Math Circles. Group Theory. Problem Set 3.

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Problems:

1. Determine all the groups of order 4.
2. Determine all groups of order 5.
3. Draw out the multiplication table of S_3.

$\cdot$	id	(132)	(123)	(12)	(13)	(23)
id						
(132)						
(123)						
(12)						
(13)						
(23)						

4. We know that D_3, S_3 and $(\mathbb{Z}_6, +)$ are groups of order 6. Are they isomorphic? are all of them non-isomorphic?
5. Is $\{0, 5, -5\}$ a subgroup of $(\mathbb{Z}, +)$?
6. Find all the subgroups of S_3.
7. What are the possible orders for a subgroup of $(\mathbb{Z}_{12}, +)$? For each order, can you find a subgroup of that order?
8. Prove that the order of an element divides the order of the group.

9. Find all the subgroups of D_4 (The group of symmetries of the square). Here is the multiplication table that may help you.

$\cdot$	e	R	R^2	R^3	H	V	D	D'
e	e	R	R^2	R^3	H	V	D	D'
R	R	R^2	R^3	e	D'	D	H	V
R^2	R^2	R^3	e	R	V	H	D'	D
R^3	R^3	e	R	R^2	D	D'	V	H
H	H	D	V	D'	e	R^2	R	R^3
V	V	D'	H	D	R^2	e	R^3	R
D	D	V	D'	H	R^3	R	e	R^2
D'	D'	H	D	V	R	R^3	R^2	e

10. Prove that inverses are unique. In other words, prove that if $ab = ba = e = ac = ca$ then $c = b$.