Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Grade 8 solutions follow the Grade 7 solutions

2005 Gauss Contests

(Grades 7 and 8)

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Solutions

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Grade 7

Grade 8 solutions follow the Grade 7 solutions

1. Calculating the numerator first, $\dfrac{3 \times 4}{6} = \dfrac{12}{6} = 2$.

   Answer: (B)

2. Calculating, $0.8 - 0.07 = 0.80 - 0.07 = 0.73$.

   Answer: (E)

3. Since the arrow is pointing between 9.6 and 9.8, it is pointing to a rating closest to 9.7.

   Answer: (C)

4. Twelve million is written as 12 000 000 and twelve thousand is written as 12 000, so the sum of these two numbers is 12 012 000.

   Answer: (A)

5. To figure out which number is largest, we look first at the number in the tenths position.
   Since four of the given numbers have a 1 in the tenths position and 0.2 has a 2, then 0.2 is the largest.

   Answer: (B)

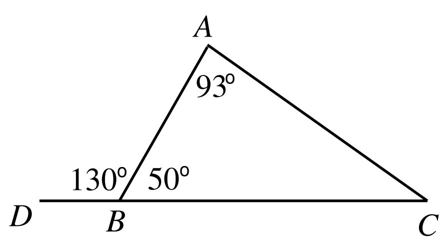
6. Since Meghan chooses a prize from 27 in the bag and the probability of her choosing a book is $\frac{2}{3}$, then $\frac{2}{3}$ of the prizes in the bag must be books.
   Therefore, the number of books in the bag is $\frac{2}{3} \times 27 = 18$.

   Answer: (E)

7. Since 83% in decimal form is 0.83, then the number of people who voted for Karen is equal to $0.83 \times 1\,480\,000 = 1\,228\,400$.

   Answer: (B)

8. Since $\angle ABC + \angle ABD = 180^\circ$ (in other words, $\angle ABC$ and $\angle ABD$ are supplementary) and $\angle ABD = 130^\circ$, then $\angle ABC = 50^\circ$.

   Since the sum of the angles in triangle ABC is 180° and we know two angles 93° and 50° which add to 143°, then $\angle ACB = 180^\circ - 143^\circ = 37^\circ$.

   Answer: (B)

9. There are six odd-numbered rows (rows 1, 3, 5, 7, 9, 11).
   These rows have $6 \times 15 = 90$ seats in total.
   There are five even-numbered rows (rows 2, 4, 6, 8, 10).
   These rows have $5 \times 16 = 80$ seats in total.
   Therefore, there are $90 + 80 = 170$ seats in total in the theatre.

   Answer: (D)

10. When it is 5:36 p.m. in St. John's, it is 90 minutes or $1\frac{1}{2}$ hours earlier in Smiths Falls, so it is 4:06 p.m. in Smiths Falls.
When it is 4:06 p.m. in Smiths Falls, it is 3 hours earlier in Whitehorse, so it is 1:06 p.m. in Whitehorse.

Answer: (A)

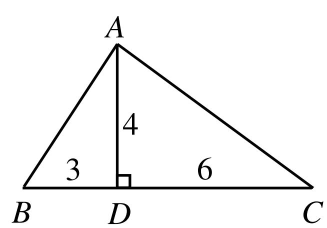
11. On each day, the temperature range is the difference between the daily high and daily low temperatures.
On Monday, the range is $6 - (-4) = 10$ degrees Celsius.
On Tuesday, the range is $3 - (-6) = 9$ degrees Celsius.
On Wednesday, the range is $4 - (-2) = 6$ degrees Celsius.
On Thursday, the range is $4 - (-5) = 9$ degrees Celsius.
On Friday, the range is $8 - 0 = 8$ degrees Celsius.
The day with the greatest range is Monday.

Answer: (A)

12. Since the bamboo plant grows at a rate of 105 cm per day and there are 7 days from May 1st and May 8th, then it grows $7 \times 105 = 735$ cm in this time period.
Since 735 cm $= 7.35$ m, then the height of the plant on May 8th is $2 + 7.35 = 9.35$ m.

Answer: (E)

13. Since $BD = 3$ and DC is twice the length of BD, then $DC = 6$.



Therefore, triangle ABC has a base of length 9 and a height of length 4.
Therefore, the area of triangle ABC is $\frac{1}{2}bh = \frac{1}{2}(9)(4) = \frac{1}{2}(36) = 18$.

