



Grade 6 Math Circles

March 3/4, 2015

Proportions

Powerful Proportions

Today, we're going to talk about different ways to express and apply *proportions*. A proportion is a relationship of one thing to another in terms of quantity, size, or number. Everything we will look at today is related and has a real-world application.

Warm-Up

Try to answer the following 11 questions in 4 minutes. You may only use a calculator for questions 9 through 11.

1. Express $\frac{20}{100}$ as a fraction in lowest terms.
 2. Express $\frac{20}{100}$ as a decimal.
 3. Express $\frac{20}{100}$ as a percent.
 4. Express 25% as a fraction in lowest terms.
 5. Express 0.02 as a fraction in lowest terms.
 6. Express $\frac{2}{3}$ as a ratio.
 7. If a \$100 coat is on sale where the discount is 10%, what is the sale price?
 8. Mike purchased a bike that cost \$120 plus 10% tax. How much did Mike pay?
 9. Ian makes a 15% commission on all sales. If he sells \$300 worth of product, how much commission did he make?
 10. Target Canada is closing its doors. Everything must go! During its clearance sale, Target sells a \$30 item for \$24. What was the discount rate?
 11. If 12 eggs cost \$1.20, how much do 8 eggs cost?
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Terms

A *ratio* is a comparison of quantities with the same units. They are expressed using the colon (:) symbol.

A *fraction* is a part of a whole. They are expressed using a fraction bar or a slash (/) symbol. The *numerator* is the number on the top of the fraction. The *denominator* is the number on the bottom of the fraction.

A *percent* is part per 100. Cent means 100 in French.

A *decimal* is a number based on 10 that may include a decimal point. In this lesson we consider a decimal to be a number with a decimal point.

Conversions

All four of the quantities mentioned above are closely related. It is important to know how to go from one representation to another.

Converting Decimals to Percentages:

Multiply the decimal by 100 or move the decimal two places to the right. Then put a percent sign after the number.

Converting Percentages to Decimals:

Divide the percent by 100 or move the decimal two places to the left. Make sure to remove the percent sign.

Converting Fractions to Decimals:

Divide the numerator by the denominator.

Converting Percentages to Fractions:

Multiply the percent by 100. This is the numerator. Now write the fraction bar and make the denominator 100. Now write this fraction in lowest terms. Make sure you've removed the percent sign.

Converting Fractions to Ratios:

Simply replace the fraction bar with a colon, and write the numerator to the left of the denominator.

Converting Ratios to Fractions:

Simply replace the colon with a fraction bar, and write the number to the left as the numerator and the number to the right as the denominator.

Try it out:

1. Convert $\frac{3}{5}$ to a ratio, percent, and decimal.
2. Convert 5% to a decimal, fraction, and ratio.

Mixing Problems

We can apply percentages to real-life scenarios in determining the concentration of solutions and determining the quantities of different objects or substances needed.

When solving these problems it is easiest to organize the information in a chart. This allows you to clearly see what information you have and need.

$$\left(\begin{array}{c} \text{Final Solution} \\ \text{Amount} \end{array}\right) \left(\begin{array}{c} \text{Percent of} \\ \text{Final Solution} \end{array}\right) = \left(\begin{array}{c} \text{Amount of} \\ \text{Solution 1} \end{array}\right) \left(\begin{array}{c} \text{Percent of} \\ \text{Solution 1} \end{array}\right) + \left(\begin{array}{c} \text{Amount of} \\ \text{Solution 2} \end{array}\right) \left(\begin{array}{c} \text{Percent of} \\ \text{Solution 2} \end{array}\right)$$

Example:

Jolene is creating 2050mL of fruit punch for her friend's birthday. She adds 1000mL of apple juice that is 84% concentrated. Jolene also adds 1050mL of orange juice. If the fruit punch has a concentration of 88% what is the concentration of the orange juice? Round your answer to the nearest hundredth.

Solution: Let c be the concentration of the orange juice.

$$\begin{aligned} (2050)(0.88) &= (1000)(0.84) + (1050)c \\ 1804 &= 840 + (1050)c \\ 964 &= (1050)c \\ c &= \frac{964}{1050} \\ c &\doteq 0.9181 = 91.81\% \end{aligned}$$

The concentration of the orange juice is about 91.81%.

