



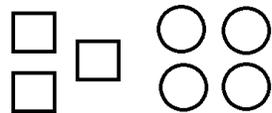
Grade 6 Math Circles
November 18/19, 2014
Ratios and Rates

Ratio

What is a ratio? A ratio is a comparison of two things. We can use ratios to compare the number of girls to boys, the number of people wearing shorts to people wearing pants or anything else really. When writing down a ratio mathematically we write:

(The number of some item):(The number of some other item)

So looking at:



Here we have 3 squares and 4 circles. So we can say there are 3 square to 4 circles or the ratio of squares to circles is 3:4

Some times we can have large numbers in a ratio that can be simplified to smaller numbers. For example:



The the figure above the ratio of stars to triangles is 6:9. However, some of you might have noticed that we can group these into smaller groups of 2 stars and 3 triangles. This gives us a ratio of 2:3.

We can also see this mathematically. We have the ratio 6:9. What are the factors of 6? 1,2,3,6. What are the factors of 9? 1,3,9. Both these numbers share a common factor of 3. This means that in our ratio of 6:9 we can divide both numbers by 3 and the ratio stays the same. $6 \div 3 = 2$ and $9 \div 3 = 3$. This now give us our new ratio of 2:3.

Example: Simplify the following ratios

- $12:20 = 3:5$
- $80:40 = 2:1$
- $21:28 = 3:4$
- $48:144 = 1:3$

Rate Example: Unit Cost Method

You decide you want to make some cookies. The recipe calls for 2 cups of chocolate chips and 3 ounces of milk chocolate. Because chocolate is so good you decide to put in 5 cups of chocolate chips. How many ounces of milk chocolate will you need so that the ratio of chocolate chips to milk chocolate is the same as in the recipe?

Solution

How do we solve this? First we will look for the **unit cost**. That is how many ounces of milk chocolate for each 1 cup of chocolate chips.

Our ratio: 2 cups of chocolate chips : 3 ounces of milk chocolate (2:3)

To find the unit cost of a cup of chocolate chips we will divide both sides by 2 since our initial ratio has 2 cups.

Unit cost ratio: 1 cup of chocolate chips : 1.5 ounces of milk chocolate (1:1.5)

In the question originally we wanted the number of ounces for 5 cups. since we have a unit ratio, we can multiply both sides of this ratio by 5 to get our answer.

Answer 5 cups of chocolate chips : 7.5 ounces of milk chocolate. Therefore we can say that we need 7.5 ounces of milk chocolate to maintain the ratio from the recipe

Similar Problem, Different Method: Cross Multiply

Your parents send you to the store to buy 4 pounds of apples. When you get to the store the sign says they cost 3 dollars for every 6 pounds. How much will the 4 pounds cost you?

What is the ratio? 3 dollars : 6 pounds

What ratio do we want? x dollars : 4 pounds

Cross Multiply Method

The method we are going to use involves writing the ratios as fractions.

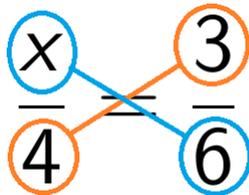
So we will write 3:6 as $\frac{3}{6}$

Similarly we will write $x:4$ as $\frac{x}{4}$

Now we want to solve

$$\frac{x}{4} = \frac{3}{6}$$

To cross multiply we take the numbers on the denominator and multiply them with the numerator on the opposite side:



From here we have $6x = 12$ so we divide both sides by 6 to get $x = 2$

Your parents send you to the store to buy 4 pounds of apples. When you get to the store the sign says they cost 3 dollars for every 6 pounds. How much will the 4 pounds cost you?

Well we just showed the answer is 2. So we can say it will cost you 2 dollars to buy the 4 pounds of apples.

Here is what we did summarized.

Example Summarized

Your parents send you to the store to buy 4 pounds of apples. When you get to the store the sign says they cost 3 dollars for every 6 pounds. How much will the 4 pounds cost you?

