Math Circles
18 October 2017
Logic and Problem Solving, Part 2
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EXERCISES

1. Each letter represents a different digit, and pairs of letters (e.g., AC) represent two-digit numbers.

   We know:

$$\begin{array}{ccc} A \times B = B & B \times C = AC & C \times D = BC \\ D \times E = CH & E \times F = DK & F \times H = CJ \\ H \times J = KJ & J \times K = E & K \times L = L \\ & A \times L = L & \end{array}$$

   Find the values of all the letters with explanation.

2. Harold and Tabby are playing a coin game.

   There are 12 coins in one pile, and each player can take either two or three coins from the pile. The player who takes the last coin loses.

   Harold goes first, and then they alternate turns.

   Each player makes a move that allows them to win, if possible. If there is no way to win, then the player will make a move that allows a tie, if possible.

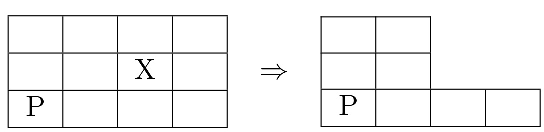
   Show that there is a winner (i.e., that there is no tie), demonstrate who the winner is and what the winning strategy is. Give some reasoning and analysis why the strategy works.

3. Chomp is a simple two-player game played on an $n \times m$ grid, which you can imagine is a chocolate bar.

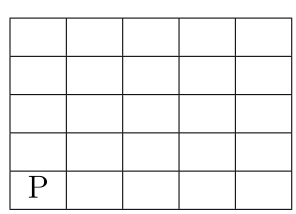
   Players take turns picking a square, and "eating it", removing that square and all squares to the right and above this square.

   In the bottom-left corner is a poisoned square: the player that eats this square loses.

   For example, on the 3×4 board, the first player selects the cell labelled X, which results in the board shown on the right:



   (a) Who wins in the 5×5 game, shown below (where P marks the poison block), and what is the winning strategy?



(b) Who wins in the 2×8 game, shown below (where P marks the poison block), and what is the winning strategy?

P							

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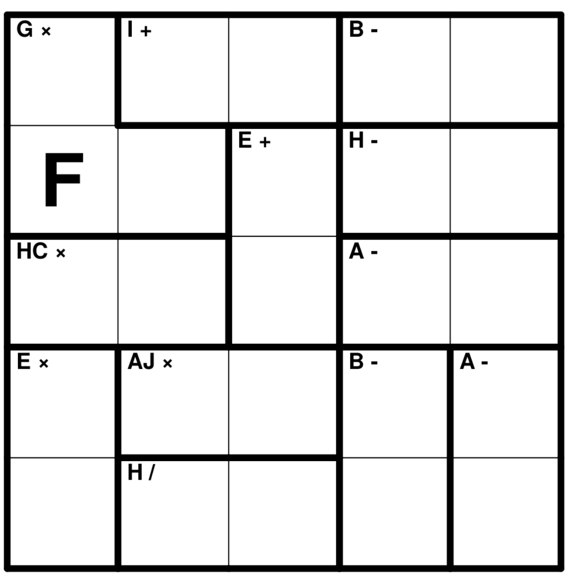
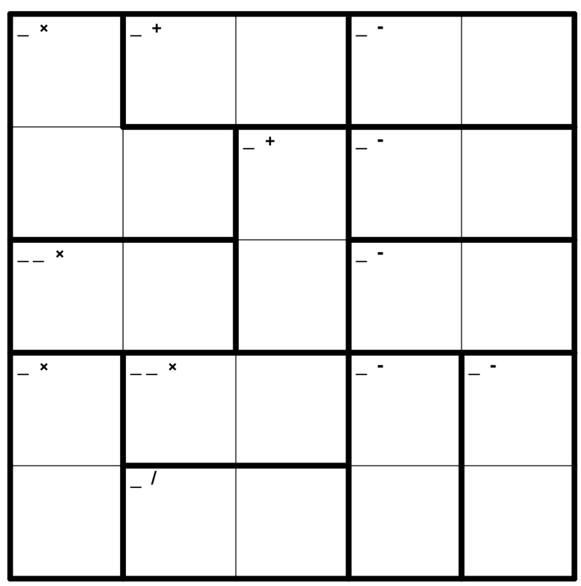
P							

(c) Suppose the starting board is larger than a 1×1 rectangle (i.e., it has more than one square). Prove that there is a winning strategy for one of the players. Hint: you don't need to show or construct the winning strategy, but just prove that there exists one. Hint 2: use contradiction.

4. Below is an encrypted KenKen puzzle. Each digit has been replaced with a single letter. In the solved puzzle, each row and each column will contain all of the digits from 1 through 5. As in "normal" Kenken, the heavy lines indicate "cages" which contain numbers which can be combined (in any order) to produce the result shown in the cage. For example, $AB/$ means that the numbers in the cage can be arranged, in some order, to divide to give the number AB. Note that numbers within a cage may repeat, so long as the repetition is not in the same row or the same column.

Fill in the 5×5 grid, as well as the 10 digits in the table below. Additionally, explain how H, A and C were determined. (Hint: start with H, A and C.)





A	B	C	D	E	F	G	H	I	J