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Centre for Education in
Mathematics and Computing

Grade 6 Math Circles
Review Challenge

November 24/25, 2015

Today marks the end of our Fall Math Circles journey. As such, this week does not include new topics but is a review challenge for you to tackle. You will need to use everything that you have learned over the past 7 weeks and more to complete this challenge. Should you get stuck, review the lessons from previous weeks to see if there are similar problems.

There are 7 'locks 'with each lock representing one week's Math Circles topics. The topics are in sequential order - so the lock 1 covers the topics from week 1. Solving these problems will give you the combinations to these 7 locks. The goal of this week is to find the combinations to each of the 7 locks.

Note: The questions on Estimation involved estimating the height/armspan of the instructor. This can only be done in person, and so an accurate estimate of height is included. The rest of the problem can be completed using the hints provided

Lock 1 - Algorithms

(a) You are given the following list of numbers: $66, 48, 18, 2, 60, 6, 22, 64, 86, 52$.

   i. Find the 1st, 3rd, and 5th numbers that would be inserted using insertion sort. Add these numbers together.

   ii. Find the 1st, 3rd, and 5th numbers from the list created by sorting these numbers from smallest to largest. Add these numbers together.

   iii. Take the larger of these numbers and subtract the smaller one. Divide the result by 2.

   iv. Call this number Q. Remember it. You will need it again later.

(b) You are given the following set of input and output values from a code-breaking machine.

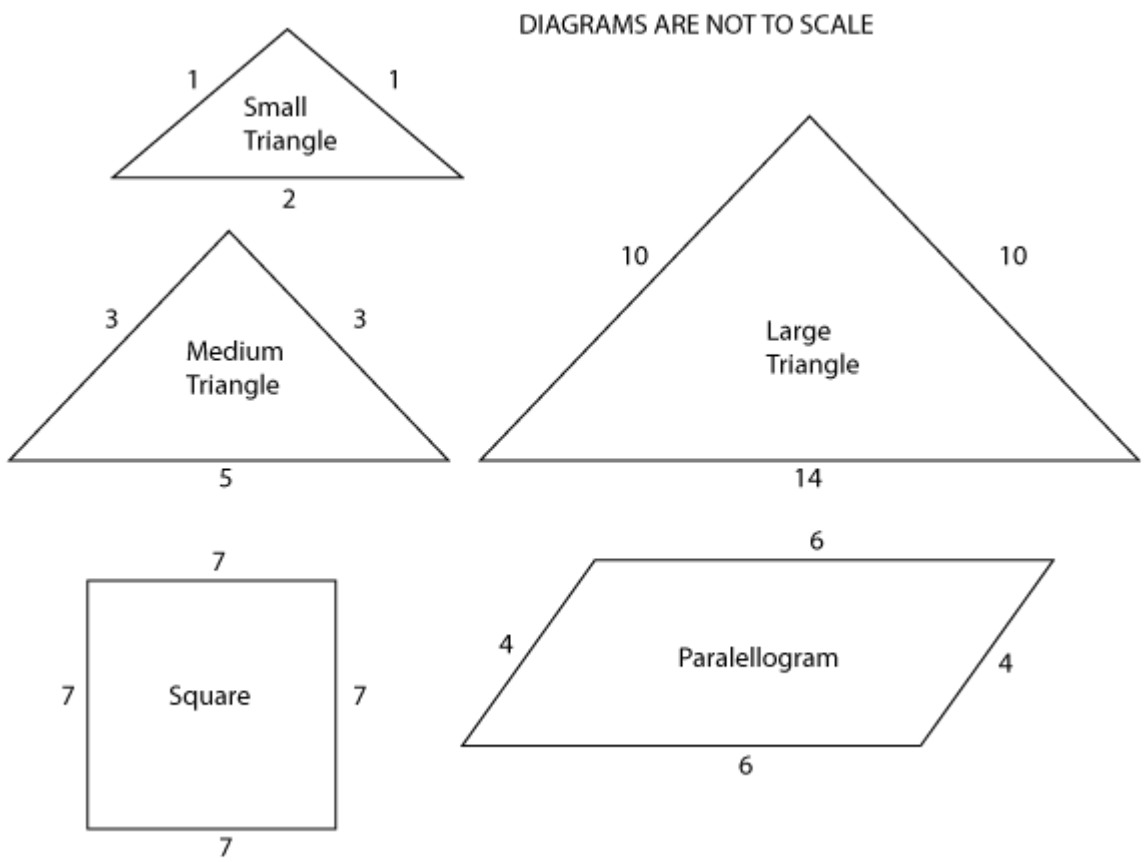
Input	Output
1	4
3	18
5	40
9	108
10	130

i. Determine the algorithm used to convert these inputs into the given output.
ii. Calculate the outputs for inputs of 2, 4, and 6
iii. Multiply these values together and then divide by 2
iv. Call the result B. Remember it. You will need it again later.