Given ratio: 3 dollars : 6 apples

Target ratio: x dollars : 4 apples

Set up as fractions: $\frac{x}{4} = \frac{3}{6}$

Solve for x : Cross multiply and divide to get $x = 2$

An other example:

A recent study found that for every kid that plays baseball in Canada there are 3 kids who play Soccer. If there are 42 kids who play soccer, how many are playing baseball?

Ratio:

Unit cost ratio:

Answer

One more example

The ratio of Taylor Swift fans to Queen fans is 20:4. If there are 50 Taylor Swift fans, how many Queen fans are there?

Ratio:

Unit cost ratio:

Answer

So far we looked at ratios and how to solve questions involving them. Now we will look at **rates**. A rate is very similar to a ratio except it has a time element. For example if Joe was running at 6 m/s we have a rate of 6 meters per second. We can also see this as a ratio 6 meters : 1 second. We solve rates much the same way we solve ratios find the rate for a single unit, then multiply by the number of units we want.

Example:

Its summer and you want to fill your pool. If you can fill your pool at a rate of 8 liters every 3 minutes. How long will it take to fill your pool with 10 000 liters of water?

Rate: 8 liters : 3 minutes

How many minutes per liter? $3 \div 8 = 0.375 \Rightarrow 1 \text{ liter} : 0.375 \text{ minutes}$

How much time for 10 000 liters?: $10000 \times 0.375 = 3750 \text{ minutes}$

Another Example

Flash decides to run a marathon. If he moves at 240 meters per second how long will it take him to finish? (Note: a marathon is 42 km)

Rate: 240 meters per second

How many second to move 1 meter?: $1 \div 240 \approx 0.004 \text{ seconds per meter}$

How many meters in 42 km? There are 1000 meters in a km, so we have 42000 meters

Answer: $0.004 \times 42000 = 168 \text{ seconds or } 2 \text{ minutes and } 48 \text{ seconds.}$

Multiple Rates

Tails thinks he can out run Sonic the Hedgehog so he Sonic challenges to a race. If Sonic can run at 25 meters per second and Tails can run at 21 meters per second. After a minute and a half how far apart will they be?

Solution

Here we have two rates, one for Sonic and one for Tails. How do we answer this? We could

calculate the distance covered by Sonic and then the distance covered by Tails and subtract the two. however there is a faster way.

If we look at the difference between their speeds, we may notice this the rate at which they are separating.

This means the distance between Sonic and Tail is increasing by $25 - 21 = 4$ meters per second.

Rate: 4 meters per second

How many seconds in a minute and a half? there are 90.

Answer We can say that Sonic will be $4 \times 90 = 360$ meters apart.

Something Different

Tom can paint a house in 5 hours. His friend Andy can paint a house in 8 hours. If they work together, how long will it take them to paint a house?

Solution: How do we approach this one? First lets find how much of a house each of them paints in 1 hour.

Tom: 5 hours for 1 house means $1 \div 5 = 0.2$ houses per hour.

Andy: 8 hours for 1 hours means $1 \div 8 = 0.125$ houses per hour.

Rate: Together they paint $0.2 + 0.125 = 0.325$ houses per hour. (0.325:1). however we want the number of hours per house.

Answer: $1 \div 0.325 = 3.08$ hours to paint a house.

PROBLEMS

1. Draw a line from the right side to the left side matching up equivalent ratios.

- | | |
|--------|----------|
| • 2:3 | • 18:16 |
| • 21:4 | • 63:12 |
| • 9:8 | • 75:100 |
| • 3:4 | • 24:36 |

Answers in pairs (R,L) where R is the row for the right column and L is row for the left column. (1,4), (2,2), (3,1) , (4,3)

2. As we all know a cow has 4 legs. You decide to go cow counting but they are behind a fence and you can only see their legs under the fence. You count 88 legs. How many cows are there?

