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Centre for Education in
Mathematics and Computing

Grade 6 Math Circles

October 17/18, 2017

Ancient Mathematics

Introduction

Have you ever wondered where mathematics came from? Who came up with the idea of numbers? Today we will go through a brief history of the different methods and discoveries of different mathematicians before the modern era.

Prehistoric Mathematics (20,000 BC)

Prehistoric mathematics refer to mathematics which have not yet went down in history (hence 'prehistoric'). This is because they are so old that nobody can actually confirm whether or not they are actual mathematics! Nevertheless, prehistoric mathematics can still be very interesting and thought provoking.

The Ishango Bone

The Ishango Bone is one of the earliest signs of mathematics which has ever been discovered. This famous bone has carvings in it which appear to be some sort of a tally system. Some people believe that these tallies were used as a simple counting technique, as well as for basic addition and subtraction.

The Ishango Bone

The most intriguing part on the entire bone is a sequence of four groups of tallies. These groups seem to contain the numbers 11, 13, 17 then 19. These numbers are very interesting because they are all of the prime numbers between 10 and 20.

Do you think that whoever put the carvings into this bone knew about these special numbers, or is this all a coincidence? Unfortunately, nobody knows the answer to this question. The first mathematics which we can guarantee are *actually* mathematics came from around 16,000 years later!

Babylonians (2,500 BC)

Babylonian mathematics are some of the earliest mathematics which have been discovered and have also been agreed upon as being legitimate mathematics.

Achievements

A few big accomplishments that the Babylonians made were:

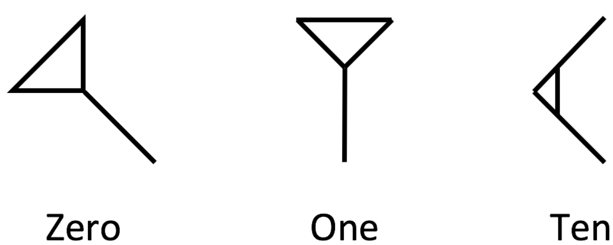
- Estimating the value of π to be 3.125.
- Finding the value of $\sqrt{2}$ to five decimal places.
- Used the first ever confirmed numeral system.

Numeral System

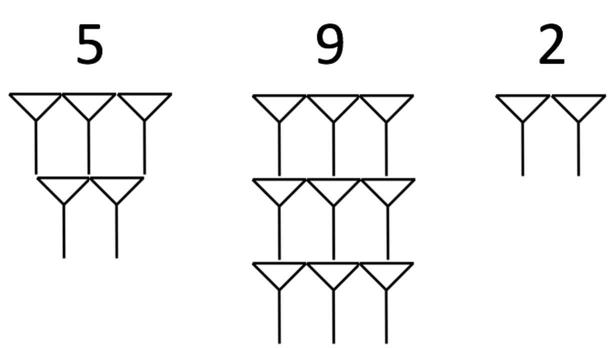
What is a numeral system? A numeral system is the system of numbers that you use to represent mathematics. For example, today almost everyone uses a base 10 numeral system. That means that we have 10 different numbers (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9) that we use to represent every single possible number.

The Babylonians used a base 60 numeral system. That means they have a different representation for all of the numbers from 0 all the way up to 59. This this might seem very strange, but there is a very common base 60 numeral system that we use everyday which actually originated from the Babylonians. This common system is time (seconds, minutes, hours)!

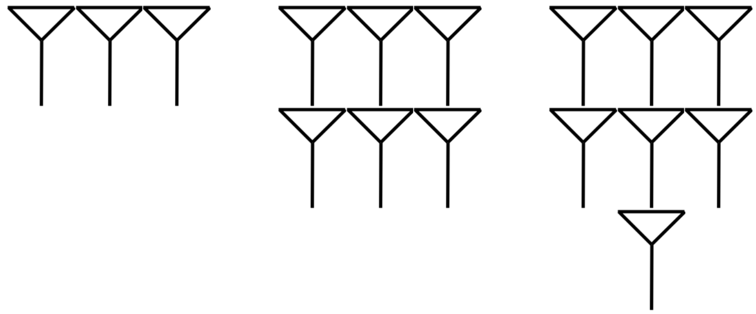
To represent all the numbers 0 to 59, the Babylonians used combinations of these 3 symbols.



To create the numbers 1 to 9, the Babylonians would arrange the 'one' symbol in rows of 3. They would use 1 symbol to represent the number one, 2 to represent the number two, 3 to represent the number three etc.