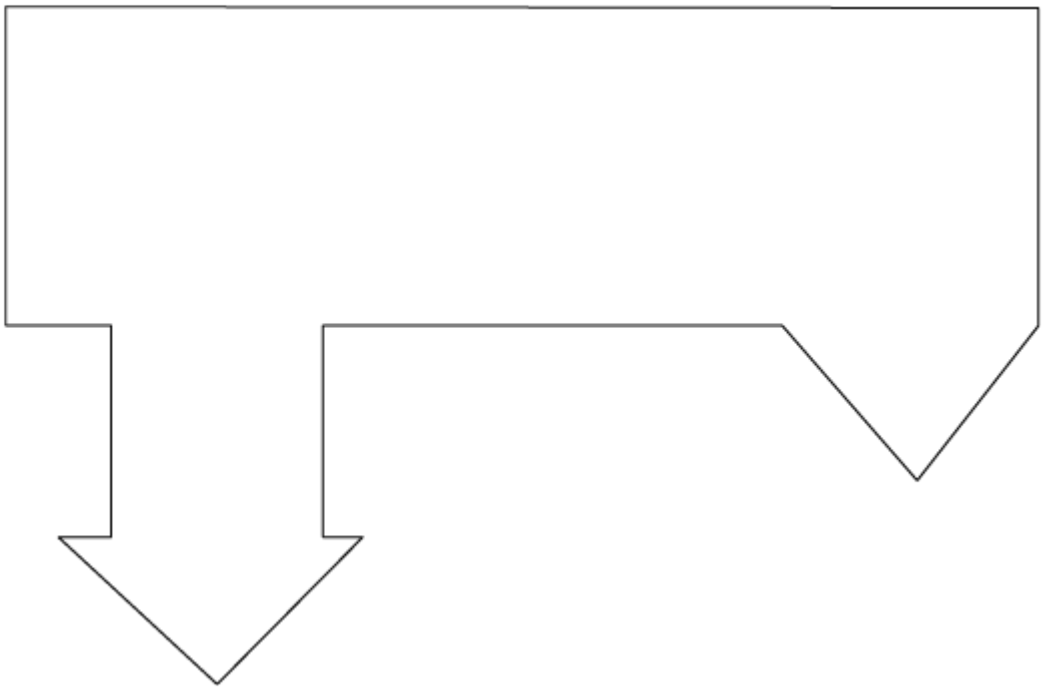
(c) Compute the following $(Q + B) - (\frac{Q}{B}) \times (B - Q) + (\frac{Q}{B} \times Q)$. Round your answer to the nearest whole number. This value is the combination for lock number 1.

Lock 2 - Tessellations

A set of tangrams is a set of 7 tiles. There is a square, a parallelogram, 2 small triangles, one medium triangle, and 2 large triangles. You will need a physical set of tangrams to complete these questions. A sample set is included at the end of this handout to print out to use. Each edge of the tangrams has been assigned a value. These values are shown below.



i. Build a square using all 7 tangrams. Multiply the values assigned to each outer edge to get a number.
ii. Build the shape found on the image below using all 7 pieces. Note that the image is not to scale. Multiply the values assigned to each outer edge to get a number. Partial edges count as a full edge, but only count them once.



iii. Add the above two numbers. This is the combination for lock number 2.

Lock 3 - Estimation

(a) Estimate my height in inches. *Use 74 inches*

(b) Estimate my arm span in centimetres.

Hint: An inch is 2.4cm

Hint: As in Leonardo Da Vinci's Vitruvian Man drawing, my arm span and my height could make up two side of a square

(c) Multiply these two numbers together, and round to the nearest hundred. Add 3.14159. This is the combination for lock 3.