There are 22 cows

3. Jimmy and Timmy went trick or treating together. After 6 houses they have 9 pieces of candy. If they go trick or treating at 48 houses how many pieces of candy can they expect to get?

House to candy ratio 6:9.

We want 48:x.

They will have 72 pieces of candy

4. Bob and Rob, friends of Jimmy and Timmy, go trick or treating together. After 8 houses they have 11 pieces of candy. If they were to trick or treat at the same number of houses as Jimmy and Timmy who would have more candy?

House to candy ratio 8:11.

We want 48:x.

They will have 66 pieces of candy.

Jimmy and Timmy will get more candy than Bob and Rob

5. At store A you can buy 4 bananas for 5 dollars and at store B you can buy 5 bananas for 6 dollars. which store has cheaper bananas?

Store A: Banana to Dollar ratio 4:5. Unit cost 1:1.25

Store B: Banana to Dollar ratio 5:6. Unit cost 1:1.2

Bananas at store B are cheaper.

6. There are 15 boys and 21 girls in a class. What is the ratio of girls to boys?(Remember to simplify the ratio if you can)

The ratio of girls to boys is 21:15 and can be simplified to 7:5

7. A ranger wants to know how many deer are in his park. He catches 50 deer and tags them all of them. A few days later he captures 30 deer randomly and 10 of them are tagged. Approximately how many deer are in the park?

Initially the ranger catches and tags 50 out of the whole population of deer. Let call the population x , so the ratio of tagged to all deer is $50:x$. Later when we catches 30 randomly there are 10 tagged so the ratio of tagged to population is $10:30$. so we get

$$\frac{50}{x} = \frac{10}{30}$$
$$x = 150$$

8. In a class the ratio of boys to girls is 3:5. If there are 32 kids in the class, how many boys and girls are there?

If the ratio of boys to girls is 3:5 that means the ratio of boys to other kids is 3:8. This can be seen as for every 8 kids there are 3 boys. From here we solve :

$$\frac{x}{32} = \frac{3}{8}$$
$$x = 12$$

Therefore there are 12 boys in the class and $32-12=20$ girls in the class.

9. Ted runs at a speed of 5 meters per second. Fred runs at a speed of 16 meters every 3 seconds. Who runs faster?

Ted's speed is already at a unit cost so lets look at Fred's.

If he runs a 16 meters : 3 seconds that means he runs at

$16 \div 3$ per second. Since $16 \div 3 = 5\frac{1}{3} > 5$ we say Fred runs faster than Ted.

10. Two cars leave the same point in opposite direction one traveling at 90 km/hr and the other at 115 km/hr. How far apart are they after 2 hours?

The two cars are separating at a speed of $90 + 115 = 205$ km/hr. After 2 hours they will be $2 \times 205 = 410$ km apart.

11. You decide one day you want to fill up a 5 liter bucket using a mini water-gun. The water-gun can fill the bucket at 2 milliliters per second (0.002 L per second). How long will it take you to fill the bucket?

We want to solve: $\frac{0.002}{1} = \frac{5}{x}$

$$x = \frac{5}{0.002}$$

$$x = 2500 \text{ seconds or } 41 \text{ minutes and } 40 \text{ seconds}$$

12. Jimmy and Timmy are back and they want to build a Lego spaceship. Jimmy can build the spaceship in 3 hours and Timmy can build it in 4 hours. If they work together how fast can they built the spaceship?

First we must find how much of the ship each can make in 1 hour.

Jimmy can make $\frac{1}{3}$ of the ship in 1 hour

Timmy can make $\frac{1}{4}$ of the ship in 1 hour

Together they can make $\frac{1}{3} + \frac{1}{4} = 0.3333333 + 0.25 = 0.5833333$ of the ship in 1 hour

$$\frac{0.58333 \text{ ship}}{1 \text{ hour}} = \frac{1 \text{ ship}}{x \text{ hours}}$$

$$x = \frac{1}{0.583333}$$

They can build the ship in 1.7 hours working together.