Try it out:

Gary makes a 100mL solution in chemistry. He adds 25mL of a 63% acidic solution and 75mL of a 43% acidic solution. How acidic is the final solution?

Discounts/Markdowns

When new products come in, stores want to clear the older products. To make the older products more attractive, stores lower the price. The lowering of the price of products is called a discount (a percentage), or markdown (a dollar amount). With discounts, a percentage is taken off.

Example:

Suppose we have a 15% discount on a \$463 suit. How much money do you save? What is the discounted price?

Solution:

Start by converting the percent to a decimal. Well, $15\% = 0.15$.

Now we can find the discount. This is simply 463×0.15 . Thus, the discount is \$69.45.

\therefore you save \$69.45.

The discounted price is $463 - 69.45 = 393.55$.

\therefore the discounted price is \$393.55.

Alternatively, you can directly find the discounted price.

Since the discount is 15%, the final price is $100\% - 15\% = 85\%$ of the original cost.

\therefore the discounted price is $463 \times 0.85 = \$393.55$.

Try it out:

1. Sajeev wants to buy a bike whose price tag is labeled \$275. There's a blowout sale at the bike store today. All bikes are 8% off. How much will Sajeev have to pay for the bike today?
2. Jiminy wants to buy a tiny violin to play during sad occasions. Normally, tiny violins cost \$40. But today, there's a worldwide sale on tiny violins. Jiminy saved \$12. What was the discount percentage?
3. Johann wants buy some potatoes. **After** a 17% discount, his potatoes come to a total of \$4.98. How much were they before the discount?

Commission

In order to motivate sales, companies pay employees by commission. This means that employees get paid a certain amount (usually a percentage) for each product they sell. The percentage at which the employee gets the commission is called the *commission rate*. The more products you sell, the more you get paid. People who get commission usually get an hourly rate as well (just in case they aren't able to sell anything).

To encourage employees to sell even more products, companies can introduce a *graduated commission*. If an employee sells more than a certain amount, the commission rate increases, so that the employee gets more pay per sale.

Example:

Lebron sells basketballs. He gets 1% commission on the first \$2000 of his sales, 3% on the next \$3000 of his sales, and 5% for any more sales after that. How much commission will he receive if he sells \$4000 worth of basketballs?

Solution:

Since Lebron sold more than the \$2000, the first commission level is "filled up". That means there is $4000 - 2000 = \$2000$ remaining to fill up the second commission level.

$$\begin{aligned}(2000)(0.01) + (2000)(0.03) &= 20 + 60 \\ &= 80\end{aligned}$$

\therefore Lebron will receive \$80 if he sells \$4000 worth of basketballs.

Try it out:

How much commission will Lebron receive if he sells \$6500 worth of basketballs?

Taxes

Taxes are a real-life application of percentages. When using percentages, it is important to remember their relationship with decimals and fractions.

When you buy something from a store in Ontario, you must pay the price on the tag, *plus* tax. This is an extra amount stated by the government in terms of a percentage. Currently, we have a Harmonized Sales Tax (HST) in Ontario of 13% on most items. Certain items, such as books, only have a 5% tax. The moral of this story is that you should spend more time reading!

Let's do an example. Fleur wants to buy some flowers. The price tag says \$5.00. The tax where Fleur lives is 10%. How much will she have to pay?

To solve this problem, recall that $10\% = 0.1 = \frac{1}{10}$. Multiplying by one tenth is the same as dividing by 10. You should understand that multiplying or dividing by 10 results in simply moving the decimal place. This is thanks to the fact that we work in the *decimal* system! You may have also heard it being called "base 10" in your math class. So, $5.00 \times \frac{1}{10} = 0.50$. But we are not done yet! We have only found that the tax itself is \$0.50. We must add this to the original price to find the total cost. Thus, $\$5.00 + \$0.50 = \$5.50$. So, Fleur must pay \$5.50.

This problem was pretty easy to solve without a calculator since the tax was 10%. What if the tax was 15% or 13% (like in Ontario)? For the sake of simplicity, you may use a calculator to solve tax problems when the tax is not 10% in this lesson.

There is an alternate approach. For example, let the tax be 13%. Well, this means the final cost of the item is $100\% + 13\% = 113\%$ of the pre-tax cost. Thus, you can multiply the pre-tax cost by 1.13 to find the after-tax price.

