Delta ACE\end{aligned}$$

So we lastly consider ΔACE.

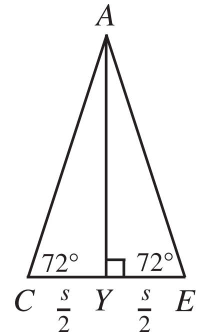


Joining A to Y, the midpoint of CE, we see that AY is perpendicular to CE, $CY = YE = \frac{1}{2}s$ and $\angle ACE = \angle AEC = 72^\circ$.

Therefore, $AY = \frac{1}{2}s(\tan 72^\circ)$, and thus

$$\text{Area } \Delta ACE = \frac{1}{2}s\left[\frac{1}{2}s(\tan 72^\circ)\right] = \frac{1}{4}s^2(\tan 72^\circ) = \frac{\tan 72^\circ}{4\sin 54^\circ \cos 54^\circ} \approx$$

and so the area of the pentagon is closest to $2 + 1.618$ or 3.62.



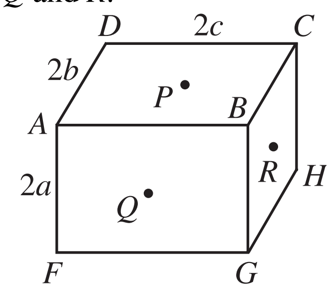
ANSWER: (A)

23. Let the edge lengths of the box be $2a$, $2b$ and $2c$. We label the vertices of the box $ABCDEFGH$.

Let the centres of the three faces meeting at a corner be P, Q and R.

Then we can say that $PQ = 4$, $QR = 5$, and $PR = 6$.

We would like to try to express these distances in terms of the edge lengths of the box.



We will start by letting M be the midpoint of edge AB, and joining P to M, Q to M and P to Q.

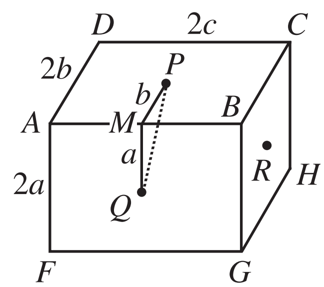
Since P is the centre of face $ABCD$ and PM is perpendicular to AB, then $PM = \frac{1}{2}AD = b$. Similarly, $MQ = a$.

Since MP and MQ are both perpendicular to AB and they lie in perpendicular planes $ABCD$ and $ABGF$, then MP is indeed perpendicular to MQ, and so ΔQMP is right-angled.

By Pythagoras, $PQ^2 = MP^2 + MQ^2$ or $16 = a^2 + b^2$.

Similarly, looking at PR, $36 = a^2 + c^2$, and, looking at QR, $25 = b^2 + c^2$.

So we now have $16 = a^2 + b^2$, $36 = a^2 + c^2$, and $25 = b^2 + c^2$, and we want to determine the volume of the box, which is $V = (2a)(2b)(2c) = 8abc$.



Adding the three equations, we obtain $77 = 2(a^2 + b^2 + c^2)$, or $a^2 + b^2 + c^2 = \frac{77}{2}$.

Subtracting the above equations one by one from this last equation yields

$$c^2 = \frac{45}{2}$$

$$b^2 = \frac{5}{2}$$

$$a^2 = \frac{27}{2}$$

Therefore, $a^2b^2c^2 = \frac{(27)(5)(45)}{8} = \frac{6075}{8}$ and so $abc = \sqrt{\frac{6075}{8}} = \frac{45\sqrt{3}}{2\sqrt{2}}$.

Thus, the volume is $V = 8\left(\frac{45\sqrt{3}}{2\sqrt{2}}\right) = \left(2\sqrt{2}\right)\left(45\sqrt{3}\right) = 90\sqrt{6}\text{ cm}^3$.

ANSWER: (C)

24. We first of all rewrite the given expression as

$$\left[(1+x)\left(1+2x^3\right)\left(1+4x^9\right)\left(1+8x^{27}\right)\left(1+16x^{81}\right)\left(1+32x^{243}\right)\left(1+64x^{729}\right)\right]^2$$

$$=\left(1+2^0x^{3^0}\right)^2\left(1+2^1x^{3^1}\right)^2\left(1+2^2x^{3^2}\right)^2\left(1+2^3x^{3^3}\right)^2\left(1+2^4x^{3^4}\right)^2\left(1+2^5x^{3^5}\right)^2\left(1+2^6x^{3^6}\right)^2$$

We note that $\left(1+2^r x^{3^r}\right)^2 = 1+2^r x^{3^r} + 2^r x^{3^r} + 2^{2r} x^{2\cdot 3^r}$, so each of the four terms in this expansion has the form $2^{mr} x^{m3^r}$ where m is 0, 1 or 2.

When we multiply the expansions of the seven factors, each of the four terms in each expansion will be multiplied by each of the four terms in each other expansion.

So when this complicated-looking expression is multiplied out, every term is of the form

$$2^{0a+1b+2c+3d+4e+5f+6g} x^{a3^0+b3^1+c3^2+d3^3+e3^4+f3^5+g3^6}$$

where the values of a, b, c, d, e, f, g can each be 0, 1, 2, telling us to take the first term, one of the middle terms, or the last term, respectively.

