Pascal Contest (Grade 9)
Wednesday, February 22, 2006

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Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C. There is no penalty for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have sixty minutes of working time.
Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. What is the value of $\frac{550 + 50}{5^2 + 5}$?
   (A) 32  (B) 40  (C) 12  (D) 65  (E) 20

2. What is the value of $\sqrt{36 + 64} - \sqrt{25} - 16$?
   (A) 5  (B) 7  (C) 13  (D) 11  (E) 9

3. How many positive whole numbers, including 1 and 18, divide exactly into 18?
   (A) 3  (B) 4  (C) 5  (D) 6  (E) 7

4. If $A + B = 5$, then the value of $B - 3 + A$ is
   (A) 2  (B) 8  (C) 7  (D) 15  (E) 13

5. In the diagram, the rectangular solid and the cube have equal volumes. The length of each edge of the cube is
   (A) 2  (B) 4  (C) 8  (D) 16  (E) 32

6. Ravindra and Hongshu made a pizza together. Ravindra ate $\frac{2}{5}$ of the pizza. Hongshu ate half as much as Ravindra. What percentage of the original pizza was left?
   (A) 20  (B) 30  (C) 40  (D) 50  (E) 60

7. In the diagram, two equal-armed balances are shown.
   How many $\bigcirc$ would it take to balance $\square \square \square \square$?
   (A) 2  (B) 1  (C) 4  (D) 5  (E) 3

8. The areas of three squares are 16, 49 and 169. What is the average (mean) of their side lengths?
   (A) 8  (B) 12  (C) 24  (D) 39  (E) 32

9. In the diagram, the rectangle has a width of $w$, a length of 8, and a perimeter of 24. What is the ratio of its width to its length?
   (A) 1 : 4  (B) 1 : 3  (C) 1 : 2  (D) 3 : 8  (E) 2 : 3
10. In the subtraction shown, \(M\) and \(N\) each represent a single digit. What is the value of \(M + N\)?
   \[
   \begin{array}{c}
   \phantom{0}M \\
   \phantom{0}N
   \end{array}
   \begin{array}{c}
   4 \\
   - \phantom{1}3
   \end{array}
   \begin{array}{c}
   \phantom{0}1 \\
   \phantom{0}6
   \end{array}
   \]
   (A) 14  (B) 12  (C) 15  (D) 13  (E) 11

Part B: Each correct answer is worth 6.

11. When \(x = 9\), which of the following has the largest value?
   \[
   \begin{array}{c}
   (A) \sqrt{x} \\
   (B) \frac{x}{2} \\
   (C) x - 5 \\
   (D) \frac{40}{x} \\
   (E) \frac{x^2}{20}
   \end{array}
   \]

12. The lengths of the three sides of a triangle are 7, \(x + 4\) and \(2x + 1\). The perimeter of the triangle is 36. What is the length of the longest side of the triangle?
   (A) 7  (B) 12  (C) 17  (D) 15  (E) 16

13. If Corina had added the numbers \(P\) and \(Q\) correctly, the answer would have been 16. By mistake, she subtracted \(Q\) from \(P\). Her answer was 4. What is the value of \(P\)?
   (A) 4  (B) 5  (C) 8  (D) 10  (E) 16

14. If \(\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{n}{12} = 2\), the value of \(n\) is
   (A) -4  (B) 13  (C) 18  (D) 4  (E) 1

15. From 7:45 p.m. to 9:30 p.m., Jim drove a distance of 84 km at a constant speed. What was this speed, in km/h?
   (A) 60  (B) 80  (C) 112  (D) 63  (E) 48

16. An unusual die has the numbers 2, 2, 3, 3, 5, and 8 on its six faces. Two of these dice are rolled. The two numbers on the top faces are added. How many different sums are possible?
   (A) 6  (B) 7  (C) 8  (D) 9  (E) 10

17. In the diagram, point \(E\) lies on line segment \(AB\), and triangles \(AED\) and \(BEC\) are isosceles. Also, \(\angle DEC\) is twice \(\angle ADE\). What is the size of \(\angle EBC\)?
   \[
   \begin{array}{c}
   (A) 75^\circ \\
   (B) 80^\circ \\
   (C) 60^\circ
   \end{array}
   \begin{array}{c}
   (D) 55^\circ \\
   (E) 45^\circ
   \end{array}
   \]

18. In the diagram, the grid is made up of squares. What is the area of the shaded region?
   \[
   \begin{array}{c}
   2 \\
   \phantom{0}4
   \end{array}
   \begin{array}{c}
   \phantom{0}12
   \end{array}
   \]
   (A) 19  (B) 24  (C) 14  (D) 12  (E) 8
19. The sum of ten consecutive integers is $S$. Ten times the smallest of these integers is $T$. What is the value of $S - T$?

(A) 45 (B) 55 (C) 10 (D) 9 (E) 66

20. Five identical rectangles are arranged to form a larger rectangle $PQRS$, as shown. The area of $PQRS$ is 4000. The length, $x$, of each of the identical rectangles is closest to

(A) 35 (B) 39 (C) 41
(D) 37 (E) 33

Part C: Each correct answer is worth 8.

21. In each row of the table, the sum of the first two numbers equals the third number. Also, in each column of the table, the sum of the first two numbers equals the third number. What is the sum of the nine numbers in the table?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>4</td>
<td>$m + 4$</td>
</tr>
<tr>
<td>8</td>
<td>$n$</td>
<td>$8 + n$</td>
</tr>
<tr>
<td>$m + 8$</td>
<td>$4 + n$</td>
<td>6</td>
</tr>
</tbody>
</table>

(A) 18 (B) 42 (C) −18
(D) −6 (E) 24

22. In the diagram, each of the three identical circles touch the other two. The circumference of each circle is 36. What is the perimeter of the shaded region?

(A) 18 (B) 6 (C) 36
(D) 12 (E) 24

23. Ben and Anna each have some CDs. If Anna gives six of her CDs to Ben, he would then have twice as many CDs as Anna. If, instead, Anna takes six CDs from Ben, then both would have the same number of the CDs. What is the total number of CDs that Ben and Anna have?

(A) 42 (B) 30 (C) 72 (D) 18 (E) 36

24. A bag contains eight yellow marbles, seven red marbles, and five black marbles. Without looking in the bag, Igor removes $N$ marbles all at once. If he is to be sure that, no matter which choice of $N$ marbles he removes, there are at least four marbles of one colour and at least three marbles of another colour left in the bag, what is the maximum possible value of $N$?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

25. John writes a number with 2187 digits on the blackboard, each digit being a 1 or a 2. Judith creates a new number from John’s number by reading his number from left to right and wherever she sees a 1 writing 112 and wherever she sees a 2 writing 111. (For example, if John’s number begins 2112, then Judith’s number would begin 111112112111.) After Judith finishes writing her number, she notices that the leftmost 2187 digits in her number and in John’s number are the same. How many times do five 1’s occur consecutively in John’s number?

(A) 182 (B) 183 (C) 184 (D) 185 (E) 186
Canadian Mathematics Competition

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