University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 6 Math Circles
Mar.7th, 2012
Measurement and Geometry

This class we'll focus on problem solving in traditional (Euclidean) geometry.

Why? Because it's a lot of fun. Geometry also has a lot uses in our real world problems such as home hardware design, architecture, and general construction. Plus, you get this really good feel of satisfaction after you solve a very challenging problem, like, "Oh! That's how it is! Now I get it!" sort of feeling.

First let us review some basic geometry concepts and formulas.

Formulas:

Area of Triangle: $A = (b \times h) \div 2$

Area of Square, Rectangle, Parallelograms: $A = b \times h$

Area Trapezoids: $A = [(t + b) \times h] \div 2$

Area of Circle: $A = \pi r^2$

Perimeter of Circle: $P = 2\pi r = d\pi$

Strategy for doing geometry problems:

In general, when doing geometry problems, you should try to:

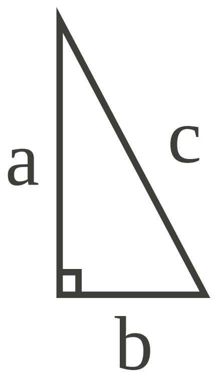
1. **Interpret the question.**
2. **Draw a diagram of the shape described if the diagram isn't given to you.**
3. **Do not rely solely on the diagram given to you if they do give you a diagram. They are usually not to scale and some are misleading.**
4. **Look for any clues that would give you the measurement that you require to further your solution to the problem.**

Today, we will learn a several very useful theorems in geometry that will be constantly used in problem solving. Additionally, we will explore some very interesting problems in measurement.

There is a very important theorem in geometry that deals with properties of right-angled triangles that everyone should know since grade 1:

The Pythagorean Theorem:

On any right angled triangle, the sum of the square of the two sides (or base and side, you may call it that) is equal to the square of the hypotenuse.



$$a^2+b^2=c^2$$

Practice 1: Given that a right-angled triangle has base 3 and height 4, find its hypotenus.

$$\begin{aligned} 3^2+4^2 &= hypotenuse^2 \\ 25 &= hypotenuse^2 \\ hypotenuse &= \sqrt{25} \\ hypotenuse &= 5 \end{aligned}$$

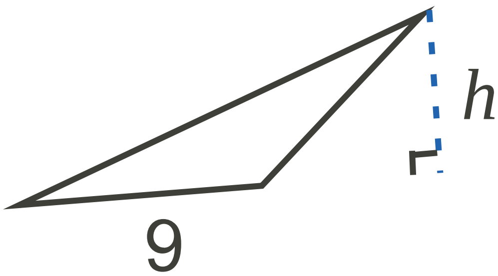
Practice 2: Given that a right-angled triangle has hypotenus 20, one of the side 8, find the length of the other side.

$$\begin{aligned} 20^2-8^2 &= side^2 \\ 336 &= side^2 \\ side &= \sqrt{336} \\ side &\approx 18.33 \end{aligned}$$

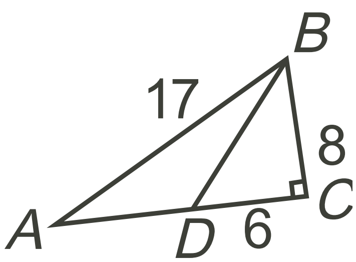
Area of Triangle:

A very important aspect to remember when calculating the area of a triangle is to find the height and the base. The height of a triangle given a chosen base is the perpendicular to that base, touching the corner opposite the base. Which one is the height of this triangle?

Example 1:
Where is the height of this triangle?



Example 2:
Determine the area of $\triangle ABC$ and $\triangle ABD$:



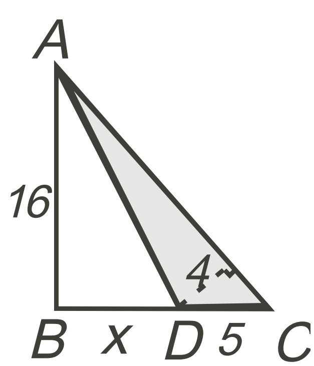
First, we need to find the length of AC. By the Pythagorean Theorem:

$$AC = \sqrt{17^2 - 8^2} = \sqrt{225} = 15$$

$$\begin{aligned} Area\triangle ABC &= \frac{15 \times 8}{2} \\ &= 60 \end{aligned}$$

$$\begin{aligned} Area\triangle ABD &= \frac{(15-6)\times 8}{2} \\ &= \frac{9 \times 8}{2} \\ &= 36 \end{aligned}$$

Example 3:
Determine the measure of x, given that the area of the small shaded triangle is 40.