Since we are looking for the coefficient of x^{2003}, we need to look at the equation

$a3^0+b3^1+c3^2+d3^3+e3^4+f3^5+g3^6=2003$

where each of the coefficients can be 0, 1 or 2.

Let's look first at the value of g. Could g be 0 or 1? If this is the case then the largest possible value of the left side $2\cdot 3^0+2\cdot 3^1+2\cdot 3^2+2\cdot 3^3+2\cdot 3^4+2\cdot 3^5+3^6=1457$.

But this is too small, so $g=2$.

Substituting and simplifying, we get $a3^0+b3^1+c3^2+d3^3+e3^4+f3^5=545$.

Next, we look at the value of f. If f was 0 or 1, then

$a3^0+b3^1+c3^2+d3^3+e3^4+f3^5 \le 2\cdot 3^0+2\cdot 3^1+2\cdot 3^2+2\cdot 3^3+2\cdot 3^4+3^5=485$

So $f=2$.

In a similar fashion, we can determine that $e=0$, $d=2$, $c=0$, $b=1$, and $a=2$.

These values are uniquely determined.

[Alternatively, we could notice that the equation

$$a3^0+b3^1+c3^2+d3^3+e3^4+f3^5+g3^6=2003$$

is asking for the base 3 expansion of 2003, which we could calculate as 2202012, thus obtaining the same result.]

So each term containing x^{2003} is of the form

$$2^{0(2)+1(1)+2(0)+3(2)+4(0)+5(2)+6(2)}x^{2003}=2^{29}x^{2003}.$$

To determine the coefficient of x^{2003}, we need to determine the number of terms of the form $2^{29}x^{2003}$ that occur before we simplify the expansion.

We note that each term of the form $2^{29}x^{2003}$ has a contribution of one term from each of the seven factors of the form $\left(1+2^r x^{3^r} + 2^r x^{3^r} + 2^{2r} x^{2\cdot 3^r}\right)$.

Since $a=2$, the expansion $1+x+x+x^2$ of $(1+x)^2$ contributes the term x^2 to the product. Similarly, since $d=2$, $f=2$, and $g=2$, the expansions of $\left(1+2^3x^{3^3}\right)^2$, $\left(1+2^5x^{3^5}\right)^2$ and $\left(1+2^6x^{3^6}\right)^2$ each contribute their last term.

Since $c=0$ and $e=0$, the expansions of $\left(1+2^2x^{3^2}\right)^2$ and $\left(1+2^4x^{3^4}\right)^2$ each contribute their first term, namely 1.

Since $b=1$, the expansion of $\left(1+2^1x^{3^1}\right)^2$ contributes the two middle terms $2^1x^{3^1}$ and $2^1x^{3^1}$. There are thus two terms of the form $2^{29}x^{2003}$ in the product, and so their sum is $2\cdot 2^{29}x^{2003}=2^{30}x^{2003}$.

The coefficient of x^{2003} is 2^{30}.

ANSWER: (C)

25. According to the requirements, the three quantities $4+2112=2116$, $n+2112$, and $4n+2112$ must be perfect squares.

Now $2116=46^2$ (a perfect square), so the conditions say that we must have $n+2112=x^2$ and $4n+2112=y^2$ for some positive integers x and y. Since n is a positive integer, then each of x and y must be greater than 46.

We now have 3 variables, but can easily eliminate the n by manipulating the two equations to obtain $4n+4(2112)=4x^2$ and $4n+2112=y^2$, and thus $4x^2-y^2=3(2112)$.

We can factor both sides of this equation to get $(2x+y)(2x-y)=3(11)(192)=2^6 3^2 11$.

So we must now determine the possibilities for x and y, and hence the possibilities for n.

We could do this by direct trial and error, but we will try to reduce the work we have to do somewhat by analyzing the situation.

Suppose $2x+y=A$ and $2x-y=B$, with $AB=2^6 3^2 11$. Then solving for x and y, we obtain $x=\dfrac{A+B}{4}$ and $y=\dfrac{A-B}{2}$.

Since AB is even, then at least one of A and B is even.

Since $y=\dfrac{A-B}{2}$ is an integer and one of A and B is even, then both of A and B are even.

Since A and B contain in total 6 powers of 2, then at least one of them contains 3 powers of 2, ie. is divisible by 8 (and thus 4).

Since $x=\dfrac{A+B}{4}$ is an integer and one of A and B is divisible by 4, then both A and B are divisible by 4.

So let $A=4a$ and $B=4b$, and so $x=a+b$ and $y=2a-2b$ and $ab=2^2 3^2 11$.

[We may assume that y is positive, because if y is negative this reverses the role of a and b.]

Since y is positive, then $a>b$. How many possibilities for a and b are there given that $ab=2^2 3^2 11$? The integer $2^2 3^2 11$ has $(2+1)(2+1)(1+1)=18$ factors, so there are 9 possibilities for the pair (a,b), namely

$$(396,1),\ (198,2),\ (132,3),\ (99,4),\ (66,6),\ (44,9),\ (36,11),\ (33,12),\ (22,18)$$

Of these, only the first 7 give values for x and y bigger than 46 (ie. positive values for n). Therefore, there are 7 possible values for n.

[For completeness, it is worth determining these values:

$$n = x^2 - 2112 = (a+b)^2 - 2112$$

so the corresponding values of n are 155497, 37888, 16113, 8497, 3072, 697, and 97.]

ANSWER: (B)