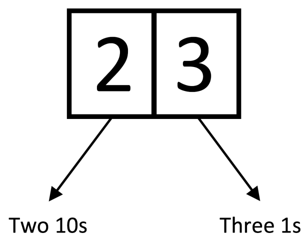
Create the Babylonian numbers 3, 6 and 7.



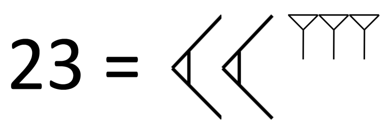
Once numbers become larger than 9, the ‘ten’ symbol must be used. The first step to writing a number greater than 9 in Babylonian terms is to see how many tens are in your number.

Example

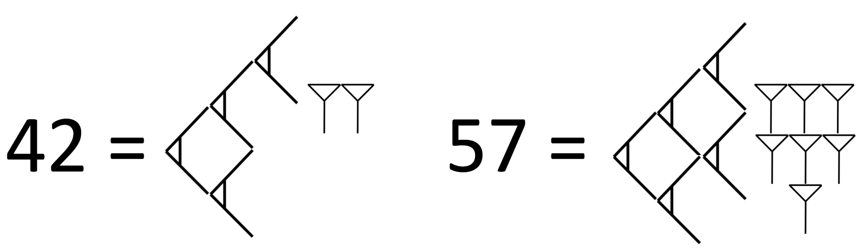
Write the number 23 in Babylonian numerals.



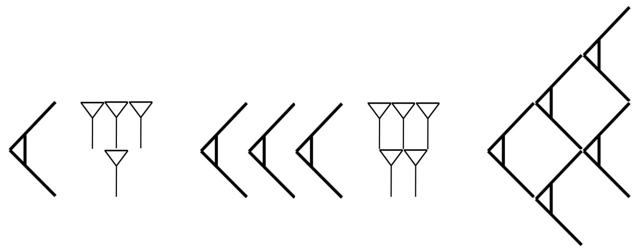
This means that to write 23 in Babylonian terms, we need two of the ‘ten’ symbol, and 3 of the ‘one’ symbol.



The ‘ten’ symbols are all written in a row like how they are above, unless if you have more than 3 of them. If you do have more than 3, they are written in a rotated ‘V’ formation. Here are a few examples of how it looks.



Create the Babylonian numbers 14, 35 and 50.



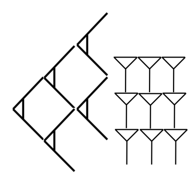
You may have noticed that I didn't give any examples with numbers bigger than 59. If you did, that is because of the fact that this is a base 60 numeral system. What will happen for numbers greater than 59? To get the hang of working with a base 60 numeral system, lets first start by using time as an example.

Example

When using a stopwatch, time increases by 1 second at a time. When 60 seconds is reached, 1 is added to the minutes column and the seconds column resets and begins counting from 0 again. When 60 minutes is reached, the same thing happens except 1 is added to the hours column instead.

This is the exact same way Babylonian numbers work! Once you reach 60, you have to add 1 to the column on the left, and reset your first column to 0! Here is how it looks:

$00_h : 00_m : 59_s$

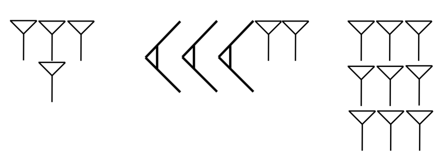


$00_h : 01_m : 00_s$

$00_h : 01_m : 01_s$

$00_h : 01_m : 02_s$

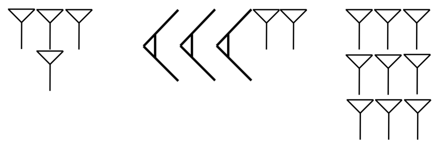
Express 4 hours, 32 minutes and 9 seconds as a Babylonian number.



The last thing we have to do now is to determine the *actual* value of a Babylonian number. One way of doing this is to think of your Babylonian as an amount of time, and however many seconds are in that amount of time will be the value of the Babylonian number!

Example

What is the value of this Babylonian number?



- First, convert the Babylonian number into time.

 $= 04_h : 32_m : 09_s$

- Second, find out how many seconds there are in this amount of time. Start with converting the numbers of hours into a number of minutes.

$$4\, hours \times 60\, minutes\; per\; hour = 240\, minutes$$

Also there are another 32 minutes, so in total there are:

$$240\, minutes + 32\, minutes = 272\, minutes$$

Now we can change the total number of minutes into a number of seconds.