Lock 4 - Sequences

(a) The numbers $1, 4, 7, 10, ...$ form an arithmetic sequence.

i. Find the common difference between the terms of this sequence. Call this number X

ii. Find the next 3 terms in the sequence. Add these 3 terms together and multiply by 7. Call the result Y

(b) 27 and 18 are the 1st and 4th terms of an arithmetic sequence respectively.

i. Find the 2nd and 3rd terms of the sequence. Add these numbers and then multiply the sum by the common difference of the sequence. Call the result Z.

ii. Calculate the sum of the first 7 terms of the sequence. Call this sum V.

(c) Calculate $(X + Y) + (X + 2Z) + (X + 3V) - (Y \times Z) - \frac{V}{7}$. This is the combination to the 4th lock.

Lock 5 - Combinatorial Games

(a) Consider the the Nim game $*1 + *7 + *9 + *7 + *9 + *1$

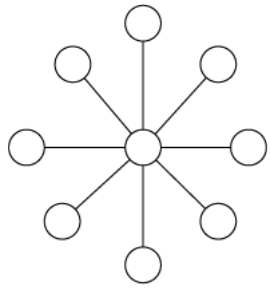
i. Which player (1 or 2) will win this game?

ii. Assuming the players follow a copycat strategy, determine the number of chips the winning player will take.

iii. Multiply the number of chips taken by the winning player by the winning player's number. Call the result S.

(b) Consider the Nim game $*9 + *3 + *3$. Determine the number of the winning player (1 or 2). Call this number H.

(c) In the Sun game, two players take turns placing discs numbered 1 to 9 in the circles on the board (shown below). Each number can only be used once. The object of the game is to be the first to place a disc so that the sum of the numbers along a line through the centre circle is 15.



What number, when placed in the middle circle, can guarantee player 1 a win? Multiply this number by 11 and call this N.

(d) In the Nim game $*1 + *7 + *1 + *9$ Player 1 can guarantee a win. Number the piles from left to right (so the first $*1$ is pile 1, and the $*9$ pile is pile 4). Divide the pile number by the number of chips removed from that pile. Call the result A.

(e) Arrange the numbers you have to form the word NASH. (e.g. if you had A = 10 and B = 24, then ABBA would be 10242410). This number is the key to the 5th lock.

Lock 6 - Logic

Use the grids at the end of this handout for questions (a) and (b)

(a) Expensive Homes: Three members of a secret society live in homes worth more than $ 1 million. From the following clues, determine each person's last name, what street they live on, and how much their home is worth.

i. Agent K's home is worth more than the home of the agent whose last name is Holloway

ii. The Elm Tree Road agent owns the least valued home

iii. Agent A does not live on Treelined Boulevard, but in the most valuable home.

iv. Holt is not the last name of the Wattle Grove resident

(b) Jigsaws: The National Jigsaw Puzzle Society has recently discovered a number of codes hidden in some of their jigsaw puzzles. Each puzzle featured a different theme and was produced by a different company. None of the puzzles were released in the same year. Determine the number of pieces in each puzzle, what the theme was, who produced the puzzle, and what year it was released. Use the following clues that the National Society has left for you:

i. The jigsaw puzzle with 1050 pieces doesn't have the railroad theme

ii. The puzzle with 1300 pieces wasn't released in 1980

iii. The puzzle made by Eduka has 500 more pieces than the puzzle released in 1991

iv. The puzzle with the orchard theme is either the jigsaw puzzle made by Hasbro or the puzzle released in 1980

v. The jigsaw puzzle released in 1984 is either the puzzle made by Ceaco or the puzzle featuring cows

vi. 3 of the puzzles consist of the puzzle made by Eduka, the jigsaw puzzle with 1050 pieces, and the puzzle with 800 pieces

vii. Of the jigsaw puzzle with 800 pieces and the jigsaw puzzle made by Hasbro, one has the orchard theme and the other has the plantation theme

