Math Circles. Group Theory. Problem Set 2.

Diana Carolina Castañeda Santos
dccastan@uwaterloo.ca
University of Waterloo

March 27, 2019

Problems:

1. Find all the elements of $(\mathbb{Z}_{12}^*, \cdot)$ and draw out the multiplication table for this group.

2. Find all values of x in $\mathbb{Z}_{10}$ that satisfy the equation $3x + 9 = 1 \pmod{10}$.

3. Does the equation $x^2 = -1 \pmod 5$ have solutions in $(\mathbb{Z}_5^*, \cdot)$?

4. Determine the order of the following groups:

   (a) $|D_5|$.

   (b) $|(\mathbb{Z}_{12}, +)|$.

   (c) $|(\mathbb{Z}_{12}^*, \cdot)|$.

   (d) $|S_4|$

   (e) $|(\mathbb{Z}_p^*, \cdot)|$ where p is prime.

   (f) $|S_n|$ where $n \in \mathbb{N}$

5. Determine the order of the following elements

   (a) $|i|$ in $\mathcal{Q}_8$.

   (b) $|3|$ in $(\mathbb{Z}_8, +)$.

   (c) $|3|$ in $(\mathbb{Z}_8^*, \cdot)$.

   (d) $|a|$ for each a in $(\mathbb{Z}_5^*, \cdot)$.

   (e) $|HV|$ in D_4.

6. Determine all the groups of order 4.

7. Determine all groups of order 5.

8. Draw out the multiplication table of S_3.

$\cdot$	id	(132)	(123)	(12)	(13)	(23)
id						
(132)						
(123)						
(12)						
(13)						
(23)						

9. We know that D_3, S_3 and $(\mathbb{Z}_6, +)$ are groups of order 6. Are they isomorphic? are all of them non-isomorphic?

10. Draw out the multiplication table of the group of quaternions $(\mathcal{Q}_8, \cdot)$.

$\cdot$	1	-1	i	$-i$	j	$-j$	k	$-k$
1								
-1								
i								
$-i$								
j								
$-j$								
k								
$-k$								

11. Prove that inverses are unique. In other words, prove that if $ab = ba = e = ac = ca$ then $c = b$.