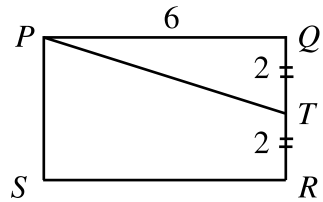
Answer: (D)

14. *Solution 1*
Since the sum of the numbers on opposite faces on a die is 7, then 1 and 6 are on opposite faces, 2 and 5 are on opposite faces, and 3 and 4 are on opposite faces.
On the first die, the numbers on the unseen faces opposite the 6, 2 and 3 are 1, 5 and 4, respectively.
On the second die, the numbers on the unseen faces opposite the 1, 4 and 5 are 6, 2 and 3, respectively.
The sum of the missing numbers is $1 + 5 + 4 + 6 + 2 + 3 = 21$.

Solution 2
The sum of the numbers on a die is $1 + 2 + 3 + 4 + 5 + 6 = 21$ and so the sum of the numbers on two die is $2 \times 21 = 42$.
Since there is a sum of 21 showing on the six visible faces, the sum of the numbers on the six unseen faces is $42 - 21 = 21$.

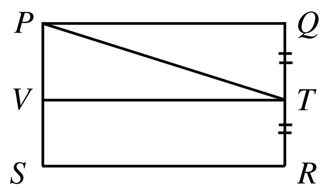
Answer: (C)

15. *Solution 1*
Since the area of rectangle $PQRS$ is 24, let us assume that $PQ = 6$ and $QR = 4$.
Since $QT = QR$, then $QR = 2QT$ so $QT = 2$.



Therefore, triangle PQT has base PQ of length 6 and height QT of length 2, so has area $\frac{1}{2}(6)(2) = \frac{1}{2}(12) = 6$.
So the area of quadrilateral $PTRS$ is equal to the area of rectangle $PQRS$ (which is 24) minus the area of triangle PQT (which is 6), or 18.

Solution 2
Draw a line through T parallel to PQ across the rectangle parallel so that it cuts PS at point V.



Since T is halfway between Q and R, then V is halfway between P and S.
Therefore, $SVTR$ is a rectangle which has area equal to half the area of rectangle $PQRS$, or 12.
Similarly, $VPQT$ is a rectangle of area 12, and $VPQT$ is cut in half by PT, so triangle PVT has area 6.
Therefore, the area of $PTRS$ is equal to the sum of the area of rectangle $SVTR$ and the area of triangle PVT, or $12 + 6 = 18$.

ANSWER: (A)

16. Nicholas sleeps for an hour and a half, or 90 minutes.
Since three sleep cross the road per minute, then $3 \times 90 = 270$ sheep cross while he is asleep.
Since 42 sheep crossed before he fell asleep, then $42 + 270 = 312$ sleep have crossed the road in total when he wakes up.
Since this is half of the total number of sheep in the flock, then the total number in the flock is $2 \times 312 = 624$.

ANSWER: (D)

17. *Solution 1*
When we calculate the value of the symbol, we add the product of the numbers on each of the two diagonals.
The product of the entries on the diagonal with the 1 and the 6 is 6.
Since the symbol is evaluated as 16, then the product of the entries on the other diagonal is 10.
Since one of the entries on the other diagonal is 2, then the missing entry must be 5.

Solution 2
Let the missing number be x.
Using the definition for the evaluation of the symbol, we know that $2 \times x + 1 \times 6 = 16$ or $2x + 6 = 16$ or $2x = 10$ or $x = 5$.

Answer: (E)

18. When a die is rolled, there are six equally likely possibilities (1 through 6).
In order for the game to be fair, half of the six possibilities, or three possibilities, must be winning possibilities.
In the first game, only rolling a 2 gives a win, so this game is not fair.
In the second game, rolling a 2, 4 or 6 gives a win, so this game is fair.
In the third game, rolling a 1, 2 or 3 gives a win, so this game is fair.
In the fourth game, rolling a 3 or 6 gives a win, so this game is not fair.
Therefore, only two of the four games are fair.

Answer: (C)

19. At each distance, two throws are made: the 1st and 2nd throws are made at 1 m, the 3rd and 4th are made at 2 m, and so on, with the 27th and 28th throws being made at 14 m.
Therefore, the 29th throw is the first throw made at 15 m.
At each distance, the first throw is made by Pat to Chris, so Chris misses catching the 29th throw at a distance of 15 m.

