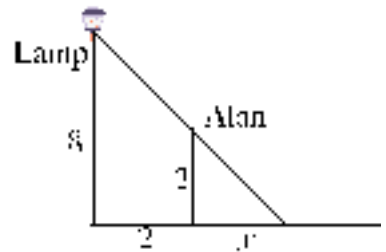


2004 Hypatia Contest (Grade 11)
Thursday, April 15, 2004

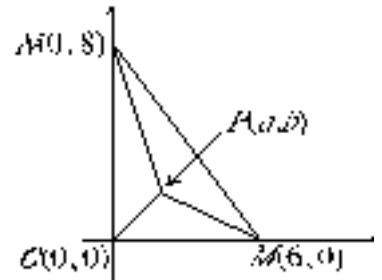
1. (a) Find all values of x which are roots of the equation $x^2 + 5x + 6 = 0$.
- (b) The roots of $x^2 + 5x + 6 = 0$ are each increased by 7. Find a quadratic equation that has these new numbers as roots.
- (c) The roots of $(x - 4)(3x^2 - x - 2) = 0$ are each increased by 1. Find an equation that has these new numbers as roots.
2. Two basketball players, Alan and Bobbie, are standing on level ground near a lamp-post which is 8 m tall. Each of the two players casts a shadow on the ground.

- (a) In the diagram, Alan is standing 2 m from the lamp-post. If Alan is 2 m tall, determine the value of x , the length of his shadow.

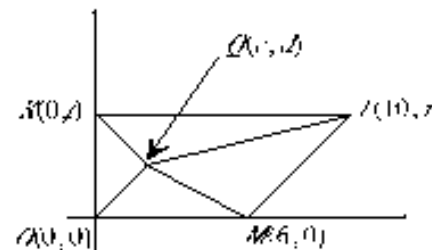


- (b) Bobbie is 1.5 m tall and is standing on the opposite side of the lamp-post from Alan. How far from the lamp-post should she stand so that she casts a shadow of length 3 m?

3. (a) In the diagram, triangle OMN has vertices $O(0,0)$, $M(6,0)$ and $N(0,8)$. Determine the coordinates of point $P(a,b)$ inside the triangle so that the areas of the triangles POM , PON and PMN are all equal.



- (b) In the diagram, quadrilateral $OMLK$ has vertices $O(0,0)$, $M(6,0)$, $L(10,t)$, and $K(0,t)$, where $t > 0$. Show that there is no point $Q(c,d)$ inside the quadrilateral so that the areas of the triangles QOM , QML , QLK , and QKO are all equal.



4. (a) 1 green, 1 yellow and 2 red balls are placed in a bag. Two balls of *different* colours are selected at random. These two balls are then removed and replaced with one ball of the *third* colour. (Enough extra balls of each colour are kept to the side for this purpose.) This process continues until there is only one ball left in the bag, or all of the balls are the same colour. What is the colour of the ball or balls that remain at the end?
- (b) 3 green, 4 yellow and 5 red balls are placed in a bag. If a procedure identical to that in part (a) is carried out, what is the colour of the ball or balls that remain at the end?
- (c) 3 green, 4 yellow and 5 red balls are placed in a bag. This time, two balls of different colours are selected at random, removed, and replaced with *two* balls of the third colour. Show that it is impossible for all of the remaining balls to be the same colour, no matter how many times this process is repeated.