## Answers to Practice Set Number 3

**Pascal**

1) E 2) D 3) D 4) A 5) C 6) C 7) E 8) B 9) B 10) E

**Cayley**

1) C 2) E 3) C 4) E 5) B 6) C 7) C 8) A 9) D 10) B

**Fermat**

1) D 2) E 3) B 4) B 5) C 6) C 7) A 8) E 9) A 10) A

**Hints, suggestions, and some solutions:**

**Pascal**

1. $\left(\frac{6}{5}\right)\left(\frac{7}{6}\right)\left(\frac{8}{7}\right) = \left(\frac{8}{5}\right)$.
2. Since the three angles add to 180, $2b + b + 3b = 180$ and $b = 30$. So $3b = 90$ and we have a right triangle.
3. Trial and error shows the numbers are 45, 47, 49.
4. Since most of these cancel in pairs we just add $24 + 25 + 26 \ldots + 31$. These 8 integers average $(24 + 31)/2$ and so the total is $4 \times 55 = 220$.
5. Think of the suits as 100 groups of 2 expensive and 1 cheap suit. The average of each group is $(24 + 31)/2$ and so the total is $4 \times 55 = 220$.
6. The numerator must be increased by 1700.
7. The area is an $a$ by $b$ rectangle plus a triangle of height $c - a$ and base $b$. The area is $ab + a(c - b)/2 = ab/2 + ac/2 = a(b + c)/2$.
8. The areas of triangles with the same altitudes are in the same ratio as their bases. So if the required area is $A$ then $\frac{A}{54} = \frac{12}{36}$ and $A = 18$.
9. If the distance is $D$ then $\frac{D}{60} - \frac{D}{100} = 2$ and $D = 300$. So the required speed is $\frac{300}{4} = 75$ km/h.
10. Join $E$ to the midpoint $M$ and $AC$. Then the triangle $CEM$ is equilateral of side 1 and triangle $EMA$ has a 120 angle at $M$ and $MA = ME = 1$. Bisect angle $M$ in the triangle to form two 30-60-90 triangles. Using the ratio of the sides of a 30-60-90 triangle $AE = 2\left(\frac{\sqrt{3}}{2}\right)$.

**Cayley**

1. The answer is approximately $14 \times 365 \times 24 \times 60 \times 60/1000000 = 400$.
2. The perimeter is just the same as a 9 by 27 rectangle!
3. Using the standard formula $2(8x15 + 4x15 + 8x4)$.

4. $x + 47 = 2(25)$ and $11 + y = 28$ so $x + y = 20$.

5. Solve $32x + 72(25 - x) = 64(25)$.

6. Since the correct answer involves multiplying and then adding 5 the error must involve reversing these operations. So $x = 5$ and the correct answer is $95 + 5 = 100$.

7. Divide the area into two semicircles around a rectangle!

8. Only decimals in lowest terms whose denominators involve only powers of 2 and 5 terminate. But $144 = 9 \times 16$ so $\frac{9}{144} = \frac{1}{16}$ is the first terminating decimal.

9. First $CZ = 6$, $BY = k$ and $CY = 8 - k$. Using Pythagoras

$$PA^2 + PB^2 + PC^2 = PX^2 + 9 + PY^2 + k^2 + PZ^2 + 36 = PX^2 + 25 + PY^2 + (8 - k)^2 + PZ^2 + 4$$

Thus $16k = 48$ and $k = 3$.

10. $2004 = 2 \times 2 \times 3 \times 167$. Let $n = k^2$ and $n + 2004 = m^2$ so $m^2 - k^2 = 2004 \Rightarrow (m + k)(m - k) = 2004$. But $m + k$ and $m - k$ must have same parity and so must be even. So $\{m + k = 2(501)$ and $m - k = 2(1)\}$ or $\{m + k = 2(167)$ and $m - k = 2(3)\}$. Thus $k = 500$ or 164, leading to just 2 values for $n$.

**Fermat**

1. Since the triangle is scalene the best we can do is 75, 74, 31.

2. A little arithmetic shows that for $a$, $b$, $c$ and $d$ to be integers $a$ must be a multiple of 9.

3. $(a - b)^2 = a^2 + b^2 - 2ab = 9$.

4. $432 = 16 \times 27$. Since 16 is a perfect square we need only make 27 one also.

5. Since the ratio $16 : 24 = 24 : 36 = 2 : 3$ the triangles are similar and their areas are in ratio $4 : 9$.

6. The number '*ddd*' has a factor $111 = 3 \times 37$. So 37 must be one number. The other is $9 \times 3 = 27$ and indeed $27 \times 37 = 999$.

7. Using sum and product $ab = b$ and $-a = a + b$. Since there are two roots, $a$ and $b$ are not zero so solving these equations $a = 1$ and $b = -2$.

8. The areas are: a) 12.25 b) 12 c) 12 d) $4\sqrt{5}$ e) $\frac{9}{2}\pi$

9. Since $30^{30} = 2^{30} \cdot 3^{30} \cdot 5^{30}$ and perfect squares have even exponents the answer is $16 \times 16 \times 16$.

10. Note triangle $ABC$ is a $30, 60, 90$ triangle! Let the other point of circle intersection be $D$. The required area is then sector $ACD$ plus sector $BCD$ minus twice triangle $ABC$.
Since the sector angles are 60 and 120 we get $\frac{1}{3}(3\pi) + \frac{1}{6}(9\pi) = 2\left(\frac{1}{2}3\sqrt{3}\right) = \frac{5}{2}\pi - 3\sqrt{3}$.