Answer: (A)

20. Since Sally's car travels 80 km/h, it travels 80 000 m in one hour.
Since there are 60 minutes in an hour, the car travels $\frac{1}{60} \times 80\,000$ m in one minute.
Since there are 60 seconds in a minute, the car travels $\frac{1}{60} \times \frac{1}{60} \times 80\,000$ in one second.
Therefore, in 4 seconds, the car travels $4 \times \frac{1}{60} \times \frac{1}{60} \times 80\,000 \approx 88.89$ m.
Of the possible choices, this is closest to 90 m.

Answer: (E)

21. Since the price of the carpet is reduced by 10% every 15 minutes, then the price is multiplied by 0.9 every 15 minutes.
At 9:15, the price was \$9.00.
At 9:30, the price fell to $0.9 \times \$9.00 = \$8.10$.
At 9:45, the price fell to $0.9 \times \$8.10 = \$7.29$.
So the price fell below \$8.00 at 9:45 a.m., so Emily bought the carpet at 9:45 a.m.

Answer: (A)

22. *Solution 1*
We start by assuming that there are 20 oranges. (We pick 20 since the ratio of apples to oranges is $1 : 4$ and the ratio of oranges to lemons is $5 : 2$, so we pick a number of oranges which is divisible by 4 and by 5. Note that we did not have to assume that there were 20 oranges, but making this assumption makes the calculations much easier.)
Since there are 20 oranges and the ratio of the number of apples to the number of oranges is $1 : 4$, then there are $\frac{1}{4} \times 20 = 5$ apples.
Since there are 20 oranges and the ratio of the number of oranges to the number of lemons is $5 : 2$, then there are $\frac{2}{5} \times 20 = 8$ lemons.
Therefore, the ratio of the number of apples to the number of lemons is $5 : 8$.

Solution 2
Let the number of apples be x.
Since the ratio of the number of apples to the number of oranges is $1 : 4$, then the number of oranges is $4x$.
Since the ratio of the number of oranges to the number of lemons is $5 : 2$, then the number of lemons is $\frac{2}{5} \times 4x = \frac{8}{5}x$.
Since the number of apples is x and the number of lemons is $\frac{8}{5}x$, then the ratio of the number of apples to the number of lemons is $1 : \frac{8}{5} = 5 : 8$.

ANSWER: (C)

23. *Solution 1*
If 4 □ balance 2 ○, then 1 □ would balance the equivalent of $\frac{1}{2}$ ○.
Similarly, 1 △ would balance the equivalent of $1\frac{1}{2}$ ○.
If we take each of the answers and convert them to an equivalent number of ○, we would have:
(**A**): $1\frac{1}{2} + 1 + \frac{1}{2} = 3$ ○
(**B**): $3\left(\frac{1}{2}\right) + 1\frac{1}{2} = 3$ ○
(**C**): $2\left(\frac{1}{2}\right) + 2 = 3$ ○
(**D**): $2\left(1\frac{1}{2}\right) + \frac{1}{2} = 3\frac{1}{2}$ ○
(**E**): $1 + 4\left(\frac{1}{2}\right) = 3$ ○
Therefore, 2 △ and 1 □ do not balance the required.

Solution 2
Since 4 □ balance 2 ○, then 1 ○ would balance 2 □.
Therefore, 3 ○ would balance 6 □, so since 3 ○ balance 2 △, then 6 □ would balance 2 △, or 1 △ would balance 3 □.
We can now express every combination in terms of □ only.
1 △, 1 ○ and 1 □ equals $3 + 2 + 1 = 6$ □.
3 □ and 1 △ equals $3 + 3 = 6$ □.
2 □ and 2 ○ equals $2 + 2 \times 2 = 6$ □.
2 △ and 1 □ equals $2 \times 3 + 1 = 7$ □.
1 ○ and 4 □ equals $2 + 4 = 6$ □.
Therefore, since 1 △, 1 ○ and 1 □ equals 6 □, then it is 2 △ and 1 □ which will not balance with this combination.

Solution 3
We try assigning weights to the different shapes.
Since 3 ○ balance 2 △, assume that each ○ weighs 2 kg and each △ weighs 3 kg.
Therefore, since 4 □ balance 2 ○, which weigh 4 kg combined, then each □ weighs 1 kg.
We then look at each of the remaining combinations.
1 △, 1 ○ and 1 □ weigh $3 + 2 + 1 = 6$ kg.
3 □ and 1 △ weigh $3 + 3 = 6$ kg.
2 □ and 2 ○ weigh $2 + 2 \times 2 = 6$ kg.
2 △ and 1 □ weigh $2 \times 3 + 1 = 7$ kg.
1 ○ and 4 □ weigh $2 + 4 = 6$ kg.
Therefore, it is the combination of 2 △ and 1 □ which will not balance the other combinations.