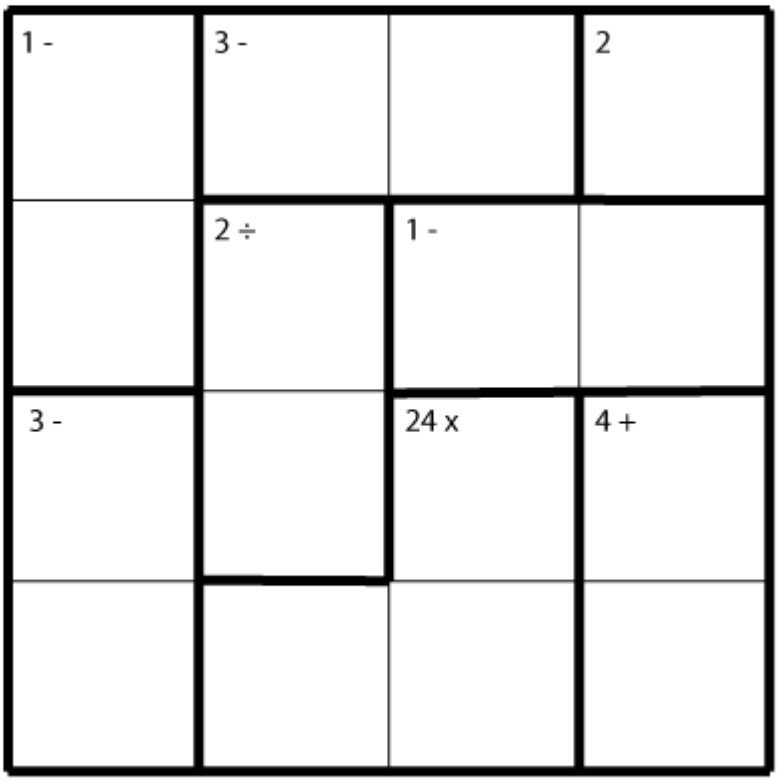
viii. The jigsaw puzzle made by Buffalo has 750 more pieces than the jigsaw puzzle released in 1990

ix. The jigsaw puzzle with the orchard theme has 250 fewer pieces than the puzzle depicting cows

x. Hasbro did not make a puzzle of 550 pieces

xi. The puzzle with the plantation theme had more pieces than the puzzle featuring cows

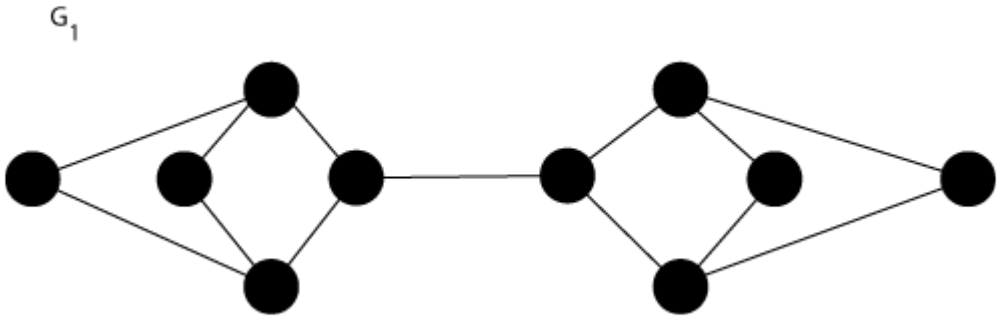
(c) Solve the following 4x4 KenKen Puzzle

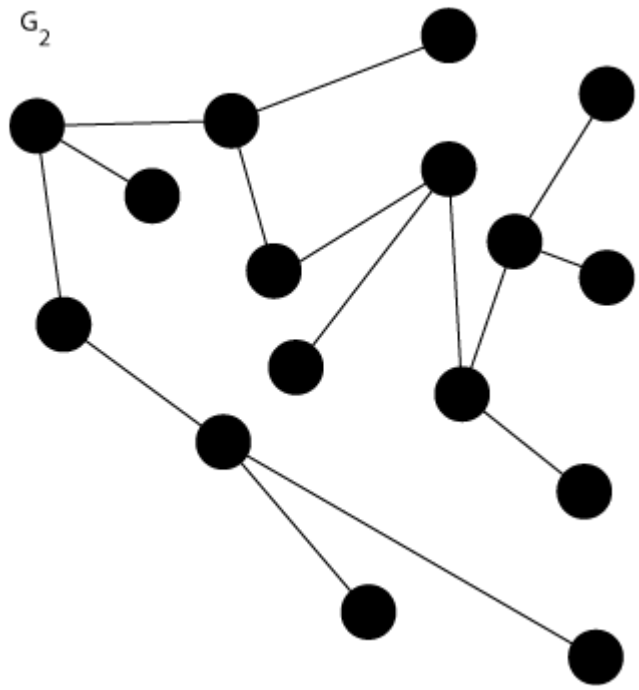


(d) Divide Agent K's house value by the number in the bottom left corner of the KenKen grid. Multiply the number of pieces in the railroad themed puzzle by the year the Hasbro puzzle was released. Add these two numbers. This is the combination for lock number 6.