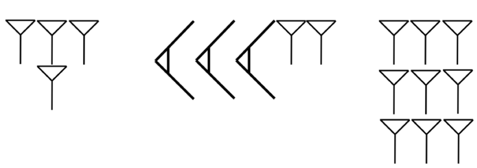
$$272\, minutes \times 60\, seconds\; per\; minute = 16,320\, seconds$$

Finally there are also another 9 seconds, so in total we get:

$$16,320\, seconds + 9\, seconds = 16,329\, seconds$$

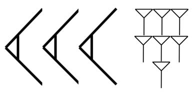
This means the value of the Babylonian number will be 16,329!

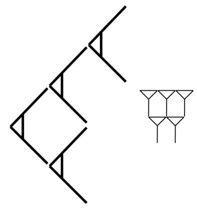
$04_h : 32_m : 09_s$ = 16,329 seconds

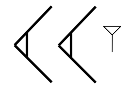
 = 16,329

Problems

1. Express the following Babylonian numerals in terms of modern day numerals.

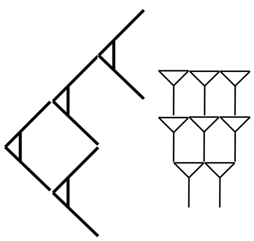
a)  = 37

b)  = 45

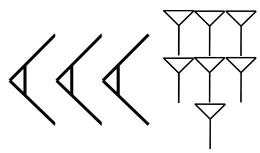
c)  = 21

2. Express the following modern day numerals in terms of Babylonian numerals.

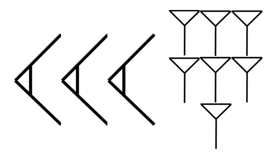
a) 48



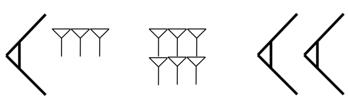
b) 37



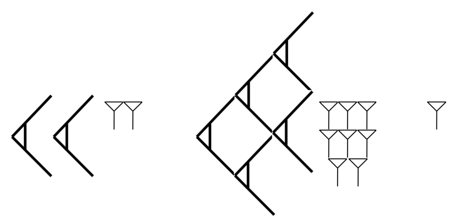
c) 68



3. Convert these large Babylonian numbers into modern day numbers.

a) 

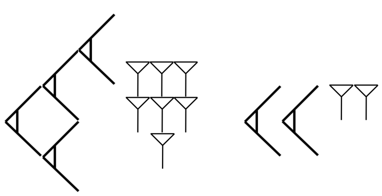
= 47, 180

b) 

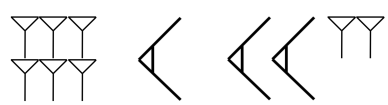
= 82, 681

4. Convert these large modern day numbers into Babylonian numbers.

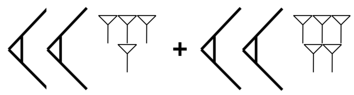
a) 2842

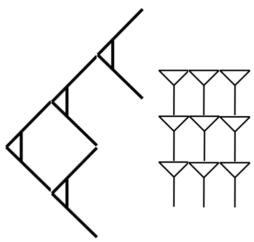


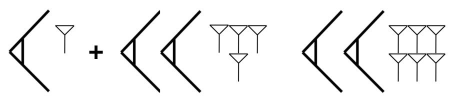
b) 22222

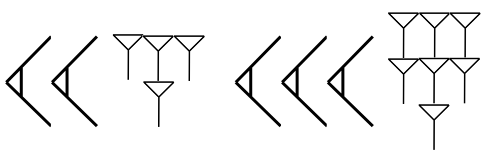


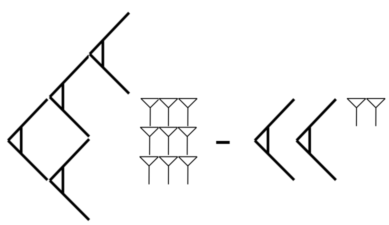
5. Try doing the following calculations in Babylonian numerals!

