UNIVERSITY OF
WATERLOO
Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Grade 6 Math Circles
Working with Sets
February 5/6, 2013

Organizing

If you went home and opened up your fridge what would you see? Maybe orange juice, butter, some strawberries, ketchup and leftover pizza. You certainly wouldn't expect to see a dirty pair of socks or a computer in there! If you get thirsty you would go to the fridge to get some milk, not to your closet.

Organization is very important in life! We can use mathematical concepts to organize information. In the above examples, objects were organized based on whether or not they could be found in a refrigerator. We also knew that certain objects would not be found in some places (like your closet).

Suggest a rule that was used to make each group below. Add more objects to each group which follow your rule. There may be many correct answers.

Carrots	Grass
Potatoes	A four-leaf clover
Broccoli	Asparagus
______________	______________
Hannah	2
Herman	4
Haley	6

Sets

Writing Sets

You have already started working with *sets*. Sets just hold objects (like your refrigerator). The objects or *elements* of a set are what the set holds. Can you identify the elements in the above sets?

As you can see elements can be **anything**: numbers, food, people, TV shows, games, animals, ANYTHING!

You show something is a set by putting curly brackets '{' and '}' around a list of elements. To show the above examples were sets we would write: {carrots, potatoes, broccoli} and {2,4,6}.

This seems easy enough but what if you wanted to make a set of names of people who are in the room? It would take some time and a large part of your paper but it is doable.

Now make a set of everyone in the world! This would be much harder. So instead, we can write sets based on the rules of what elements are in the set. {everyone in the world} is much nicer than listing out 7 billion names! When reading, we say "*the set of* everyone in the world."

We can also give a name to sets. Often we just use a capital letter, but we can use words or symbols too. For example we could name the set of everyone in the world as W. We show this by saying W={everyone in the world}. Now we can talk about set W, which is much faster to say and write {everyone in the world}.

Size of a Set

The *size* of a set is the number of elements in the set. The size of F={people in my family} would be five, since there are five people in my family. A line on both ends of a set means "what is the size of this set?" $|F|$=5 is saying, "what is the size of set F" which is 5.

What is the size of each of the following sets?

1. $|\{$letters in the alphabet$\}|=$______
2. $|\{$people in your family$\}|=$______
3. $|\{$your favourite colours$\}|=$______
4. $|\{$people in this room who have green hair with purple and orange polka-dots$\}|=$______

Hopefully everyone got the same answer for question 1. For questions 2 & 3 there probably were many different answers. What about question 4? Try writing the set by listing all its elements. It should look like this: { } since there is no one that belongs in such a set. This set has a special name; it's called the *empty set*. We can also write the empty set as $\emptyset$ (0 with a line through it).

There is one other special size to look at. What is $|\{$odd numbers$\}|$? This set never ends so we say it is infinitely large (shown by ∞)! It is possible to write this set another way. We start by writing the first few elements of the set. Then we show that the pattern continues using three dots. So {odd numbers} = {1,3,5,7,...}. We do this when a *pattern* exists with the elements, and to show that there are many more elements which belong to the set.

Elements of a Set

When an element is in a set we say it is a *member* of the set. We use the symbol $\in$ to say the element is a member of a set. For example, Dumbledore $\in$ {Characters from Harry Potter}. We also can show an element is not a member of a set using $\notin$.
Scooby Doo $\notin$ {Characters from Harry Potter}.

We say two sets are *equal* if all the elements are members of both sets.

Suppose you make a set of food in your fridge by listing everything you see starting at the top shelf and working your way down. Then I come by, and make my set of food in the fridge listed alphabetically. Are these two sets equal? Be sure to remember the definition of *equal*.

As you can hopefully see, in a set the order of the elements *does not matter*! So {11,4,7,3,}={3,4,7,11}. Usually some sort of order (alphabetical, lowest to highest, etc.) is nice when writing a set, but is not required and it has no meaning!

Elements cannot be repeated in a set. Either an element is in a set once or not at all. For example, suppose you have three cartons of milk. In the set of things in your fridge, milk can only be listed once even though there is actually more than one carton.

However, if two items are similar but have different names then both items may be listed once each. For example, a carton of milk and a bag of milk are different items and may both be listed in a set if needed.

If two people have the same first name, we could list both first and last names in {People at Math Circles} so that |{People at Math Circles}|= the number of people present at Math Circles.

Using Sets

With numbers we can do more than just count; there are important operations we use on multiple numbers. There are also operations we can use to compare sets and make new sets.

Union

There are 11 students in Ms. DaGoal's grade 6 class. The following sets were made:
H={students who play hockey}={Tom, Becky, Nathan, Sarah, Billy, Fred}
S={students who play soccer}={Susan, Kyle, Matt, Becky, Peter, Fred}

What do these sets tell you about Sarah? Peter? Becky? What about William?

Now make set A, of students who play at least one of soccer **or** hockey (or both). You can easily look at both sets to find A={Tom, Becky, Nathan, Sarah, Billy, Fred, Susan, Kyle, Matt, Peter}.

But what if you're dealing with sets with hundreds or thousands of elements? We can write A as the *union* of H and S. The union takes all the elements from H and all elements from S, and puts them into one set. Remember elements cannot be in a set twice! We write the union as '$\cup$'. So $A = H \cup S$.

You can also think of union as '**or**'. If an element is in H **or** S **or** both, then it belongs in A.

Intersection

Using the hockey and soccer example from above, what do you notice about Becky and Fred?