Lock 7 - Graph Theory

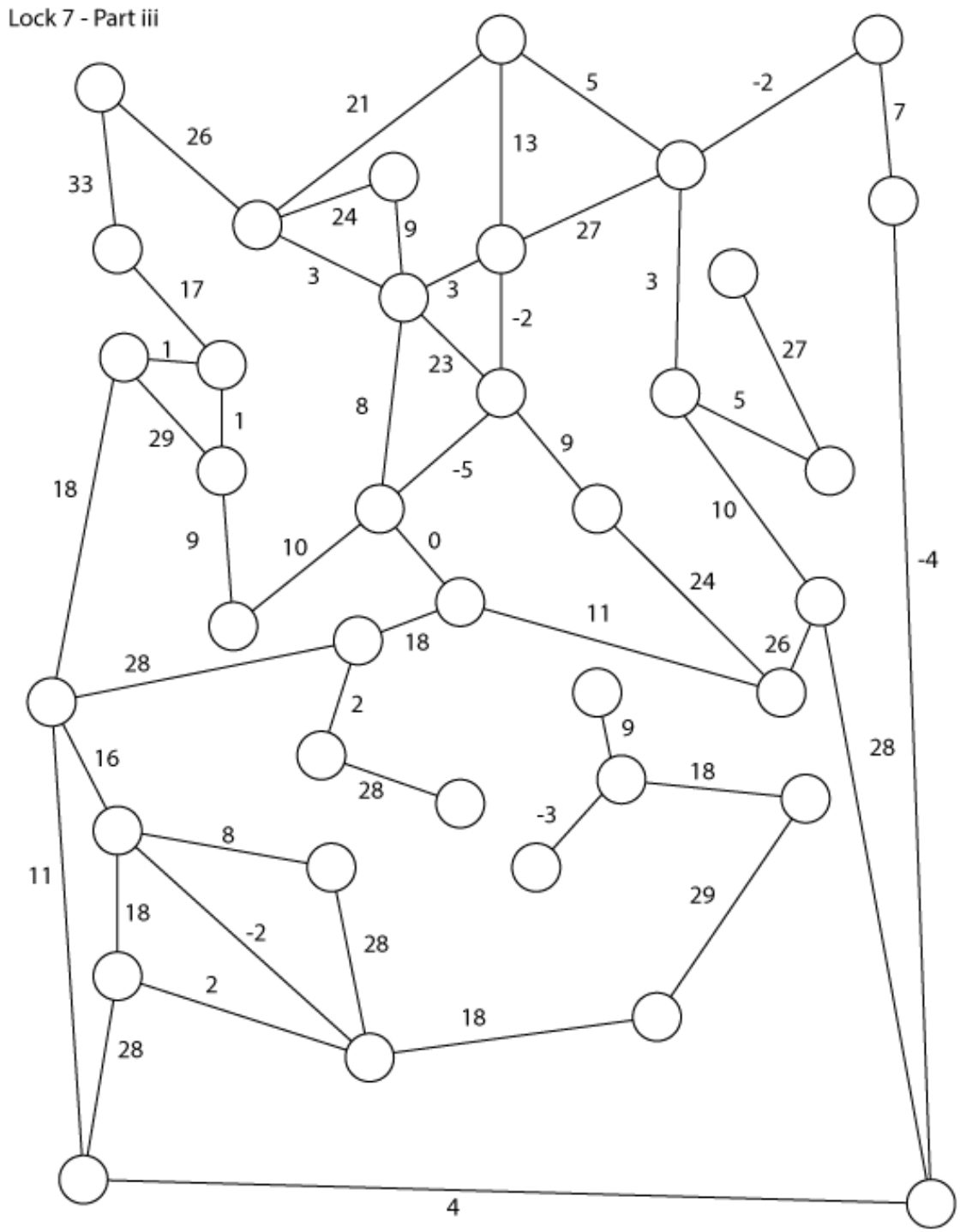
(a) Count the number of edges and vertices in each of the two graphs (G_1 & G_2) below. Add all of those numbers. Call the result E.



