UNIVERSITY OF
WATERLOO
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# Senior Math Circles

November 5, 2013

## *Surfaces - Problem Set 2*

Questions with a * are important.

1. Make a list of all the surfaces you can think of that aren't equivalent. Can you make a list of all of them? How do you know any two on your list aren't equivalent?

2. Find a planar model for the two holed torus.

3. Find a way to represent connected sum using planar models. For example, can you find a planar model for $T\#T$? How about $KB\#PP$ or $PP\#PP$. How about any two surfaces in general?

4. Calculate the Euler characteristic of the following surfaces.

   (a) The Klein bottle.
   (b) The projective plane.
   (c) $PP\#PP$.
   (d) $KB\#PP$.
   (e) $T\#PP$.
   (f) A two holed torus.
   (g) *An $n$-holed torus.

5. *Suppose that $S$ and $T$ are two surfaces. Find a formula for $\chi(S\#T)$ in terms of $\chi(S)$ and $\chi(T)$.

6. *Is $KB$ equivalent to $PP\#PP$? Why or why not? Is $KB\#PP$ equivalent to $T\#PP$? Why or why not?

7. What happens to a surface when you take its connected sum with a sphere? Can you prove your answer?

8. Decide whether or not the following are orientable.

   (a) A Klein bottle
   (b) $T\#T$.
   (c) *$T\#T\#\cdots\#T$ (that is, $n$ tori connected summed together).
   (d) $PP\#T$.
   (e) $KB\#T$.
   (f) *$PP\#A$ for any surface $A$.

9. How can you tell by just looking at the word of a surface whether or not it is orientable? Is $abca^{-1}b^{-1}c^{-1}$ orientable? How about $abcab^{-1}c^{-1}$?

10. *Make a list of all your conjectures about surfaces and keep them safe. Light will be shed on a lot of things by the end and you can see if your guesses are correct.

11. Do any questions from last week's sheet.