13. * Molly and Holly help their mother plant a garden. Molly can plant a garden in 12 hours, Holly in 10 hours and their mother in 8 hours. Working together, how long will it take for them to plant a garden? (Note the lowest common multiple of 8,10,12 is 120. This will help if you chose to deal with the fractions.)

First find how much of a a garden they can plant in 1 hour.

Molly can plant $\frac{1}{12}$ of the garden in 1 hour

Holly can plant $\frac{1}{10}$ of the garden in 1 hour

Mom can plant $\frac{1}{8}$ of the garden in 1 hour

Together they plant $\frac{1}{12} + \frac{1}{10} + \frac{1}{8} = \frac{37}{120} = 0.30833333$

$$\frac{0.30833333 \text{ garden}}{1 \text{ hour}} = \frac{1 \text{ garden}}{x \text{ hours}}$$

$$x = \frac{1}{0.30833333} = 3.243$$

They can plant the garden in 3.243 hours working together.

14. ** If Joe can paint a house in 6 hours and Joe and Moe together can paint a house in 1.5 hours. How fast can Moe paint a house alone?

$$\begin{aligned} \text{Joe} + \text{Moe} &= \frac{1 \text{ house}}{1.5 \text{ hours}} \\ \frac{1 \text{ house}}{6 \text{ hours}} + \frac{1 \text{ house}}{x \text{ hours}} &= \frac{1 \text{ house}}{1.5 \text{ hours}} \end{aligned}$$

Together they can paint 0.66666 of a house in 1 hour or $\frac{2}{3}$ of a house

We know Joe paint one house in 6 hours. So he paints $\frac{1}{6}$ houses in 1 hour

if we call Moe's rate x we get:

$$\frac{1}{6} + x = \frac{2}{3} \text{ Now put everything on the same denominator.}$$

$$x = \frac{4}{6} - \frac{1}{6}$$

$$x = \frac{3}{6}$$

$$x = \frac{1}{2}$$

This means that Moe can paint $\frac{1}{2}$ of a house in 1 hour. Equivalently we can say that it takes Moe 2 hours to paint a house.

15. *** 7 men and 2 boys can finish a job in 4 days, while 5 boys and 7 women can finish the same job in 3 days. In how many days can 1 man, 1 boy and 1 woman working together finish the same job (assume constant rates each for men, women, and boys)?

There are many ways to approach this problem. I will present the unit cost method.

Let's call the group of 7 men and 2 boys, Group A. Group A takes 4 days to finish the job.

Group B is the 5 boys and 7 women. Group B takes 3 days to finish the job.

The key here is to find how much of a job each group finishes in 1 day.

Group A: 1 job : 4 days $\rightarrow \frac{1}{4}$ job in 1 day.

Group B: 1 job : 3 days $\rightarrow \frac{1}{3}$ job in 1 day.

If Group A and Group B work together, then they finish $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$ of a job in 1 day. But Group A and Group B together is 7 men, 7 women, (2 + 5 =) 7 boys.

So 7 men, 7 women and 7 boys finish $\frac{7}{12}$ of a job in 1 day. We want to find how

LONG it takes them to finish 1 entire JOB. So set up the ratio $1 : \frac{7}{12}$, and divide.

$1 \div \frac{7}{12} = \frac{12}{7}$. This means it takes 7 men, 7 women, and 7 boys $\frac{12}{7}$ days to finish 1 job.

How long does it take 1 man, 1 woman, and 1 boy? 7 men, 7 women, and 7 boys work 7 times FASTER than 1 man, 1 woman, 1 boy.

So it will take 1 man, 1 woman, and 1 boy 7 times as long, or $7 \times \frac{12}{7} = 12$ days.

Therefore, 1 man, 1 woman, and 1 boy take 12 days to finish the job.