ANSWER: (D)

24. Since Alphonse and Beryl always pass each other at the same three places on the track and since they each run at a constant speed, then the three places where they pass must be equally spaced on the track. In other words, the three places divide the track into three equal parts.
We are not told which runner is faster, so we can assume that Beryl is the faster runner.
Start at one place where Alphonse and Beryl meet. (Now that we know the relative positions of where they meet, we do not actually have to know where they started at the very beginning.)
To get to their next meeting place, Beryl runs farther than Alphonse (since she runs faster than he does), so Beryl must run $\frac{2}{3}$ of the track while Alphonse runs $\frac{1}{3}$ of the track in the opposite direction, since the meeting placed are spaced equally at $\frac{1}{3}$ intervals of the track.
Since Beryl runs twice as far in the same length of time, then the ratio of their speeds is $2:1$.
Answer: (D)

25. *Solution 1*
We want to combine 48 coins to get 100 cents.
Since the combined value of the coins is a multiple of 5, as is the value of a combination of nickels, dimes and quarters, then the value of the pennies must also be a multiple of 5.

Therefore, the possible numbers of pennies are 5, 10, 15, 20, 25, 30, 35, 40.
We can also see that because there are 48 coins in total, it is not possible to have anything other than 35, 40 or 45 pennies. (For example, if we had 30 pennies, we would have 18 other coins which are worth at least 5 cents each, so we would have at least $30 + 5 \times 18 = 120$ cents in total, which is not possible. We can make a similar argument for 5, 10, 15, 20 and 25 pennies.)

It is also not possible to have 3 or 4 quarters. If we did have 3 or 4 quarters, then the remaining 45 or 44 coins would give us a total value of at least 44 cents, so the total value would be greater than 100 cents. Therefore, we only need to consider 0, 1 or 2 quarters.

<u>Possibility 1:</u> 2 quarters
If we have 2 quarters, this means we have 46 coins with a value of 50 cents.
The only possibility for these coins is 45 pennies and 1 nickel.

<u>Possibility 2:</u> 1 quarter
If we have 1 quarter, this means we have 47 coins with a value of 75 cents.
The only possibility for these coins is 40 pennies and 7 nickels.

<u>Possibility 3:</u> 0 quarters
If we have 0 quarters, this means we have 48 coins with a value of 100 cents.
If we had 35 pennies, we would have to have 13 nickels.
If we had 40 pennies, we would have to have 4 dimes and 4 nickels.
It is not possible to have 45 pennies.

Therefore, there are 4 possible combinations.

Solution 2
We want to use 48 coins to total 100 cents.
Let us focus on the number of pennies.
Since any combination of nickels, dimes and quarters always is worth a number of cents which is divisible by 5, then the number of pennies in each combination must be divisible by 5, since the total value of each combination is 100 cents, which is divisible by 5.

Could there be 5 pennies? If so, then the remaining 43 coins are worth 95 cents. But each of the remaining coins is worth at least 5 cents, so these 43 coins are worth at least $5 \times 43 = 215$ cents, which is impossible. So there cannot be 5 pennies.
Could there be 10 pennies? If so, then the remaining 38 coins are worth 90 cents. But each of the remaining coins is worth at least 5 cents, so these 38 coins are worth at least $5 \times 38 = 190$ cents, which is impossible. So there cannot be 10 pennies.
We can continue in this way to show that there cannot be 15, 20, 25, or 30 pennies.
Therefore, there could only be 35, 40 or 45 pennies.

If there are 35 pennies, then the remaining 13 coins are worth 65 cents. Since each of the remaining coins is worth at least 5 cents, this is possible only if each of the 13 coins is a nickel. So one combination that works is 35 pennies and 13 nickels.

