UNIVERSITY OF WATERLOO
University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 7 & 8 Math Circles
October 27, 2010
Pascal's Triangle

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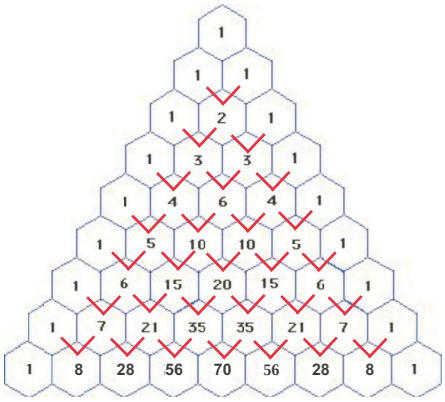
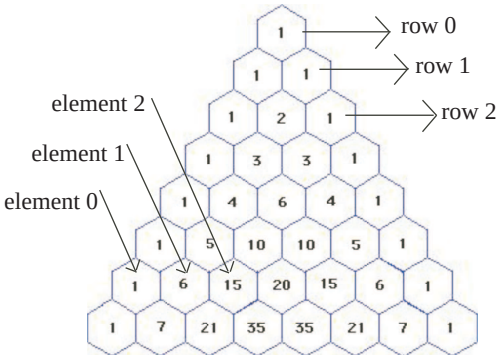
Creating Pascal's Triangle



Rows and Elements



'n^{th} row'

'r^{th} element in the row'

Patterns in Pascal's Triangle:

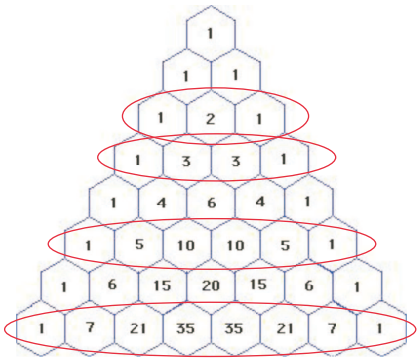
1. Sum of Rows

Row	Sum
0	1
1	2
2	4
3	8
4	16

The sum of the rows is equal to 2^n, where n is the row number.

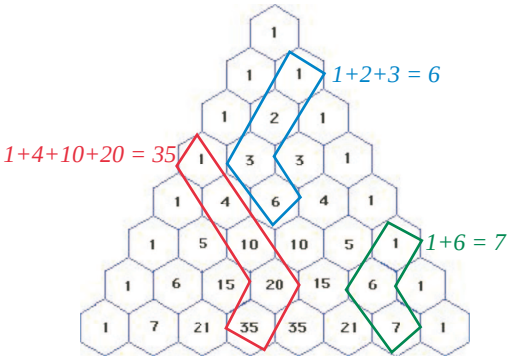
2. Prime Numbers

A **prime number** is a positive whole number greater than one that has two factors, one and itself.



IF the **first** element in each row is a prime number, then all other numbers in the row excluding one are multiples of the prime.

3. Hockey Stick Pattern



Numbers selected on a diagonal starting at an outside one have a sum of the number below that is not on the same diagonal.

4. Magic 11's

When there is a multiple digit element place brackets around the preceding element's ones digit and the first digit of the multiple digit element. Then add the numbers inside the brackets together.

Example

Row 8: 1,8,28,56,70,56,28,8,1
Add brackets: 1,(8+2),(8+5),(6+7),(0+5),(6+2),8,8,1
Add: 1,10,13,13,5,8,8,8,1
Add brackets: (1+1),(0+1),(3+1),3,5,8,8,8,1
Add: 2,1,4,3,5,8,8,8,1
Final number: 214358881

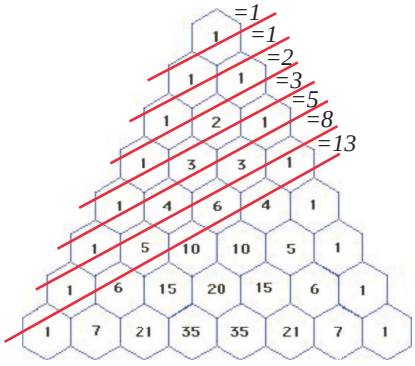
Row Number	Actual Row	Number
0	1	$1=11^0$
1	1,1	11^1
2	1,2,1	$121=11^2$
3	1,3,3,1	$1331=11^3$
4	1,4,6,4,1	$14641=11^4$
5	1,5,10,10,5,1 1,(5+1),(0+1),0,5,1 1,6,1,0,5,1	$161051=11^5$
6	1,6,15,20,15,6,1 1,(6+1),(5+2),(0+1),5,6,1 1,7,7,1,5,6,1	$1771561=11^6$

If you use the elements in a row for the digits of a number, the number is equal to 11^n where n is the row number.

5. Fibonacci Numbers

Fibonacci numbers are numbers with the first numbers being 0 and 1 and the following being the sum of the previous two. The first 10 terms of the Fibonacci Sequence are 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, and 55. The sequence can be represented by the formula below:

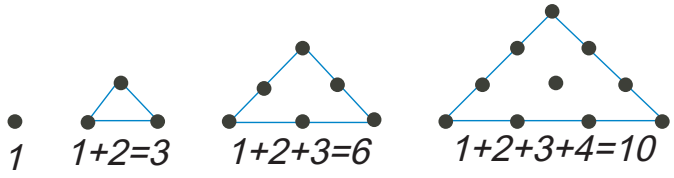
$$F_n = F_{n-1} + F_{n-2}, \text{ where } F_0 = 0,\ F_1 = 1 \text{ and } n \text{ is the } n^{th} \text{ number.}$$

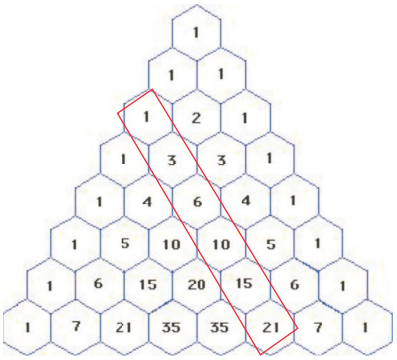


The sum of consecutive diagonals are the numbers in the Fibonacci Sequence.