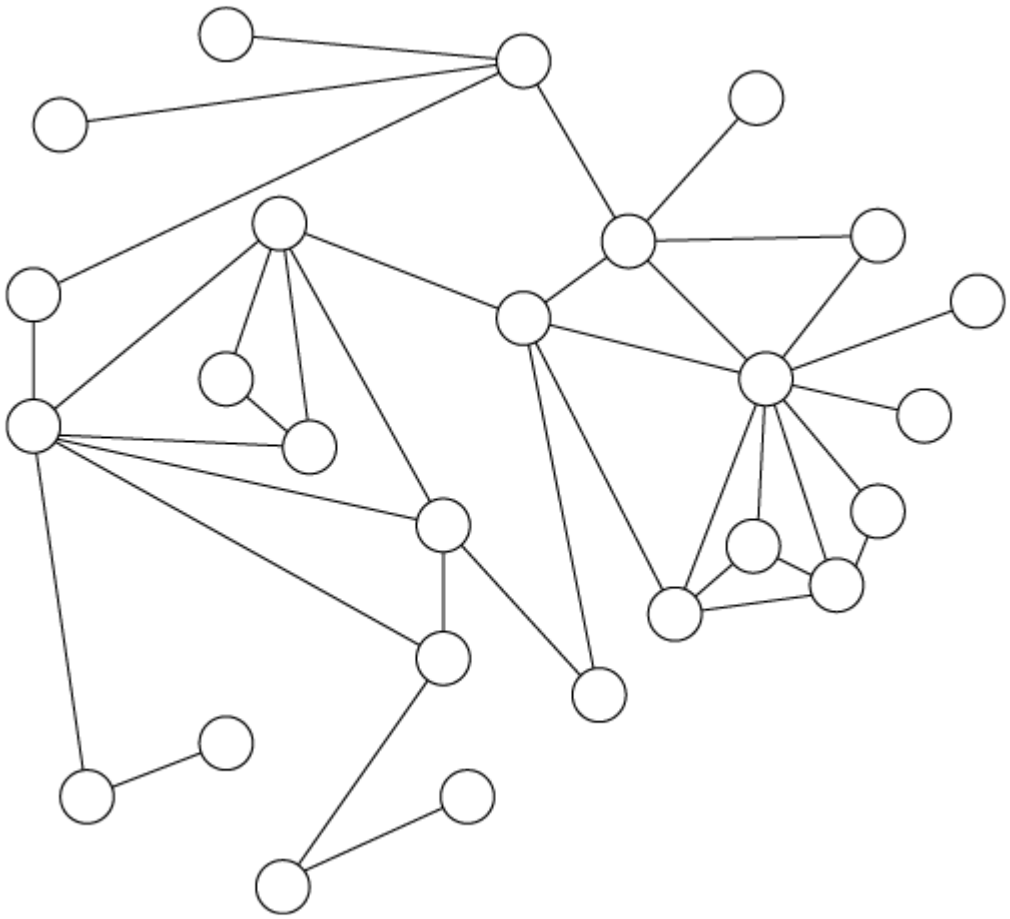


(b) Determine the total number of distinct cycles in G_1. Remember that two cycles using the same vertices that simply start in different places are NOT distinct. Call this number S.

(c) Find a minimum spanning tree in the below graph (you can start with any vertex you want). Call the weight of this spanning tree G.