We know that the area of $\triangle ADC$ is 40. The area of a triangle is calculated by $\frac{base \times height}{2}$. My height is 4, so substituting these into the area formula, I get that my base is $(40 \times 2) \div 4 = 20$. So line $AC = 20$. Now, by the Pythagorean Theorem,

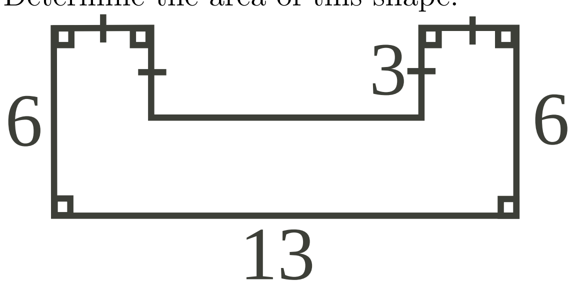
$$BC = \sqrt{20^2 - 16^2} = \sqrt{144} = 12$$

Therefore, we have that $x = 12 - 5 = 7$.

Combining Geometric Shapes

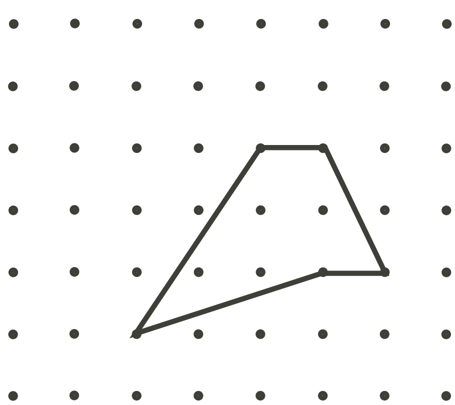
We'll do a several examples of mixed geometry of irregular figures. The key to doing these problems is to divide up the irregular object into figures that you are familiar with, then use the formulas that you know, calculate what is required.

Example 4:
Determine the area of this shape.



$$\begin{aligned} Area &= 3 \times 3 + 3 \times 3 + 13 \times 3 \\ Area &= 57 \end{aligned}$$

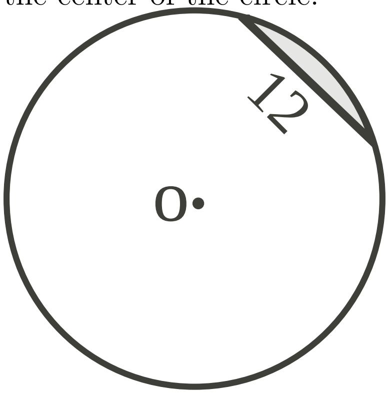
Example 5:
Determine the area of this shape.



$$\begin{aligned} Area &= 3 \times 4 - \frac{2 \times 1}{2} - \frac{3 \times 2}{2} - \frac{(1+4) \times 1}{2} \\ Area &= 12 - 1 - 3 - \frac{5}{2} \\ Area &= \frac{11}{2} \end{aligned}$$

Example 6:

Determine the perimeter around the shaded region, given that the circle has radius $r = 12$. O is the center of the circle.



Extend a chord from the center O to the two ends of the shaded region to get that both of these chords are the radius and thus have length 12. Then I have an equilateral triangle (a triangle whose 3 sides have equal length). Therefore, the inner angle $\angle O$ is $60°$. This means the arc that surrounds the shaded region has $\frac{1}{6}$ the length of the circumference.

The circumference is: $2 \times 12\pi = 24\pi$

The arc surrounding the shaded regions has length: 4π. Therefore, the perimeter around the shaded region is $12 + 4\pi$.

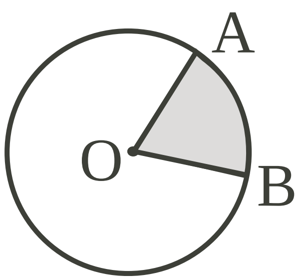
Exercises:

1. A farmer has a rectangular field whose length is 200m and width 150m. His home and barn are located diagonally cross-corner on two ends of the field. Every summer, because the farm field is flourishing with crispy delicious vegetables, the farmer has to walk around the outskirts of the farm to get from the barn to home. But in the winter, the field is a patch of snow covered dirt and so the farmer walks across the farm field, taking the closest path possible from the barn to home.