If there are 40 pennies, then the remaining 8 coins are worth 60 cents.
We now look at the number of quarters in this combination.
If there are 0 quarters, then we must have 8 nickels and dimes totalling 60 cents. If all of the 8 coins were nickels, they would be worth 40 cents, so we need to change 4 nickels to dimes to increase our total by 20 cents to 60 cents. Therefore, 40 pennies, 0 quarters, 4 nickels and 4 dimes works.
If there is 1 quarter, then we must have 7 nickels and dimes totalling 35 cents. Since each remaining coin is worth at least 5 cents, then all of the 7 remaining coins must be nickels. Therefore, 40 pennies, 1 quarter, 7 nickels and 0 dimes works.
If there are 2 quarters, then we must have 6 nickels and dimes totalling 10 cents. This is impossible. If there were more than 2 quarters, the quarters would be worth more than 60 cents, so this is not possible.

If there are 45 pennies, then the remaining 3 coins are worth 55 cents in total.
In order for this to be possible, there must be 2 quarters (otherwise the maximum value of the 3 coins would be with 1 quarter and 2 dimes, or 45 cents).
This means that the remaining coin is worth 5 cents, and so is a nickel.
Therefore, 45 pennies, 2 quarters, 1 nickel and 0 dimes is a combination that works.

Therefore, there are 4 combinations that work.

ANSWER: (B)

Grade 8

1. Using a common deminator of 8, $\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$.

   Answer: (B)

2. Calculating, $(-3)(-4)(-1) = (12)(-1) = -12$.

   Answer: (A)

3. Since $V = l \times w \times h$, the required volume is $4 \times 2 \times 8 = 64$ cm^3.

   Answer: (C)

4. The mean of these five numbers is $\dfrac{6+8+9+11+16}{5} = \dfrac{50}{5} = 10$.

   Answer: (C)

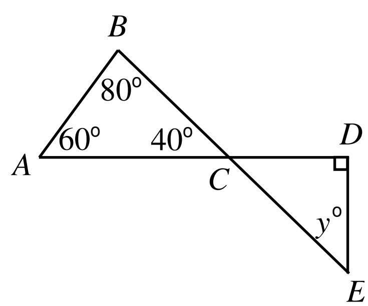
5. 10% of 10 is $0.1 \times 10 = 1$ or $\frac{1}{10} \times 10 = 1$.
   20% of 20 is $0.2 \times 20 = 4$ or $\frac{1}{5} \times 20 = 4$.
   Therefore, 10% of 10 times 20% of 20 equals $1 \times 4 = 4$.

   Answer: (E)

6. $8210 = 8.21 \times 1000$ so we must have $10^{\square} = 1000$ so the required number is 3.

   Answer: (C)

7. Since $\angle ABC + \angle BAC + \angle BCA = 180^\circ$ and $\angle ABC = 80^\circ$ and $\angle BAC = 60^\circ$, then $\angle BCA = 40^\circ$.



   Since $\angle DCE = \angle BCA = 40^\circ$, and looking at triangle CDE, we see that $\angle DCE + \angle CED = 90^\circ$ then $40^\circ + y^\circ = 90^\circ$ or $y = 50$.

   Answer: (D)

8. *Solution 1*
   There are 10 numbers (30 to 39) which have a tens digit of 3.
   There are 6 numbers (3, 13, 23, 33, 43, 53) which have a units digit of 3.
   In these two lists, there is one number counted twice, namely 33.
   Therefore, the total number of different numbers in these two lists is $10 + 6 - 1 = 15$.

   Solution 2
   We list out the numbers in increasing order: 3, 13, 23, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 43, 53. There are 15 of these numbers.

   Answer: (D)

9. Since the average monthly rainfall was 41.5 mm in 2003, then the average monthly rainfall in 2004 was $41 + 2 = 43.5$ mm.
Therefore, the total rainfall in 2004 was $12 \times 43.5 = 522$ mm.

Answer: (B)

10. Since Daniel rides at a constant speed, then, in 30 minutes, he rides $\frac{3}{4}$ of the distance that he does in 40 minutes.
Therefore, in 30 minutes, he rides $\frac{3}{4} \times 24 = 18$ km.

Answer: (D)

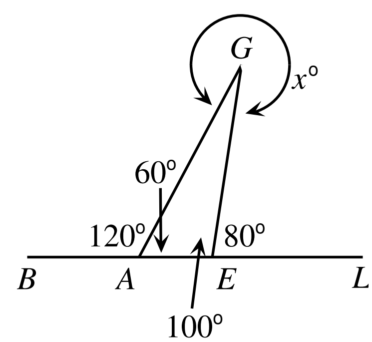
11. Triangle ABC has base AB of length 25 cm and height AC of length 20 cm.
Therefore, the area of triangle ABC is $\frac{1}{2}bh = \frac{1}{2}(25\text{ cm})(20\text{ cm}) = \frac{1}{2}(500\text{ cm}^2) = 250\text{ cm}^2$.

