

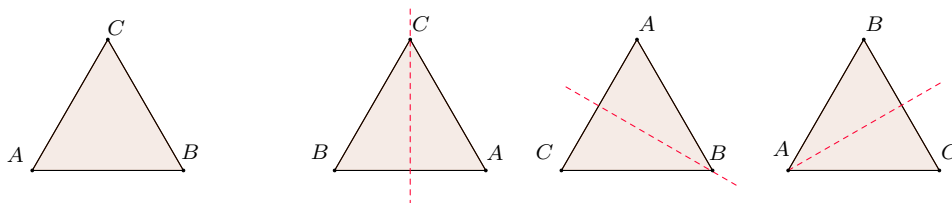
# Math Circles. Group Theory. Problem Set 1.

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## 1 Problems

- Let  $(2\mathbb{Z}, +)$  denote the set of even integers  $\{\dots, -4, -2, 0, 2, 4, 6, \dots\}$  with the usual addition.
  - What is the identity in this group?
  - What is the inverse of 2?
  - What is the inverse of  $-100$ ?
  - What is the inverse of an element  $a$  in  $2\mathbb{Z}$ ?
  - What is the relationship between  $(2\mathbb{Z}, +)$  and  $(\mathbb{Z}, +)$ ?
- Let's find the Dihedral group  $D_3$ , the group of symmetries of an equilateral triangle. Let  $e$  be the fixed rotation (or  $0^\circ$  rotation),  $R$  be the  $60^\circ$  clockwise rotation,  $R^2$  be the  $120^\circ$  clockwise rotation. To name the flips, let's use the following notation:



The first triangle is the triangle in the initial position. We call  $V$  the flip made in the second figure. Similarly we call  $D$  the flip of the third figure, and  $D'$  the

flip of the last figure.

Complete the operation table for  $D_3$ .

3. Complete the multiplication table of  $(\mathbb{Z}_{10}, +)$ .
4. Can we think on multiplication in  $\mathbb{Z}_5$ ? Complete the multiplication table for  $(\mathbb{Z}_5, \cdot)$  and figure out if it satisfy the nice properties of having an identity element and inverses. If not, can you fix it?
5. Find all values of  $x$  in  $\mathbb{Z}_{10}$  that satisfy the equation  $3x + 9 = 1 \pmod{10}$ .
6. Does the equation  $x^2 = -1 \pmod{5}$  have solutions in  $\mathbb{Z}_5$ ?
7. The natural numbers  $\mathbb{N}$  with the usual addition don't form a group. Can you guess why this is not a group?
8. Find elements  $A, B, C$  in  $D_4$  such that  $AC = CB$ , but  $A \neq B$ . This shows that the "cross cancellation" is not valid!
9. Describe the symmetries of a non-square rectangle. Draw out the multiplication table of this group.