The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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2018 Canadian Team Mathematics Contest

Individual Problems

IMPORTANT NOTES:

- Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

PROBLEMS:

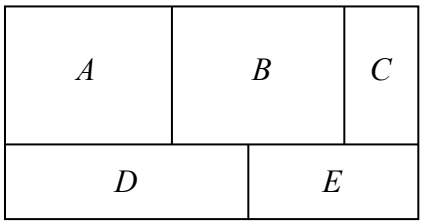
1. The point with coordinates $(a, 0)$ is on the line with equation $y = x + 8$. What is the value of a?

2. If

$$x = \left(1 - \tfrac{1}{12}\right)\left(1 - \tfrac{1}{11}\right)\left(1 - \tfrac{1}{10}\right)\left(1 - \tfrac{1}{9}\right)\left(1 - \tfrac{1}{8}\right)\left(1 - \tfrac{1}{7}\right)\left(1 - \tfrac{1}{6}\right)\left(1 - \tfrac{1}{5}\right)\left(1 - \tfrac{1}{4}\right)\left(1 - \tfrac{1}{3}\right)\left(1 - \tfrac{1}{2}\right)$$

   what is the value of x?

3. In the diagram, a large rectangle is divided into five smaller rectangles which are labelled A, B, C, D, E. In how many ways can exactly two of these five rectangles be shaded so that the shaded rectangles are not touching?



4. The length of the diagonal of a square is 10. What is the area of this square?

5. A three-digit positive integer n has digits abc. (That is, a is the hundreds digit of n, b is the tens digit of n, and c is the ones (units) digit of n.) Determine the largest possible value of n for which

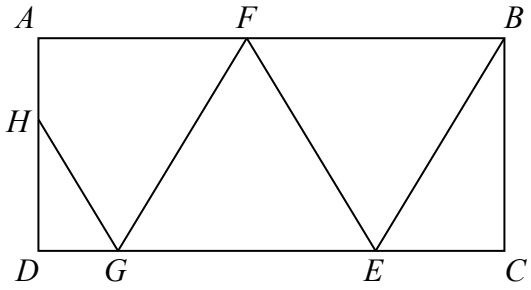
   - a is divisible by 2,
   - the two-digit integer ab (that, a is the tens digit and b is the ones (units) digit) is divisible by 3 but is not divisible by 6, and
   - n is divisible by 5 but is not divisible by 7.

6. Determine all pairs of real numbers (x, y) for which $(4x^2 - y^2)^2 + (7x + 3y - 39)^2 = 0$.

7. An arithmetic sequence has a common difference, d, that is a positive integer and is greater than 1. The sequence includes the three terms 3, 468 and 2018. What is the sum of all of the possible values of d?

   (An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant, called the common difference. For example, $3, 5, 7, 9$ are the first four terms of an arithmetic sequence with common difference 2.)

8. Rectangular room $ABCD$ has mirrors on walls AB and DC. A laser is placed at B. It is aimed at E and the beam reflects off of the mirrors at E, F and G, arriving at H. The laws of physics tell us that $\angle BEC = \angle FEG$ and $\angle BFE = \angle AFG$ and $\angle FGE = \angle HGD$. If $AB = 18$ m, $BC = 10$ m and $HD = 6$ m, what is the total length of the path $BEFGH$ travelled by the laser beam?



9. A box contains R red balls, B blue balls, and no other balls. One ball is removed and set aside, and then a second ball is removed. On each draw, each ball in the box is equally likely to be removed. The probability that both of these balls are red is $\frac{2}{7}$. The probability that exactly one of these balls is red is $\frac{1}{2}$. Determine the pair (R, B).

10. A cylindrical tank has radius 10 m and length 30 m. The tank is lying on its side on a flat surface and is filled with water to a depth of 5 m. Water is added to the tank and the depth of the water increases from 5 m to $10 + 5\sqrt{2}$ m. If the volume of water added to the tank, in m^3, can be written as $a\pi + b + c\sqrt{p}$ for some integers a, b, c and prime number p, determine the quadruple (a, b, c, p).