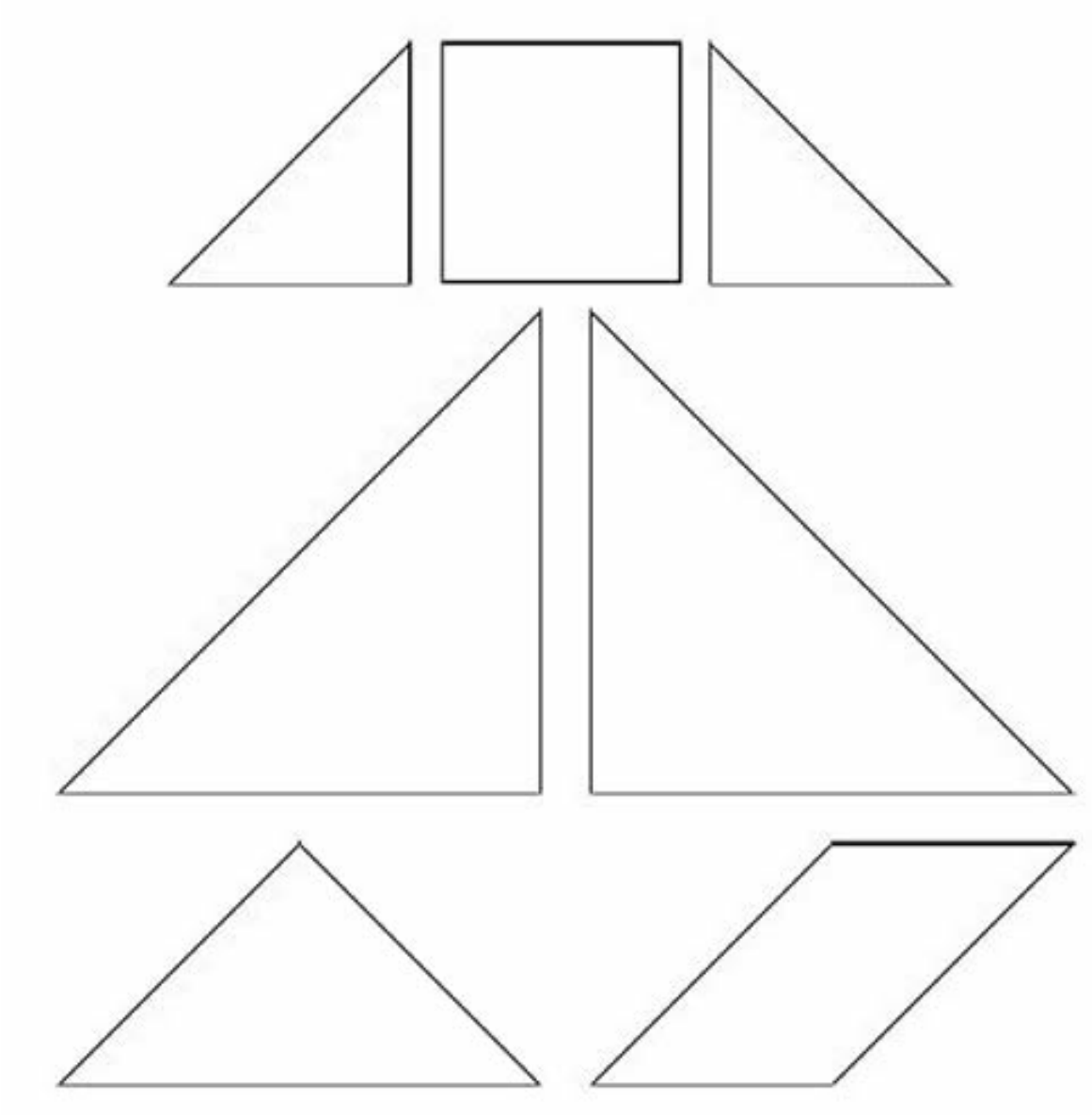


(d) Colour the graph below with the fewest colours possible. Call this number D.



(e) Arrange the numbers you have to form the word EDGES. (e.g. if you had A = 10 and B = 24, then ABBA would be 10242410). This number, divided by 2 plus the number of bridges in Königsberg is the key to the 7th lock.

Tangram Set - For Printing



Expensive Homes Logic Grid

		Last Name			Value			Street		
		Holloway	Holt	Meltosia	$1,050,000	$1,278,500	$1,499,000	Elm Tree	Treelines	Wattle Grove
Agent	Agent A									
	Agent G									
	Agent K									
Street	Elm Tree									
	Treelines									
	Wattle Grove									
Value	$1,050,000									
	$1,278,500									
	$1,499,000									

Jigsaws Logic Grid

		Company					Theme					Year Released				
		Buffalo	Ceaco	Eduka	Hasbro	Masterpiece	Cows	Dogs	Orchard	Plantation	Railroad	1980	1984	1990	1991	1996
Piece Count	300															
	550															
	800															
	1050															
	1300															
Year Released	1980															
	1984															
	1990															
	1991															
	1996															
Theme	Cows															
	Dogs															
	Orchard															
	Plantation															
	Railroad															