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Group Theory Solutions

Rubik's Cube

We will begin to learn about Group Theory by looking at one of the famous toys in history, the Rubik's cube. Invented in 1974 by Erno Rubik of Budapest, Hungary, the Rubik's cube comes prepackaged in a solved position, where each face of the cube has the same colour. However, we can scramble the cube by rotating any one of it's six faces. The goal of this particular puzzle is to return the cube back to it's original/solved position.

The Rubik's cube is of significant mathematical interest because of it's symmetrical nature. Symmetry is present everywhere in mathematics, but nowhere as studied or observed than in **Group Theory**. Can you give or think of examples of symmetry?

In the Rubik's cube,

- There are a set of actions you perform on the cube (can rotate any of it's 6 sides)
- Each action can be reversed (can rotate the opposite way to undo rotations)
- Combining actions results in another action

Groups

Using what we learned about the Rubik's Cube, we will define a **group** as follows:

Group

For a nonempty set G and a list of defined actions on elements of G, G is a *group* if:

(Inverse Element) Every action is reversible by another action.

(Identity) There is an action that does nothing.

(Closure) Consecutive actions result in an action we previously defined.

Recall that an **integer** is a whole number that can be positive, negative or zero.

Example: Let's define the set of integers as {..., -3, -2, -1, 0, 1, 2, 3, ...} and the action defined here is the addition of any two integers. Let's prove that this is a group.

- *Inverse Element*: We must first show that every action is reversible by another action. Whenever we add an integer n, we can reverse the action by __subtracting a or adding $-a$__.
- *Identity*: Clearly if we add __0__ to any integer, we are not changing anything so adding __0__ is the identity action and __0__ is the **identity element**.
- *Closure*: Adding any two integers together results in an __integer__ which is an element in our set so closure exists.

All required conditions are met so the set of integers with the action of addition is a **group**.

Non-Example: Given the set {2, 3, 4} and the action of multiplication. We know that for multiplication, if we multiply by 1 we change nothing so 1 is the **identity element** but 1 is not in this set of numbers so this is not a group as the Identity condition is not held.

Non-Example: Given the set of integers {..., -3, -2, -1, 0, 1, 2, 3, ...} and the action of multiplication, let's try to find the inverse element of multiplying by 2. That is, let's multiply $a \times 2$ by something to reverse the multiplication by 2 to get a. We know that we need to multiply by $\frac{1}{2}$ but $\frac{1}{2}$ is not in this set of integers so this is not a group as the Inverse Element Condition is not held.

Exercise Set 1

Recall that a **rational number** is a number that can be expressed as the fraction $\frac{a}{b}$ of two integers, a numerator a and a *non-zero* denominator b.

Exercise: Let $\mathbb{Q} \setminus 0$ be the set of all rational numbers **excluding 0** with the action of multiplication. Prove that this is a group.

- *Inverse Element*: We must first show that every action is reversible by another action. Whenever we add a rational number $\frac{a}{b}$, we can reverse the action by multiplying by $\frac{b}{a}$ so that $\frac{a}{b} \times \frac{b}{a} = 1$ and 1 times anything will simply be itself.
- *Identity*: Clearly if we multiply any rational number by 1, we are not changing anything so multiplying by 1 is the identity action and 1 is the **identity element**.
- *Closure*: Multiplying any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ together results in $\frac{a \times c}{b \times d}$ and since a, b, c, d are integers, $a \times c$ and $b \times d$ are integers which makes $\frac{a \times c}{b \times d}$ a rational number which is an element in our set so closure exists.

Exercise: In the exercise above, why is the set of rational numbers, $\mathbb{Q}$ (which includes 0), with the action of multiplication not a group?

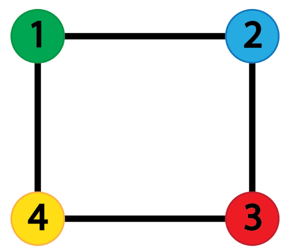
Because if we multiply a rational number by 0, we will get 0 and multiplying 0 by anything will remain 0 so we can never get back to our original rational number. This means that there is an element in the set, in this case, 0, which does not have an inverse.
As an example, $\frac{1}{2} \times 0 = 0$ and we cannot multiply 0 by anything to get back $\frac{1}{2}$ as 0 does not have an inverse.