Answer: (E)

12. To make the sum of the five consecutive even numbers including 10 and 12 as large as possible, we should make 10 and 12 the smallest of these five numbers.
Therefore, to make the sum as large as possible, the numbers should be 10, 12, 14, 16, and 18, which have a sum of 70.

Answer: (E)

13. Since the sum of the angles at any point on a line is 180°, then $\angle GAE = 180^\circ - 120^\circ = 60^\circ$ and $\angle GEA = 180^\circ - 80^\circ = 100^\circ$.



Since the sum of the angles in a triangle is 180°,
$\angle AGE = 180^\circ - \angle GAE - \angle GEA = 180^\circ - 60^\circ - 100^\circ = 20^\circ$.
Since $\angle AGE = 20^\circ$, then the reflex angle at G is $360^\circ - 20^\circ = 340^\circ$, so $x = 340$.

Answer: (A)

14. *Solution 1*
Since the numerators of the five fractions are the same, the largest fraction will be the one with the *smallest* denominator.
To get the smallest denominator, we must subtract (not add) the largest quantity from 2.
Therefore, the largest fraction is $\dfrac{4}{2 - \frac{1}{2}}$.

Solution 2
We evaluate each of the choices:

$$\begin{aligned}
\frac{4}{2-\frac{1}{4}} &= \frac{4}{\frac{7}{4}} = \frac{16}{7} \approx 2.29 \\
\frac{4}{2+\frac{1}{4}} &= \frac{4}{\frac{9}{4}} = \frac{16}{9} \approx 1.78 \\
\frac{4}{2-\frac{1}{3}} &= \frac{4}{\frac{5}{3}} = \frac{12}{5} = 2.4 \\
\frac{4}{2+\frac{1}{3}} &= \frac{4}{\frac{7}{3}} = \frac{12}{7} \approx 1.71 \\
\frac{4}{2-\frac{1}{2}} &= \frac{4}{\frac{3}{2}} = \frac{8}{3} \approx 2.67
\end{aligned}$$

The largest of the five fractions is $\frac{4}{2-\frac{1}{2}}$.

Answer: (E)

15. We first try $x = 1$ in each of the possibilities and we see that it checks for all five possibilities.
Next, we try $x = 2$.
When $x = 2$, $y = x + 0.5 = 2 + 0.5 = 2.5$.
When $x = 2$, $y = 1.5x = 1.5(2) = 3$.
When $x = 2$, $y = 0.5x + 1 = 0.5(2) + 1 = 2$.
When $x = 2$, $y = 2x - 0.5 = 2(2) - 0.5 = 3.5$.
When $x = 2$, $y = x^2 + 0.5 = 2^2 + 0.5 = 4.5$.
So only the second choice agrees with the table when $x = 2$.
When $x = 3$, the second choice gives $y = 1.5x = 1.5(3) = 4.5$ and when $x = 4$, $y = 1.5x = 1.5(4) = 6$.
Therefore, the second choice is the only one which agrees with the table.

Answer: (B)

16. If the student were to buy 40 individual tickets, this would cost $40 \times \$1.50 = \$60.00$.
If the student were to buy the tickets in packages of 5, she would need to buy $40 \div 5 = 8$ packages, and so this would cost $8 \times \$5.75 = \$46.00$.
Therefore, she would save $\$60.00 - \$46.00 = \$14.00$.

Answer: (C)

17. *Solution 1*
In this solution, we try specific values. This does not guarantee correctness, but it will tell us which answers are wrong.
We try setting $a = 2$ (which is even) and $b = 1$ (which is odd).
Then $ab = 2 \times 1 = 2$, $a + 2b = 2 + 2(1) = 4$, $2a - 2b = 2(2) - 2(1) = 2$, $a + b + 1 = 2 + 1 + 1 = 4$, and $a - b = 2 - 1 = 1$.
Therefore, $a - b$ is the only choice which gives an odd answer.

Solution 2
Since a is even, then ab is even, since an even integer times any integer is even.
Since a is even and $2b$ is even (since 2 times any integer is even), then their sum $a + 2b$ is even.
Since 2 times any integer is even, then both $2a$ and $2b$ are both even, so their difference $2a - 2b$

is even (since even minus even is even).
Since a is even and b is odd, then $a+b$ is odd, so $a+b+1$ is even.
Since a is even and b is odd, then $a-b$ is odd.
Therefore, $a-b$ is the only choice which gives an odd answer.