Try it out:

1. Tyrion wants to purchase bubbles. The price tag says the bubbles cost \$15.60. The tax where Tyrion lives is 13%. How much Tyrion will have to pay?
2. Arya wants to buy a soccer ball. The price tag says the ball costs \$160. If the tax where Arya lives is 27%, how much does she have to pay?
3. Jaimie buys a microwave for \$48 (after tax). If he pays \$8 tax, what is the tax rate?

Unit Price

The unit price (also known as the unit cost) tells you the cost per one unit of a quantity or measure. The technique to find the unit price is straightforward. You divide the cost by the quantity.

Comparing unit prices can be a good way of finding the best buy.

Example:

Which is the better deal?

- (a) 10 chocolates for \$40.00
- (b) 6 chocolates for \$27.00

Solution:

$$40.00 \div 10 = \$4.00 \text{ per chocolate}$$

$$27.00 \div 6 = \$4.50 \text{ per chocolate}$$

\therefore (a) is a better deal.

Try it out: Which is the best deal?

- (a) 2 litres of milk for \$3.80
- (b) 1.5 litres of milk for \$2.70
- (c) 0.75 litres of milk for \$1.20

Wrap-Up

Today, you looked at the relationships between different ways to express proportions. You also examined just how useful they can be in real life situations.

Problem Set

Complete the following 18 problems. You may use a calculator.

1. Convert $\frac{15}{16}$ into a:
 - (a) Ratio
 - (b) Percent
 - (c) Decimal
2. Convert 56 : 64 into a:
 - (a) Fraction
 - (b) Percent
 - (c) Decimal
3. Convert 177% into a:
 - (a) Fraction
 - (b) Ratio
 - (c) Decimal
4. For every 29 on-time flights between Chicago and San Francisco, there are 3 late flights. What percentage of flights are on time?
5. Ryan is exceptionally good at math. On his last test, he received a score of 214 out of 200 because there were bonus questions. What was his mark as a percentage?
6. Charlie's blood-juice level is 0.06. Unfortunately, Charlie is not allowed to ride a bike with this much juice in his system. A blood-juice level of 0.06 means that there are 60 mg of juice in every 100 mL of blood in his body. What is this ratio as a percentage?
7. With a blood-juice level of 0.06, Charlie is very uncoordinated. For every 5 metres he walks, he stumbles for 3 metres. If he moves 24 metres in total, what percentage of the distance did he stumble? What distance did he stumble?

8. Julian mixes a solution made of 100mL of pumpkin juice, which is 5% acidic, with 100mL of apple juice. If the resulting solution's acidity percent is 3 times the pumpkin juice, how acidic is the apple juice?
9. The sale price of a clock is \$15 after a 30% discount. What is the original price and markdown?
10. Crazy Cow discounted their toy cars by 60%. The markdown was \$42. What is the original price and sale price of the toy cars?
11. Akshayaa works at a toy car dealership. She is on a graduated commission, making 0% of the first \$150, 10% of the next \$3000 and 15% of all sales afterwards. How much must she sell in order to make \$1200?
12. From question 11, if Akshayaa switches to a single commission rate, what rate should she have so that she sells the same amount as she did to make \$1200?
13. Aishwarya used to live in New Zealand where she sold dreamcatchers for \$11. She now lives in Ontario and wants to sell dreamcatchers. If the sales tax is 15% in New Zealand and 13% in Ontario, what price should she set the dreamcatchers at so that customers in Ontario pay the same as what the customers in New Zealand paid?
14. Which is the best deal?
 - (a) 3 cans of soup for \$3.75
 - (b) 53 cans of soup for \$33.39
 - (c) 11 cans of soup for \$9.90
15. How much does a discount rate have to be so that the total (after 15% tax) comes to half the original price?
16. An item is discounted three times: 30%, 15%, then 7%. What single discount rate would reduce the price by the same amount?
17. Perry buys a platypus for \$78.40 after a 30% discount and 12% tax. If the discount is applied to the pre-tax price and the tax is applied after the discount is taken off, what was the pre-tax price?
18. Sapphire earns a 7% commission on all sales on the before-discount, before-tax price. In her store, the discount is applied to the after-tax price. If the store is having a clearance event and all items are 20% off and she has made \$1630 in sales today, how much commission did she make if the tax rate where she lives is 10%?