Optimization Problem Set

John has many activities to do during the next 5 hours (300 minutes).

The following are a list of his activities and what time do they start and finish. Time 0 corresponds to now and time 300 correspond to 5 hours from now (300 minutes).

Activity number	Start time	End time
1	0	100
2	50	150
3	180	220
4	210	260
5	120	190
6	190	270
7	140	190
8	150	180
9	200	210
10	80	170

John can only do at most one activity at a time. He wants to do as many activities as possible.

1. Give an example of a set of activities that are a **feasible solution** to John's problem.

2. What is the corresponding **objective value** of that solution (i.e. how many activities did you pick)?

3. What makes a set of activities **feasible** or not?

4. What is the largest set of activities that you can come up with that are feasible?

5. What is the logic you used?

Jessica is trying to serve her many customers. Each customer requires a certain amount of minutes of her time. Once she starts serving a customer, she must not interrupt it.

The following are a list of the times required for serving each customer

Customer number	Time required
1	30
2	10
3	20
4	15
5	30
6	100
7	50
8	90
9	10
10	60

Jessica must serve all customers. Her goal is to minimize the average waiting time of her customers. Customers wait from time zero (now) until Jessica is finished serving them.

1. What would describe a **feasible solution** to Jessica's problem? Give an example of a feasible solution.

2. What is the corresponding **objective value** of that solution?

3. What is the best solution that you can come up with?

4. What is the logic you used?

Exercises

1. Find the optimal solution to John's problem with the following activities.

Activity number	Start time	End time
1	20	120
2	10	50
3	80	290
4	210	250
5	150	190
6	190	280
7	240	300
8	0	80
9	30	130
10	90	170

2. Find the optimal solution to Jessica's problem with the following data.

Customer number	Time required
1	10
2	50
3	5
4	65
5	90
6	10
7	45
8	75
9	35
10	80

3. Consider the following other rules for John's problem:

   - Choose as the last activity the one with latest finishing time, remove all incompatible activities and repeat.
   - Choose as the last activity the one with latest starting time, remove all incompatible activities and repeat.

   Will any of these rules always produce an optimal solution? Why or why not?

4. **Challenge** Consider a variation of Jessica's problem where customers have a due time by which they want their activity completed

Customer number	Time required	Due time
1	30	50
2	10	30
3	20	100
4	15	80
5	30	200
6	100	300
7	50	90
8	90	400
9	10	300
10	60	500

The *lateness* of a customer is the difference between the time their activity finished and their due date. For instance, if customer 1 finished at time 40, then its lateness is -10. If customer 1 finished at time 70, its lateness is 20.

Develop an algorithm to solve the problem of:

- Minimizing the sum of the lateness of all customers

  Some thoughts

  - Come up with several different solutions and compare the objective value of Jessica's original problem with this new one
  - Try out with a small (say 4 or 5) customers first to get a better feel
  - If you think you have a good algorithm, can you prove it always gives the optimal solution?

- Minimizing the largest lateness out of all customers

  Some thoughts

  - Come up with several different solutions and compute their objective values
  - Try out with a small (say 4 or 5) customers first to get a better feel
  - If you have a solution (for instance customers served in order 1,2,...), how would you try to improve its objective value?
  - If you think you have a good algorithm, can you prove it always gives the optimal solution?