1. Emily’s old showerhead used 18 L of water per minute. She installs a new showerhead that uses 13 L per minute.

(a) When Emily takes a bath, she uses 260 L of water. Using the new showerhead, what length of shower, in minutes, uses 260 L of water?

(b) How much less water is used for a 10 minute shower with the new showerhead than with the old showerhead?

(c) Emily is charged 8 cents per 100 L of water that she uses. Using the new showerhead instead of the old showerhead saves water and so saves Emily money. How much money does Emily save in water costs for a 15 minute shower?

(d) How many minutes of showering, using the new showerhead, will it take for Emily to have saved $30 in water costs?

2. (a) Quadrilateral $QABO$ is constructed as shown. Determine the area of $QABO$.

(b) Point $C(0, p)$ lies on the y-axis between $Q(0, 12)$ and $O(0, 0)$ as shown. Determine an expression for the area of $\triangle COB$ in terms of $p$.

(c) Determine an expression for the area of $\triangle QCA$ in terms of $p$.

(d) If the area of $\triangle ABC$ is 27, determine the value of $p$. 
3. (a) Solve the system of equations algebraically for \((x, y)\):

\[
\begin{align*}
    x + y &= 42 \\
    x - y &= 10
\end{align*}
\]

(b) Suppose that \(p\) is an even integer and that \(q\) is an odd integer. Explain why the system of equations

\[
\begin{align*}
    x + y &= p \\
    x - y &= q
\end{align*}
\]

has no positive integer solutions \((x, y)\).

(c) Determine all pairs of positive integers \((x, y)\) that satisfy the equation \(x^2 - y^2 = 420\).

4. (a) In \(\triangle PQR\), point \(T\) is on side \(QR\) such that \(QT = 6\) and \(TR = 10\). Explain why the ratio of the area of \(\triangle PQT\) to the area of \(\triangle PTR\) is 3 : 5.

(b) In \(\triangle ABC\), point \(D\) is the midpoint of side \(BC\). Point \(E\) is on \(AC\) such that \(AE : EC = 1 : 2\). Point \(F\) is on \(AD\) such that \(AF : FD = 3 : 1\). If the area of \(\triangle DEF\) is 17, determine the area of \(\triangle ABC\).

(c) In the diagram, points \(X\), \(Y\) and \(Z\) are on the sides of \(\triangle UVW\), as shown. Line segments \(UY\), \(VZ\) and \(WX\) intersect at \(P\). Point \(Y\) is on \(VW\) such that \(VY : YW = 4 : 3\). If \(\triangle PYW\) has an area of 30 and \(\triangle PZW\) has an area of 35, determine the area of \(\triangle UXP\).