Exercise: Give two reasons why the set of integers {..., -3, -2, -1, 0, 1, 2, 3, ...} with the defined action of division is **not** a group.

- Dividing any integer by 0 is not mathematically defined so not an allowed action.
- Dividing integers creates a fraction $(\frac{1}{2})$ which is not in the set. Closure doesn't exist.
- Once you divide two integers $(\frac{1}{2})$, there is no other integer you can divide by to get back to 1. Division by any other integer will result in a smaller value.

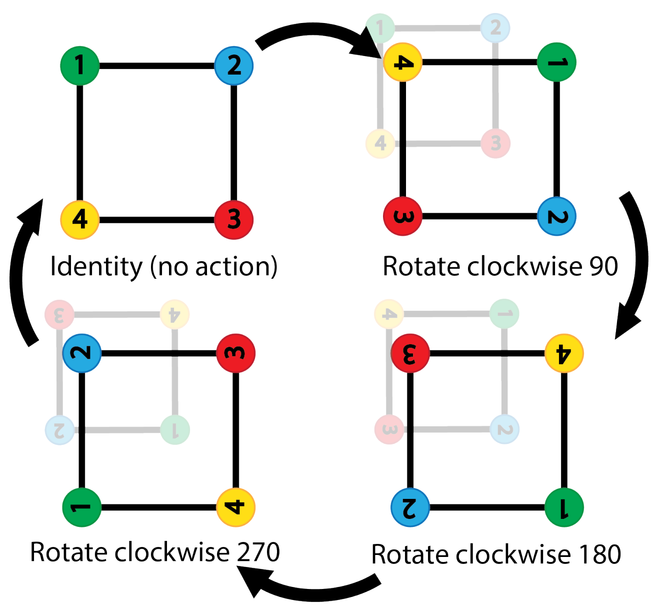
Rotations

Example 1: You are given the square below, with labels 1, 2, 3, 4 on the corners of the square and you are ONLY allowed to rotate it **clockwise** by 90°, 180°, and 270°.



We can see how this is a group:

- **4 actions**: do nothing (rotate 0°), rotate 90°, 180°, and 270° clockwise
- A rotation can be undone by another rotation. For example, if I rotate 90°, and then I rotate 270°, I'll return the square back to it's original position
- Two rotations combined is equivalent to another rotation.



To simplify the notation, we will the use the following to represent our actions:

$$\{\mathbf{I}, \mathbf{R_{90^\circ}}, \mathbf{R_{180^\circ}}, \mathbf{R_{270^\circ}}\}$$

We can use two actions to formulate a third action. For example, combining a rotation of 90 degrees and 180 degrees gives me a rotation of 270. We will write it as

$$R_{90^\circ} R_{180^\circ} = R_{270^\circ}$$

Exercise:

1. What is $R_{90^\circ} R_{90^\circ}$? Draw out how the square looks like after the two rotations.

   When we rotate 90° twice, it is equivalent as rotating it once 180°. We express this mathematically as:

   $$R_{90^\circ} R_{90^\circ} = R_{180^\circ}$$

2. What is $R_{180^\circ} R_{270^\circ}$? Draw out how the square looks like after the two rotations.

   Rotating 270° is the same as rotating 180° followed by a rotation of 90°. So we can see that $R_{180^\circ} R_{270^\circ}$ is the same as rotating 180° twice followed by a rotation by 90°. We can see that after two 180 degree rotations, we return the square back to it's original position. Then it is followed by a 90 degree rotation. Mathematically, we write this as:

   $$R_{180^\circ} R_{270^\circ} = R_{90^\circ}$$

3. If I rotated 270° 53 times, what will my square look like at the end?

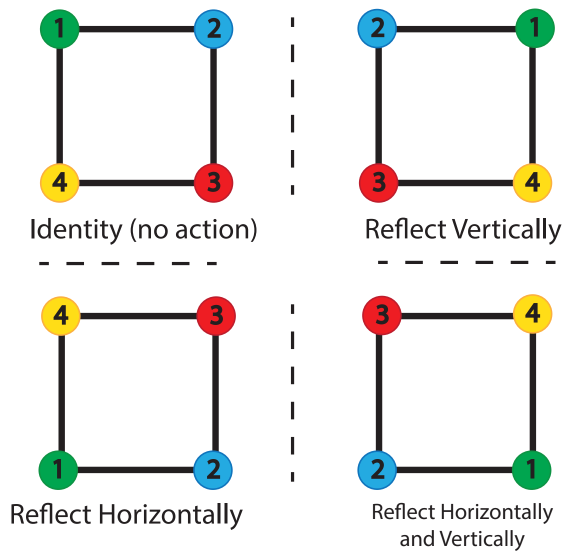
   Observe that rotating 270° 4 times returns the square back to it's original position. Now we have that $53 = 4 \times 13 + 1$. We are grouping every 4 rotations of 270°. We can see that leaves only one rotation of 270°. Therefore, the square will look like it was rotated by 270° once.

4. When we combine two rotations we always end up with another rotation. Does the order how you combine the rotation matter? For example, if I rotated 90° clockwise and then 180° is it the same as rotating 180° clockwise and then 90° clockwise? Justify your answer.

   The order of rotations does not matter. This property is known as **commutativity**.

Example 2 - (A Non-Group):

Hint: Remember that two actions must combine to from our list of allowed actions. It may be helpful to draw out every combination of the two actions.



Our problem lies when we have a horizontal reflection followed by vertical reflection or a vertical reflection followed by a horizontal reflection, we produce neither the original square, a vertical reflection, nor a horizontal reflection. Recall that when we combine actions we must produce an action that we previously allowed. Therefore, we don't have a group.

How might we fix this then?

We can add the action of: rotating by 180° or R_{180°

In total our four actions are now: $\{F_v, F_h, R_{180^\circ}, I\}$

Exercise: Check if the following actions on the square above form a group.

$$\{F_v, F_h, R_{180^\circ}, I\}$$

We can show that any combination of two actions will form an action that is in the group. We can also show the inverse of each flip and rotation is itself and we know that I (doing nothing) is the identity element so we have a group.

Organizing Group Actions: Cayley Tables

Drawing every possible combination of our permitted actions quickly becomes cumbersome. Instead, we can construct a square table to see all the possible combinations of actions performed on a square. This is called a **Cayley Table**.

Example.

Going back to our first example with the rotations. We can express all combinations succinctly the chart shown below.

Action	**I**	R_{90°	R_{180°	R_{270°
I	I	R_{90°	R_{180°	R_{270°
R_{90°	R_{90°	R_{180°	R_{270°	I
R_{180°	R_{180°	R_{270°	I	R_{90°
R_{270°	R_{270°	I	R_{90°	R_{180°

Exercise.

Construct the Cayley Table for Example 2 with our 3 actions in addition to doing nothing - Rotation Clockwise 180°, Vertical Reflection, and Horizontal Reflection.

Action	**I**	R_{180°	F_h	F_v
I	I	R_{180°	F_h	F_v
R_{180°	R_{180°	I	F_v	F_h
F_h	F_h	F_v	I	R_{180°
F_v	F_v	F_h	R_{180°	I

More on Groups

There is more to groups than the definition. Below are some properties of groups.

Order of a Group

The **order** of a group is the number of allowed actions.

Example: In the first rotation example, the defined actions were: $\{I, R_{90°}, R_{180°}, R_{270°}\}$. The **order** of this group is 4 as there are 4 actions allowed.

Exercise: What is the order of the final group in the second rotation example?

In the second rotation example, our final group had the following allowed actions:

$$\{F_v, F_h, R_{180°}, I\}$$

The order of the group is 4.

Exercise: * Recall the set of integers {...-3, -2, -1, 0, 1, 2, 3, ...} with the defined action of addition. What is the order of this group?

For this group, a defined action is to add an integer. Adding 1 is an action, adding 2 is an action, adding 3 is an action, etc. Since there is an infinite number of integers we can add, there are infinite actions defined for this group so the order is ∞.