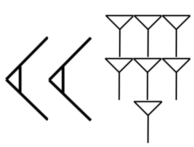
a) 

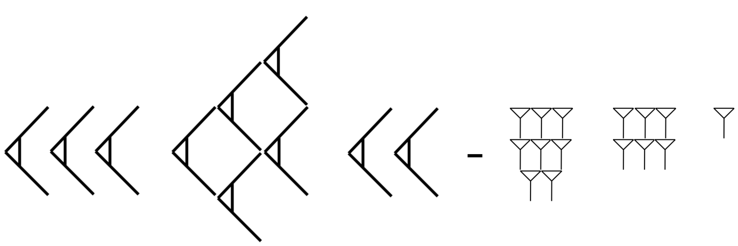
= 

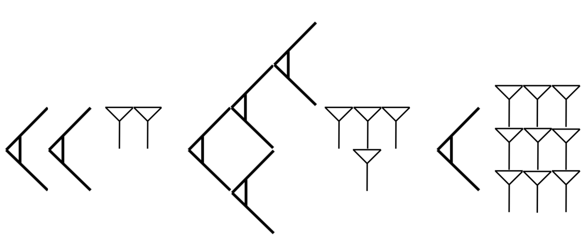
b) 

= 

c) 

= 

d) 

= 

Egyptians (2,000 BC)

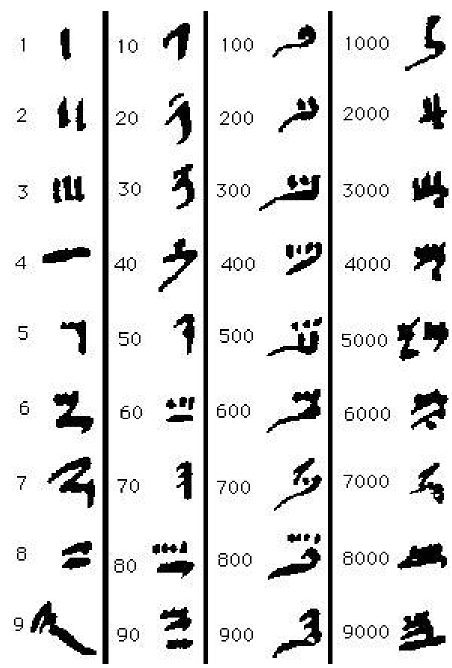
Achievements

- Derived area formulas for most common shapes.
- First mathematicians to use the idea of an average or a 'mean'.
- Solved linear equations.

Numeral System

Similar to our modern numeral system, the Ancient Egyptians used a base 10 numeral system.

As you can see to the right, there are many symbols which the Egyptians used; one for each number from 1 to 10, and 1 for each of these numbers' powers of 10.



Egyptian Hieratic Numerals
Source: aldokkan.com

The nice part about this system is that you write the numbers out the exact same way in which you would read them. For example, if I wanted to write out 4394, I would read it out as "Four thousand, three hundred and ninety four". All you have to do now is write out the symbols in the order you read them!

Some of the drawbacks of this numeral system are that it can be difficult to remember all of the symbols, and the pens used back in 2,000 BC were much different than the ones we used today. It can be challenging to draw these symbols with a modern pen or pencil. Also, because there are an infinite amount of numbers, there is a need for an infinite amount of symbols!

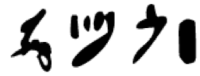
4394 written in Egyptians Hieratic Numerals

Problems

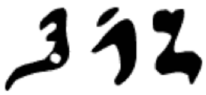
1. What numbers do these numerals represent?

a) 

= 1707

b) 

= 7441

c) 

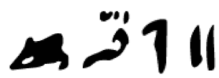
= 926

d) 

= 2208

e) 

= 565

f) 

= 8852

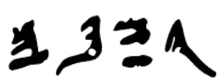
2. Write out the following numbers in Egyptian numerals:

a) 8055

b) 2017

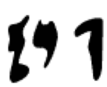
c) 1337

d) 9999



e) 4321

d) 5050

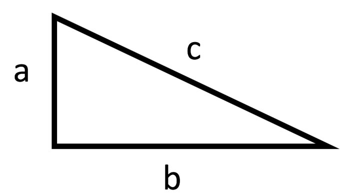


Ancient Greeks (600 BC)

Around a millennium after the era of the Egyptians ended, the Ancient Greeks began learning mathematics at a rate much greater than any other previous civilizations. From this era, there are many famous mathematicians, even a few you may have heard of!

Achievements

- Found that the value of pi must be between 3.1408 and 3.1429 (actual value is 3.1415...).
- Pythagorean theorem; an equation relating the 3 sides of a right angle triangle.



$$a^2 + b^2 = c^2 \tag{1}$$

- Archimedes' principle (an object submerged in water displaces an amount of water equal to it's own volume).
- Many other breakthroughs in all fields of mathematics!