Answer: (E)

18. *Solution 1*
For 100 to divide evenly into N and since $100 = 2^2 \times 5^2$, then N must have at least 2 factors of 2 and at least 2 factors of 5.
From its prime factorization, N already has enough factors of 2, so the box must provide 2 factors of 5.
The only one of the five choices which has 2 factors of 5 is 75, since $75 = 3 \times 5 \times 5$.
Checking, $2^5 \times 3^2 \times 7 \times 75 = 32 \times 9 \times 7 \times 75 = 151\,200$.
Therefore, the only possible correct answer is 75.

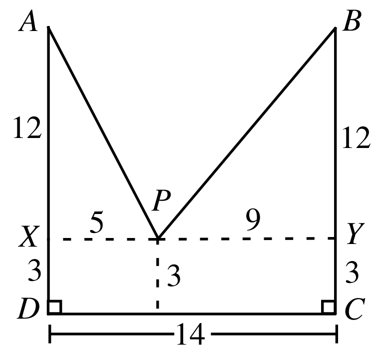
Solution 2
Multiplying out the part of N we know, we get $N = 2^5 \times 3^2 \times 7 \times \square = 32 \times 9 \times 7 \times \square = 2016 \times \square$.
We can then try the five possibilities.
$2016 \times 5 = 10\,080$ which is not divisible by 100.
$2016 \times 20 = 40\,320$ which is not divisible by 100.
$2016 \times 75 = 151\,200$ which is divisible by 100.
$2016 \times 36 = 72\,576$ which is not divisible by 100.
$2016 \times 120 = 241\,920$ which is not divisible by 100.
Therefore, the only possibility which works is 75.

Answer: (C)

19. In the diagram, B appears to be about 0.4 and C appears to be about 0.6, so $B \times C$ should be about $0.4 \times 0.6 = 0.24$.
Also, A appears to be about 0.2, so $B \times C$ is best represented by A.

Answer: (A)

20. Label the points A, B, C and D, as shown.
Through P, draw a line parallel to DC as shown.
The points X and Y are where this line meets AD and BC.
From this, we see that $AX = YC = 15 - 3 = 12$.
Also, $PY = 14 - 5 = 9$.



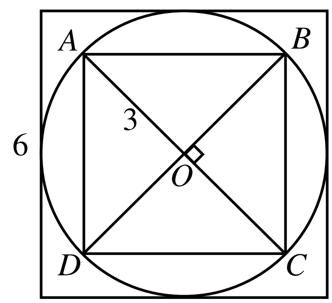
To calculate the length of the rope, we need to calculate AP and BP, each of which is the hypotenuse of a right-angled triangle.

Now, $AP^2 = 12^2 + 5^2 = 169$ so $AP = 13$, and $BP^2 = 12^2 + 9^2 = 225$, so $BP = 15$.
Therefore, the required length of rope is $13 + 15$ or 28 m.

Answer: (A)

21. *Solution 1*

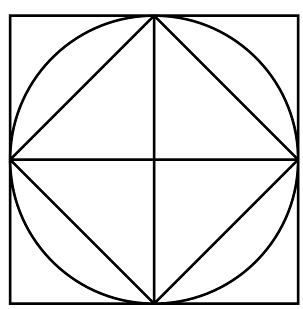
Since the area of the large square is 36, then the side length of the large square is 6.
Therefore, the diameter of the circle must be 6 as well, and so its radius is 3.
Label the four vertices of the small square as A, B, C, and D.
Join A to C and B to D.
Since $ABCD$ is a square, then AC and BD are perpendicular, crossing at point O, which by symmetry is the centre of the circle.
Therefore, $AO = BO = CO = DO = 3$, the radius of the circle.



But square $ABCD$ is divided into four identical isosceles right-angled triangles.
The area of each of these triangles is $\frac{1}{2}bh = \frac{1}{2}(3)(3) = \frac{9}{2}$ so the area of the square is $4 \times \frac{9}{2} = 18$.

Solution 2

Rotate the smaller square so that its four corners are at the four points where the circle touches the large square.
Next, join the top and bottom points where the large square and circle touch, and join the left and right points.



By symmetry, these two lines divide the large square into four sections (each of which is square) of equal area.
But the original smaller square occupies exactly one-half of each of these four sections, since each edge of the smaller square is a diagonal of one of these sections.
Therefore, the area of the smaller square is exactly one-half of the area of the larger square, or 18.