A **theorem** is a statement that can be demonstrated to be true by accepted mathematical operations and arguments. The process of showing a theorem to be correct is called a **proof**.

Theorem 1 - Uniqueness of Identity

For a defined group, there is only one **identity element** that doesn't do anything.

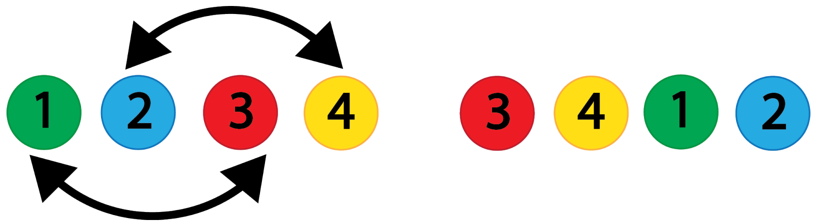
In other words, there is only one way to do nothing.

Theorem 2 - Uniqueness of Inverses

For a defined group, if b and c are both **inverses** of a then $b = c$.

Permutation Groups

For example, we have four balls, let's swap the 2^{nd} ball's location with the 4^{th} ball, and the 3^{rd} ball with the 1^{st} ball's location.



Exercise: Consider the set of 4 balls labelled 1, 2, 3, and 4 with the action of swapping the location of the balls. Is this a group? Prove why or why not.

- *Inverse Element*: For every swap, we can complete the swap once more to undo the changes the first swap made. So each swap is its own inverse.
- *Identity*: We certainly have the identity element which is to do nothing and this is simply to not do any swaps.
- *Closure*: Each swap simply moves two of the balls around to create a new arrangement of the balls and we know all arangements of the balls are in the set of possibilities for this group so closure exists. So we have shown that this is a group.

Exercise: List all the possible different ways, you can arrange the 4 balls shown above.
Hint: It may be helpful to determine the total number of different arrangements first.
Note: *You can only have one ball in each position.*

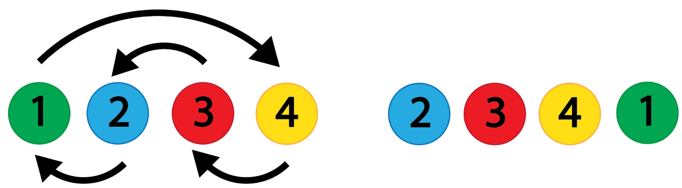
We consider the four spaces the balls can be placed in as ______, ______, ______, ______.
In the first position we can place any of the four balls. Once we've placed the first ball, we have 3 choices for the second position, 2 choices for the third position and the only remaining ball is placed in the fourth position. There are a total of $4! = 4 \times 3 \times 2 \times 1 = 24$ different arrangements/permutations.
All 24 are: 1234 1243 1324 1342 1423 1432 2143 2134 2341 2314 2431 2413 3124 3142 3214 3241 3412 3421 4132 4123 4231 4213 4321 4312

Example.

Suppose I have the 4 balls lined up from 1 to 4 in order. Instead of swapping, let's relocate each ball to a different position.

- I move the ball from the **first position** to the **fourth position**
- I move the ball from the **second position** to the **first position**
- I move the ball from the **third position** to the **second position**
- I move the ball from the **fourth position** to the **third position**

What does my final arrangement look like?



We use arrows tell us where each ball is going. After relocating each ball, we have our new arrangment/permutation of our 4 balls.

We write this mathematically as:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

Here we have an array of numbers, where the top row indicates the which position we are referring to initially, and the bottom number indicates which position we are sending the ball. For example, below the number 1 on the top row is 4. The ball that is located in position 1 is now placed in the fourth position. Similarly the ball in position 2 on it is now placed in the first position and so forth.

Exercise: What does my final arrangement look like for the following permutation:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

Permutation Groups - Multiple Rearrangements

Here is something more interesting, let's move every ball from a position **twice**. Suppose I have 4 balls as labelled above.