Famous Mathematicians

There were many famous Greek mathematicians: Pythagoras, Archimedes, Thales, Plato, Hypatia and many more. Thales of Miletus was the first of all Greek mathematicians and has been credited for being the first "modern day mathematician". He used the same deductive reasoning in his mathematics which all mathematicians use today. Hypatia of Alexandria has been recorded as the first ever female mathematician.

Arguably the most famous of all Greek mathematicians is *Eukleides of Alexandria* or more commonly 'Euclid'. Euclid is famous for revolutionizing geometry. In his textbook 'Elements', the most successful and influential textbook of all time, Euclid used just 2 tools to help build the entire foundation of geometry. These two tools were a straightedge and a compass.

Straightedge and Compass Geometry

Back in around 300 BC, there were no such things as rulers or protractors because they hadn't been invented yet. This left mathematicians with a very interesting challenge.

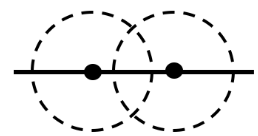
How can we correctly draw mathematical shapes with *no* error using only a straightedge and a compass? Lets start by trying a few examples.

Example

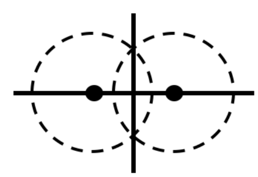
Create two *perfectly* perpendicular lines using only a straight edge and a compass. (Although we are using rulers today, you are not allowed to use them for measuring and you are not allowed to use the sides to make parallel or perpendicular lines. These are the same conditions Euclid had to follow).

1. Draw ***any*** straight line.

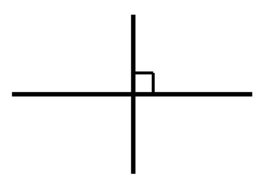
2. Draw ***any*** 2 overlapping circles which have their centers somewhere along the line.



3. Draw a line connecting the intersections of the 2 circles.



4. Well done! You have made 2 perfectly perpendicular lines!

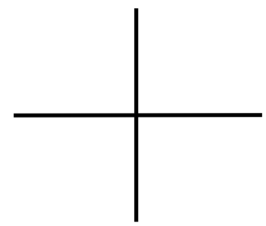


Try it here!

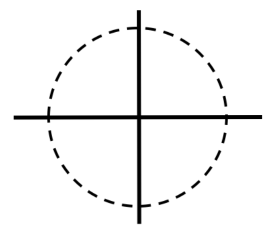
Example

Create a 45 degree angle using only a straightedge and compass.

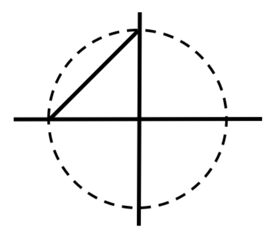
1. Create ***any*** 2 perpendicular lines.



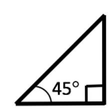
2. Draw a circle centered at the intersection of the 2 perpendicular lines.



3. Connect two points on the circle as shown. This can be done in ***any of the four quadrants*** of the circle.



4. Congratulations! You have constructed a 45-degree angle!



Try it here!

Problems

1. Create a perfect square using only a straightedge and a compass.

Step by step solutions can be found on the PowerPoint file! Begin the slideshow to view animations.

2. Create two parallel lines using only a straightedge and a compass.

Step by step solutions can be found on the PowerPoint file! Begin the slideshow to view animations.

3. Bisect an angle (i.e. split an angle into 2 equal smaller angles) using only a straightedge and a compass

Step by step solutions can be found on the PowerPoint file! Begin the slideshow to view animations.

4. Create a regular hexagon using only a straightedge and compass (i.e. all sides have the same length, and every angle is 120 degrees).

Step by step solutions can be found on the PowerPoint file! Begin the slideshow to view animations.

5*. An ancient greek artist was just about to finish his painting of the Parthenon, when he accidentally spilt 3 drops of ink on the skyline of his painting. He knew this ink would be very hard to erase, so instead of erasing it, he decided to paint a picture of the sun over the drops in such a way that it would be *just* big enough to cover all 3 of them.

Using only a straightedge and a compass, how can the artist make sure the sun he draws is a *perfect* circle that is *just* big enough to cover all 3 drops?

Step by step solutions can be found on the PowerPoint file! Begin the slideshow to view animations.

** Denotes a difficult question.*

6**. For an extra challenge, if you're a geometry master, try to make a regular pentagon using only a compass and a straightedge!

Step by step solutions can be found on the PowerPoint file! Begin the slideshow to view animations.

*** Denotes an extremely difficult problem. I'll be surprised if anyone solves this!*