6. Triangular Numbers

A **triangular number** is the number of dots in an equilateral triangle that is evenly filled.

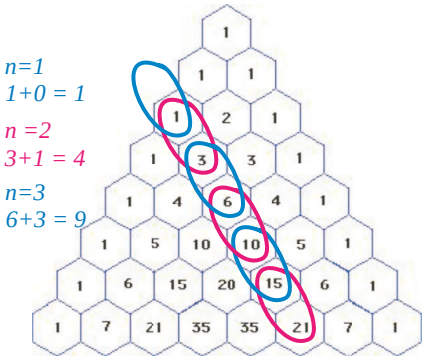




Triangular numbers are on the diagonal starting in the third row.

7. Square Numbers

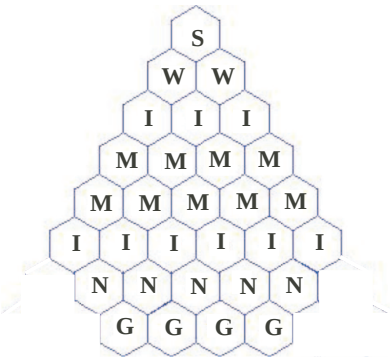
A square number is the number resulting from multiplying a number by itself.



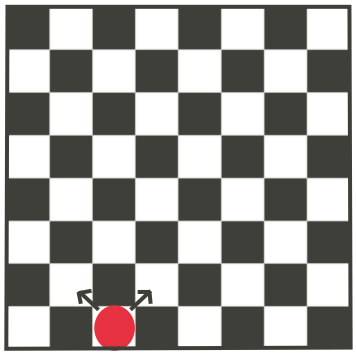
The n^{th} square number is equal to the sum of the the n^{th} and $(n-1)^{th}$ triangular number.

Problem Set

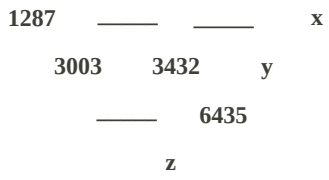
1. Starting at S at the top of the triangle and moving downwards to adjoining letters, how many ways can you spell swimming?



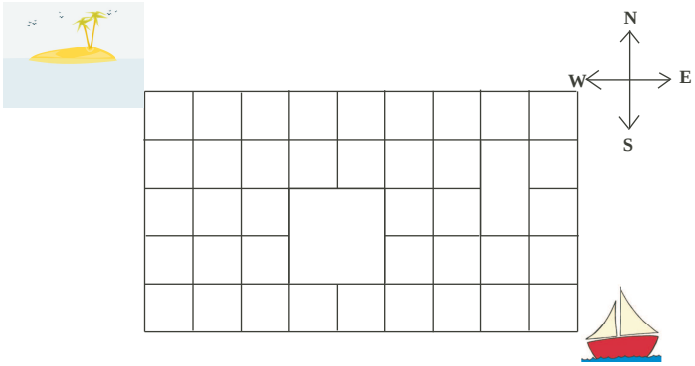
2. An eight-square checker board is shown below. A checker is placed at the third position in the bottom row. You are allowed to move the checker one square diagonally up (either left of right) at any one time. How many different ways are there to get to the opposite side of the board?



3. The following is a portion of Pascal's Triangle. Find the values of x, y, z.



4. Liam is sailing to a tropical island through many smaller channels. If he can only sail North and West how many different routes are possible for him to take?



5. A triangle can be formed having side lengths 4, 5, and 8. It is impossible to construct a triangle with side lengths 4, 5, and 9 because the length of two of the sides is equal to the third side, so no area is formed. Ron has eight sticks, each having an integer length. He observes that he cannot form a triangle using any three of these sticks as side lengths. What is the shortest possible length for the longest of the eight sticks he has?

6. $F(1) = 1, F(2) = 3$, and $F(3) = 6$, find a formula for finding the n^{th} triangular number.

7. Find the sum of the integers 2 to 298.

8. The sum of a number, its square, and its square root is 276. Find the number.

9. A prime number is called a 'Superprime' if doubling it, and then subtracting 1, results in another prime number. What is the number of Superprimes less than 15?

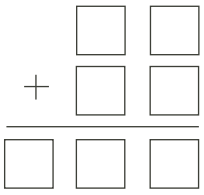
10. A number greater than 1 is said to be a prime number if its only factors are 1 and itself. Determine the smallest perfect square that has three different prime numbers as factors.

11. The two prime factors of a number are greater than 12 but less than 25. The number is greater than 300 but less than 350. What is the number? What are the factors?

12. Kelsey works at the local bird store. She has 100 birds to put in cages but no cage can have the same number of birds and each cage must contain at least one bird. What is the maximum number of cages Kelsey could use to put the birds into?

13. I am a three digit number. My second digit is four times more than my third digit. My first digit is seven less than my second digit. What number am I?

14. In the addition of two 2-digit number each blank space including those in the answer, is to be filled with one of the digits 0, 1, 2, 3, 4, 5, and 6 used exactly once. What is the ones digit of the sum?



Problem Set Solutions

1. 112
2. 89
3. x=1287, y=3003, z=12870
4. 1123
5. 21
6. $\frac{n(n+1)}{2}$
7. 44550
8. 16
9. 3
10. 900
11. Number=323. Factors=$19, 17$.
12. 13
13. 182
14. 5