   (a) How much more does he have to walk in the summer than in the winter in one trip from the barn to home?

   (b) The farmer goes to the barn from home in the morning and comes back from the barn to home at night everyday. Assuming there are 70 days of summer and 70 days of winter, how many more kilometers does the farmer walk in the summer than in the winter?

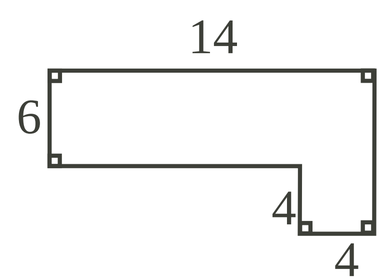
2. This circle has diameter 10. O is the center of the circle.



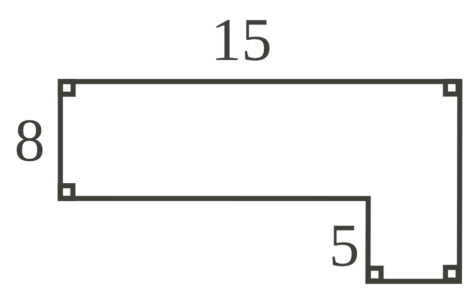
   (a) If the shaded region has 20% of the area of the circle, determine the area of the shaded region.

   (b) Determine the measure of $\angle AOB$.

3. What is the perimeter of...

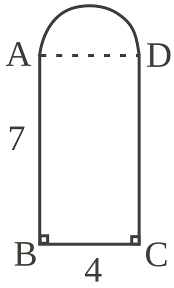


   (a) This figure?



   (b) This figure?

4. Given that the arc DA is a half circle (semicircle). What is...



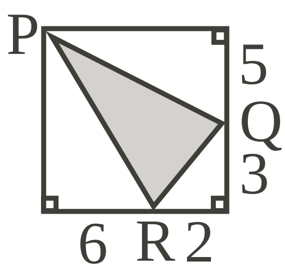
   (a) The perimeter of this figure?
   (b) The area of this figure?

5. Find the area of a trapezoid with bottom length 12, top length 6 and height 3.

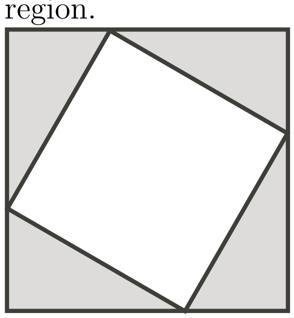
6. Find the area of a parallelogram with base length 18 and height 8.

7. Find the square that has the same area as the parallelogram from previous question. What is the side length of this square?

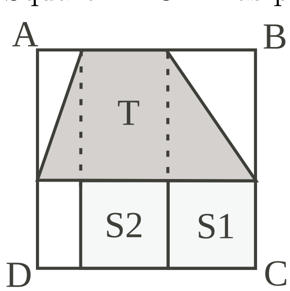
8. Find the area of triangle PQR.



9. A square of perimeter 32 is contained in a square of perimeter 48. Find the area of the shaded region.



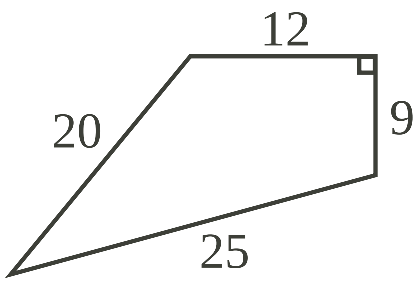
10. Square $ABCD$ has perimeter 72. The squares $S1$ and $S2$ are equal and both have area 64.



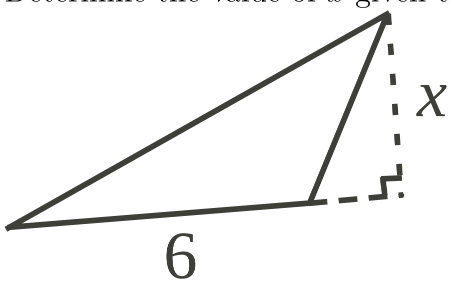
(a) What is the area of trapezoid T?

(b) What is the perimeter of trapezoid T?

11. Determine the area of this convex quadrilateral.



12. Determine the value of x given the area of the obtuse triangle is 9.



13. O is the midpoint of square $EFGH$. P is the midpoint of FG. Given that $\triangle HOP$ has area 9. What is area of the shaded region?