Answer: (E)

22. *Solution 1*
Since there were 50 students surveyed in total and 8 played neither hockey nor baseball, then 42 students in total played one game or the other.
Since 33 students played hockey and 24 students played baseball, and this totals $33 + 24 = 57$ students, then there must be 15 students who are "double-counted", that is who play both sports.

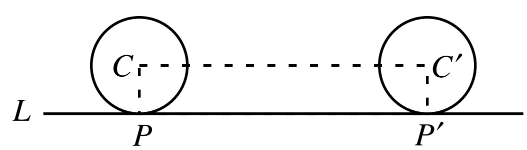
Solution 2
Let x be the number of students who play both hockey and baseball.
Then the number of students who play just hockey is $33 - x$ and the number of students who play just baseball is $24 - x$.
But the total number of students (which is 50) is the sum of the numbers of students who play neither sport, who play just hockey, who play just baseball, and who play both sports.
In other words,

$$\begin{aligned} 8 + (33 - x) + (24 - x) + x &= 50 \\ 65 - 2x + x &= 50 \\ 65 - x &= 50 \\ 65 - 50 &= x \\ x &= 15 \end{aligned}$$

Therefore, the number of students who play both sports is 15.

Answer: (D)

23. We begin by considering a point P which is where the circle first touches a line L.



If a circle makes one complete revolution, the point P moves to P' and the distance PP' is the circumference of the circle, or 2π m.
If we now complete the rectangle, we can see that the distance the centre travels is CC' which is exactly equal to PP' or 2π m.

Answer: (C)

24. Let the three positive integers be x, y and z.
From the given information $(x + y) \times z = 14$ and $(x \times y) + z = 14$.
Let us look at the third number, z, first.
From the first equation $(x + y) \times z = 14$, we see that z must be a factor of 14.
Therefore, the only possible values of z are 1, 2, 7 and 14.

If $z = 1$, then $x + y = 14$ and $xy = 13$. From this, $x = 1$ and $y = 13$ or $x = 13$ and $y = 1$. (We can find this by seeing that $xy = 13$ so the only possible values of x and y are 1 and 13, and then checking the first equation to see if they work.)

If $z = 2$, then $x + y = 7$ and $xy = 12$. From this, $x = 3$ and $y = 4$ or $x = 4$ and $y = 3$.

(We can find these by looking at pairs of positive integers which add to 7 and checking if they multiply to 12.)

If $z = 7$, then $x + y = 2$ and $xy = 7$. There are no possibilities for x and y here. (Since x and y are positive integers and $x + y = 2$ then we must have $x = 1$ and $y = 1$, but this doesn't work in the second equation.)

If $z = 14$, then $x + y = 1$ and $xy = 0$. There are no possibilities for x and y here. (This is because one of them must be 0, which is not a positive integer.)

Therefore, the four possible values for x are 1, 13, 3 and 4.

Answer: (B)

25. Suppose that there are n coins in the purse to begin with.
Since the average value of the coins is 17 cents, then the total value of the coins is $17n$.
When one coin is removed, there are $n - 1$ coins.
Since the new average value of the coins is 18 cents, then the new total value of the coins is $18(n - 1)$.
Since 1 penny was removed, the total value was decreased by 1 cent, or $17n - 1 = 18(n - 1)$ or $17n - 1 = 18n - 18$ or $n = 17$.
Therefore, the original collection of coins has 17 coins worth 289 cents.

Since the value of each type of coin except the penny is divisible by 5, then there must be at least 4 pennies.
Removing these pennies, we have 13 coins worth 285 cents. These coins may include pennies, nickels, dimes and quarters.

How many quarters can there be in this collection of 13 coins?
12 quarters are worth $12 \times 25 = 300$ cents, so the number of quarters is fewer than 12.
Could there be as few as 10 quarters?
If there were 10 quarters, then the value of the quarters is 250 cents, so the remaining 3 coins (which are pennies, nickels and dimes) are worth 35 cents. But these three coins can be worth no more than 30 cents, so this is impossible.
In a similar way, we can see that there cannot be fewer than 10 quarters.

Therefore, there are 11 quarters in the collection, which are worth 275 cents.
Thus, the remaining 2 coins are worth 10 cents, so must both be nickels.
Therefore, the original collection of coins consisted of 11 quarters, 2 nickels and 4 pennies.

Answer: (A)