We can create a new set B, of students who play both hockey **and** soccer. We know that B={Becky, Fred}. We write the intersection of two sets using '$\cap$'. So $B = H \cap S$.

Another way to think of intersection is '**and**'. If and element is a member of H **and** S, then it belongs in B.

Universe

In the sports example above, notice how we were only looking at the members of one class. We weren't interested in everyone who plays hockey in the world, just those in Ms. DaGoal's grade 6 class. *'Students in Ms. Dagoal's grade 6 class'* is the *universe* of the problem.

The *universe* is the largest possible set of elements you wish to consider for a specific problem. It is usually shown by a fancy U, $\mathbb{U}$. If Jacques is not in Ms. DaGoal's class, it doesn't matter if he plays hockey or soccer, he is not part of the universe so he cannot be in sets H or S.

Example: All the students in grade 6 at Waterloo P.S. fill out an anonymous survey. The survey asks "What is your favourite fruit?" The grade 8 student that collects the surveys will create a set containing all the unique (different) answers. What is the universe set, $\mathbb{U}$ of the survey?

Complement

What happens to all the students in Ms. DaGoal's class who do not play hockey? They are in $\mathbb{U}$ but not in H. The *complement* of set H, written $\overline{H}$, is the set of all elements in $\mathbb{U}$ (the universe) that are not in H.

You can think of compliment as '**not**'. H={students who play hockey} and $\overline{H}$={students who do **not** play hockey}. So $\overline{H}$={Susan, Kyle, Matt, Peter, William}.

For the following problems, write each set as a list of all it's elements or just write the name of the set if it has one.

What is $\overline{S}$?

How about $\overline{\mathbb{U}}$?

What is $\overline{H} \cup H$?

How about $\overline{S} \cup S$?

What is $\overline{H} \cap H$?

You have learned the basics of using sets. If you look all around you, sets are everywhere! This week, look for ways sets are used in the world and try to find the most unique ones! At the start of next class I'd love for you to share what sets you saw.

Problems

1. For the given rules, make sets showing some of the elements from each.
   (a) Games you play.
   (b) Teachers at your school.
   (c) Multiples of 3.
   (d) Words with more than 10 letters.

2. Rewrite each set according its rule. There may be many correct answers. Add 3 more elements to each set.
   (a) {piano, trumpet, flute, clarinet,...}
   (b) {exit, xylophone, Trix, lynx,...} (Trix are for kids!)
   (c) {swimming, tree, deer, Aaron, pizza,...}
   (d) {21, 49, 28, 84, 14,...}

3. Is the following statement true? {2,4,6,8}={2,4,6,8,...}. Explain.

4. What is the size of each set?
   (a) |{people in this room right now}|
   (b) |{items you brought with you}|
   (c) |{1, 4, 9, 16, 25, 36, 49,...}|
   (d) |{number of math circles classes there will be}| (Think carefully about this one)

5. Fill in the blanks with either $\in$ or $\notin$ so the statement is true.
   (a) Batman ____ {Superman, Iron-Man, The Hulk,...}
   (b) Penguins ____ {Animals that live in the Arctic}

(c) Monopoly ___ {Games you've played}

(d) 1 ___ {Sum of values on two rolled dice}

6. Find some of the elements of the resulting set for the following:

   (a) A = {countries you've visited}$\cap${countries Mr. Auckland has visited} (Find all the elements)

   (b) B = {people with the same birth-month as you}$\cup${people with their birthday on the same day of the month as you}

   (c) C = {odd numbers}$\cap${numbers divisible by 3}

   (d) D = {prime numbers}$\cup${composite numbers}

7. What is the universe, $\mathbb{U}$, and compliment of each set.

   (a) {cities/towns you've ever lived in}

   (b) {shapes with a 27° angle}

   (c) {people in this room with brown hair}

   (d) {whole numbers}

8. Let A={Carter, Ashley, Patrick, Kelly, Helen}, B={Shawn, Stephanie, Helen, Juan, Kelly}, C={Jacques, Helen, Steven, Stephanie, Carter}, and D={Ashley, Patrick, Juan, Terry, Jacques}.

   (a) What is $A \cup B$? What is $A \cup B \cup C$? What is $A \cup B \cup C \cup D$?

   (b) If $\mathbb{U} = A \cup B \cup C \cup D$, what is $\overline{A} \cap D$?

   (c) What is $|(A \cap B) \cup (C \cap D)|$? Evaluate the sets in brackets first.

   (d) What is $\overline{\overline{D}}$?

   (e) What is $\overline{A} \cap \overline{B}$ and $\overline{(A \cap B)}$? For the second part, evaluate the intersection first and then find the complement.

(f) What is $(\overline{B} \cap \overline{C}) \cap (A \cup D)$? Evaluate the sets in brackets first.

9. CHALLENGE 1: Using the sets from question 8, make an equation using A, B, C, D, complements, unions, and intersections so that the resulting set is {Shawn, Terry, Kelly}.

10. CHALLENGE 2: You all have worked with sets before. Venn Diagrams use sets. Draw a two circle Venn Diagram. One circle represents set C and the other represents set D.

    (a) What do you suppose the shared portion represents?

    (b) Fill in your Venn Diagram using sets C and D from question 8. Now what parts represent $\overline{C}$, $\overline{D}$, and $C \cup D$?

    (c) Where would names like Kelly or Shawn go? Put *all* the names in the appropriate location on your Venn Diagram. Does this change your answer for part b?

    (d) Now draw a three circle Venn Diagram, representing sets A, B, and C. Put all names in the proper location.