Fermat Contest (Grade 11)
Wednesday, February 23, 2005

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Time: 60 minutes
Calculators are permitted.

Instructions
1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is no penalty for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have sixty minutes of working time.
Scoring: There is no penalty for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of \( \frac{150 + (150 \div 10)}{15 - 5} \) is
   (A) 6       (B) 3       (C) 146      (D) 151.5      (E) 16.5

2. \( \frac{1}{2} - \frac{1}{3} + \frac{3}{9} \) equals
   (A) \( \frac{1}{4} \)       (B) \( \frac{1}{2} \)       (C) \( \frac{5}{15} \)      (D) \( \frac{1}{5} \)      (E) 0

3. If \( a = \frac{1}{2} \) and \( b = \frac{2}{3} \), then \( \frac{6a + 18b}{12a + 6b} \) equals
   (A) 9       (B) 7       (C) 10      (D) 6      (E) \( \frac{3}{2} \)

4. If \( \sqrt{4 + 9 + x^2} = 7 \), then a possible value for \( x \) is
   (A) 6       (B) 2       (C) 4       (D) 36      (E) 0

5. A Fermat coin rolls from \( P \) to \( Q \) to \( R \), as shown. If the distance from \( P \) to \( Q \) is equal to the distance from \( Q \) to \( R \), what is the orientation of the coin when it reaches \( R \)?
   (A) \( \bigcirc \)       (B) \( \bigcirc \)       (C) \( \bigcirc \)
   (D) \( \bigcirc \)       (E) \( \bigcirc \)

6. The sum of the first 2005 terms of the sequence 1, 2, 3, 4, 1, 2, 3, 4, \ldots \) is
   (A) 5011     (B) 5110     (C) 5020     (D) 5010     (E) 501

7. In triangle \( ABC \), \( \angle A \) is 21° more than \( \angle B \), and \( \angle C \) is 36° more than \( \angle B \). The size of \( \angle B \) is
   (A) 20°     (B) 41°     (C) 62°     (D) 46°     (E) 56°

8. Seven children, each with the same birthday, were born in seven consecutive years. The sum of the ages of the youngest three children is 42. What is the sum of the ages of the oldest three?
   (A) 51     (B) 54     (C) 57     (D) 60     (E) 63
9. The lines \( y = -2x + 8 \) and \( y = \frac{1}{2}x - 2 \) meet at \((4, 0)\), as shown. The area of the triangle formed by these two lines and the line \( x = -2 \) is
(A) 15  (B) 27  (C) 30
(D) 36  (E) 45

10. If 50% of \( P \) equals 20% of \( Q \), then \( P \), as a percent of \( Q \), is
(A) 60%  (B) 250%  (C) 40%  (D) 20%  (E) 30%

Part B: Each correct answer is worth 6.

11. Rectangle \( ABCD \) is made up of six squares. The areas of two of the squares are shown. The perimeter of rectangle \( ABCD \), in centimetres, is
(A) 50  (B) 44  (C) 46
(D) 52  (E) 48

12. Starting with the 2 in the centre, the number 2005 can be formed by moving from circle to circle only if the two circles are touching. How many different paths can be followed to form 2005?
(A) 36  (B) 24  (C) 12
(D) 18  (E) 6

13. A circle is drawn so that no part of it lies outside a regular hexagon. If such a circle does not touch all six sides of the hexagon, what is the maximum number of sides that it could touch?
(A) 1  (B) 2  (C) 3  (D) 4  (E) 5

14. The weight of a lioness is six times the weight of her female cub and four times the weight of her male cub. If the difference between the weights of the male and female cub is 14 kg, the weight of the lioness, in kg, is
(A) 84  (B) 252  (C) 168  (D) 140  (E) 112

15. If \((x - 4)(5x + 2) = 0\), then the two possible values of \(5x + 2\) are
(A) -4 and \(\frac{2}{5}\)  (B) 0 and -18  (C) 0 and 22  (D) 0 and 4  (E) 4 and 22
16. In the diagram, circles $C_1$ and $C_2$ each have center $O$.
The area of the shaded region is
(A) $2\pi$  (B) $3\pi$  (C) $4\pi$
(D) $6\pi$  (E) $8\pi$

17. A cylinder with radius 2 cm and height 8 cm is full of water. A second cylinder of radius 4 cm and height 8 cm is empty. If all of the water is poured from the first cylinder into the second cylinder, the depth of the water in the second cylinder will be
(A) 1 cm  (B) 2 cm  (C) 3 cm  (D) 4 cm  (E) 6 cm

18. A test has ten questions. Points are awarded as follows:
- Each correct answer is worth 3 points.
- Each unanswered question is worth 1 point.
- Each incorrect answer is worth 0 points.
A total score that is not possible is
(A) 11  (B) 13  (C) 17  (D) 23  (E) 29

19. Sam bicycles at 16 km/h and Chris bicycles at 24 km/h. At noon, Sam is 1 km north of Chris, and each begins to ride north. How many minutes will it take for Chris to catch Sam?
(A) $1 \frac{1}{2}$  (B) $2 \frac{1}{2}$  (C) $3 \frac{3}{4}$  (D) $7 \frac{1}{2}$  (E) 8

20. In triangle $ABC$, if $AB = AC = x + 1$ and $BC = 2x - 2$, where $x > 1$, then the area of the triangle is always equal to
(A) $(x - 1)\sqrt{2x^2 + 2}$
(B) $2(x - 1)$
(C) $\frac{1}{2}(x + 1)^2$
(D) $(x + 1)(x - 1)$
(E) $2(x - 1)\sqrt{x}$

Part C: Each correct answer is worth 8.

21. Four different numbers $a$, $b$, $c$, and $d$ are chosen from the list $-1, -2, -3, -4$, and $-5$. The largest possible value for the expression $a^b + c^d$ is
(A) $\frac{5}{4}$  (B) $\frac{7}{5}$  (C) $\frac{31}{32}$  (D) $\frac{10}{9}$  (E) $\frac{26}{25}$

22. In the diagram, a semi-circle has diameter $XY$. Rectangle $PQRS$ is inscribed in the semi-circle with $PQ = 12$ and $QR = 28$. Square $STUV$ has $T$ on $RS$, $U$ on the semi-circle and $V$ on $XY$. The area of $STUV$ is closest to
(A) 12  (B) 13  (C) 16
(D) 14  (E) 15
23. A solid cube of side length 4 cm is cut into two pieces by a plane that passed through the midpoints of six edges, as shown. To the nearest square centimetre, the surface area of each half cube created is

(A) 69  (B) 48  (C) 32

(D) 65  (E) 58

24. The arithmetic sequence \(a, a+d, a+2d, a+3d, \ldots, a+(n-1)d\) has the following properties:

- When the first, third, and fifth, and so on terms are added, up to and including the last term, the sum is 320.
- When the first, fourth, seventh, and so on, terms are added, up to and including the last term, the sum is 224.

What is the sum of the whole sequence?

(A) 656  (B) 640  (C) 608  (D) 704  (E) 672

25. A triline is a line with the property that three times its slope is equal to the sum of its \(x\)-intercept and its \(y\)-intercept. For how many integers \(q\) with \(1 \leq q \leq 10\,000\) is there at least one positive integer \(p\) so that there is exactly one triline through \((p, q)\)?

(A) 60  (B) 57  (C) 58  (D) 61  (E) 59

N.B. This problem has been corrected from its original version with the addition of the underlined word “positive”.
Canadian Mathematics Competition

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