



University of Waterloo
Faculty of Mathematics



Centre for Education in
Mathematics and Computing

Grade 7 & 8 Math Circles

October 20, 2010

Statistics

Definition

Statistics: is the science of collecting, organizing, and interpreting data.

In statistics there are many cases where we will be summing a large amount of numbers. Instead of writing each number individually we will use sigma notation. Sigma notation is a condensed way of expressing the addition of many numbers.

Example

$$1 + 2 + 3 + 4 + 5 + \dots + 100 = \sum_{j=1}^{100} j$$

Exercise 1

1. Expand the following:

(a) $\sum_{j=1}^{50} 2j$

(b) $\sum_{j=0}^{100} j^3$

2. Write the following sums using sigma notation:

(a) $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \dots + 99^2$

(b) $1 + 3 + 5 + \dots + n$

Measures of Central Tendency

Definitions

Mean(\bar{x}): the sum of all the numbers in a set divided by the number of numbers in the set.

Median: The middle number in a set of numbers arranged in ascending or decreasing order. If there are an even amount of numbers the median is the mean of the middle two numbers.

Mode: the number that occurs most often in a set of numbers.

Exercise 2

1. The following is a table of how many butterflies were sighted by Isabelle since 1990. Find the mean, median, and mode. When needed, round your answer to the nearest hundredth.

Year	Butterflies Sighted	Year	Butterflies Sighted	Year	Butterflies Sighted
1990	15	1997	16	2004	36
1991	45	1998	41	2005	52
1992	36	1999	25	2006	60
1993	24	2000	52	2007	42
1994	68	2001	37	2008	67
1995	37	2002	29	2009	62
1996	52	2003	61	2010	50

Measures of spread

Definition

Measure of Spread: indicates the extent to which the values in a set of data are spread out or scattered.

Measures of Spread:

1. Range

The **range** is the difference between the greatest and smallest value in a set of data.

2. Quartiles:

*To calculate the following you **must** arrange the data in ascending order*

(a) Lower Quartile(Q_1)

The median of the lower half of a set of data.

value at position $\frac{n+1}{4}$, where n is the number of numbers in the set.

(b) Upper Quartile(Q_3)

The median of the upper half of a set of data.

value at position $\frac{3(n+1)}{4}$, where n is the number of numbers in the set.

(c) Interquartile Range(IQR)

The range of the middle 50% of the data values in a set.

$$IQR = Q_3 - Q_1$$

3. Semi-Interquartile Range($SIQR$)

One half the difference between the upper and lower quartiles.

$$SIQR = \frac{IQR}{2}$$

Exercise 3

1. Given the following data find the range, lower quartile, upper quartile, interquartile range, and semi-interquartile range.

45 65 12 98 75 36 41 97 64 31 19 24 38 74 82 93 96 85 74 10 87 25 37

2. Given the following data find the range, lower quartile, upper quartile, interquartile range, and semi-interquartile range.

16 37 92 4 72 27 10 46 13 79 14 36 96 85 54 98 78 10 29 6 73 70 95 72 78 87 19 15 92 40 12

Problem Set

- Write $0 - 2 + 4 - 6 + 8 - \dots - 98 + 100$ using sigma notation.
- Expand $\sum_{x=1}^9 (x + 1)^2$.
- Given the following data and with a range of 11, what two numbers must x be between?

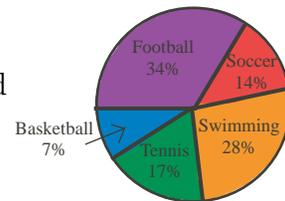
13 10 19 9 18 16 x 11 17 20 15 11

- Given the following data and with a mean of 42, what is x ?

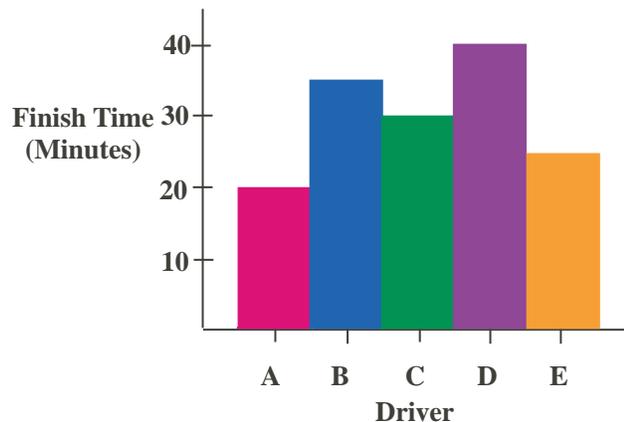
35 23 93 96 85 74 78 87 x 20 28 37 15 48 38 27 23 27 10 36 13 12

- The mean of 4 numbers is 28. If three of the numbers are 4, 8, and 9 what is the fourth number?
- In a set of 5 numbers the mean of three numbers is 9 and the mean of the other two numbers is 4. What is the mean of all five numbers?
- A set of six different positive integers has a mean of 9. What is the largest possible number in the set?
- The average of five integers is 25. If two of the integers are 10 and 19 and the others are three consecutive integers, determine these three integers.
- The scores of eight students on a quiz are 6, 7, 7, 8, 8, 8, 9, and 10. Which score should be removed to leave seven scores with the same mode and range as the original eight scores, but with a higher mean?

- According to the graph to the right, if 200 people were surveyed how many more people would prefer swimming to basketball?



- The bar graph displays the time it takes for 5 race car drivers to finish a 1500 meter race. What is the average speed of:
 - All the drivers
 - The driver that finished first



12. Two candles of the same height are lit at the same time. The first is consumed in four hours and the second in three hours. Assuming that each candle burns at a constant rate, how many hours after being lit was the first candle twice the height of the second?
13. A set of positive integers, each of which is different, has a sum of 477 and an average of 53. If one of the integers is 108, then what is the largest possible integer that could appear in the set?
14. Mike has 6 marks on his report card. The mean of the 6 marks is 74. The mode of the 6 marks is 76. The median of the 6 marks is 76. The lowest mark is 50. The highest mark is 94. Only one mark appears twice and no mark appears more than twice. Assuming all of his marks are integers, what is the number of possibilities for her second lowest mark?
15. A purse contains a collection of quarters, dimes, nickels, and pennies. The average value of the coins in the purse is 17 cents. If a penny is removed from the purse, the average value of the coins becomes 18 cents. How many nickels are in the purse?
16. In a list of 5 positive integers the only number to appear more than once is 8. The median is 9 and the average is 10. What is the largest possible integer that could appear in this list?

Problem Set Solutions

1. $\sum_{j=0}^{50} (-1)^j (2j)$
2. $4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100$
3. $9 \leq x \leq 20$
4. 19
5. 91
6. 7
7. 39
8. 31, 32, 33
9. 7
10. 42
11. (a) $50m/min$
(b) $75m/min$
12. $2.4hours$
13. 341
14. 17
15. 2
16. 15