Question 1

Consider the following graph in Figure 1.

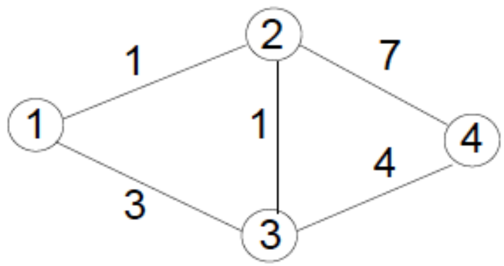

Figure 1: Shortest path exercise

(a) Use Dijkstra's algorithm to find the shortest $1-4$ path

(b) Find a set of feasible barriers that certify the optimality of the path found

Question 2

Consider the following linear program:

$$\begin{array}{llllllll}
\min & x_{12} & +3x_{13} & +x_{23} & +7x_{24} & +4x_{34} & & \\
\text{s.t.} & x_{12} & +x_{13} & & & & \geq 1 & \\
 & & +x_{13} & +x_{23} & +x_{24} & & \geq 1 & \\
 & x_{12} & & +x_{23} & & +x_{34} & \geq 1 & \\
 & & & & x_{24} & +x_{34} & \geq 1 & \\
 & x_{12} & & & & & \geq 0 & \\
 & & x_{13} & & & & \geq 0 & \\
 & & & x_{23} & & & \geq 0 & \\
 & & & & x_{24} & & \geq 0 & \\
 & & & & & x_{34} & \geq 0 &
\end{array} \tag{1}$$

Now for every 1-4 path in Figure 1, define variable $x_{ij} = 1$ if edge $\{i, j\}$ is in the path or zero otherwise. For example, for path $(1, 3, 4)$, we define $x_{13} = x_{34} = 1$ and all other x variables to be zero.

(a) Show that for every 1-4 path, if we define the x_{ij} variables as above, they are feasible for (1)

(b) Write down the dual of (1)

(c) (challenge) How can you use the barriers and weights from problem 1b to get a solution to the dual problem that has the same objective as the solution to (1) defined by the shortest path to the graph in Figure 1?

Question 3

Consider the following optimization problem:

$$\begin{array}{llllll}
\max & 3x_1 & +4x_2 & +x_3 & \\
\text{s.t.} & x_1 & +x_2 & +x_3 & \leq 6 \\
 & 4x_1 & +10x_2 & +5x_3 & \leq 30 \\
 & x_1 & & & \geq 0 \\
 & & x_2 & & \geq 0 \\
 & & & x_3 & \geq 0
\end{array} \tag{3}$$

(a) Write down the dual linear program

(b) Show that the optimal solution to (2) is $(x_1, x_2, x_3) = (5, 1, 0)$, by giving a certificate of optimality.