- I move the ball from the **first position** to the **second position**
- I move the ball from the **second position** to the **third position**
- I move the ball from the **third position** to the **fourth position**
- I move the ball from the **fourth position** to the **first position**

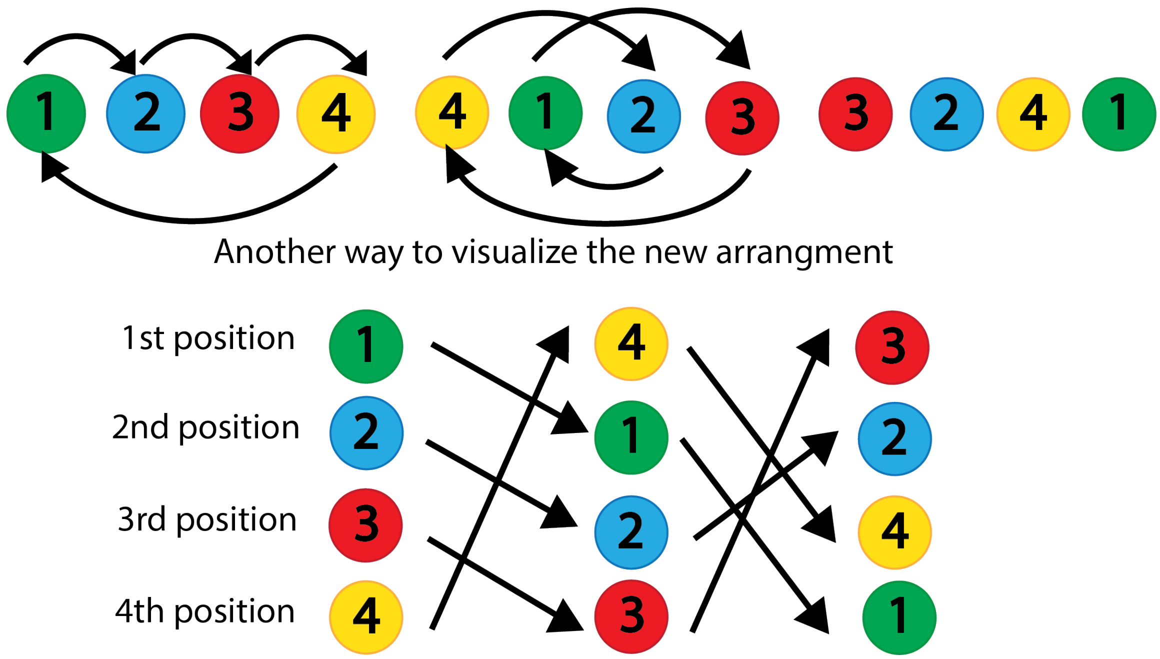
Now with the balls already moved once from their initial position. Let's move them again.

- I move the ball form the **first position** to the **third position**
- I move the ball from the **second position** to the **fourth position**
- I move the ball from the **third position** to the **second position**
- I move the ball from the **fourth position** to the **first position**

We write this mathematically as:

$$\overbrace{\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}}^{\text{Second Rearrangment}} \overbrace{\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}}^{\text{First Rearrangement}}$$

When we combine permutations, we read from **right to left**. What does the final configuration look like?



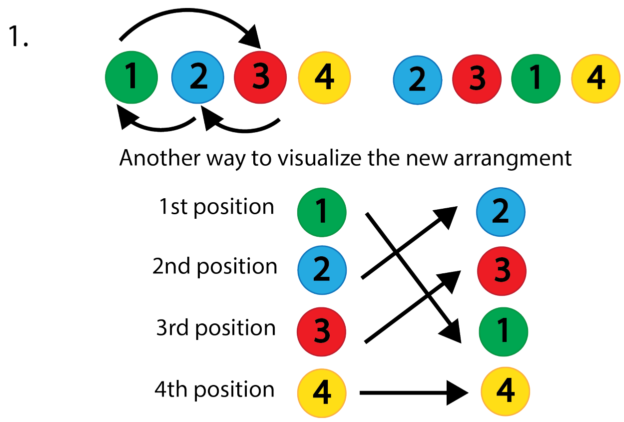
We can express our solution mathematically as

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$$

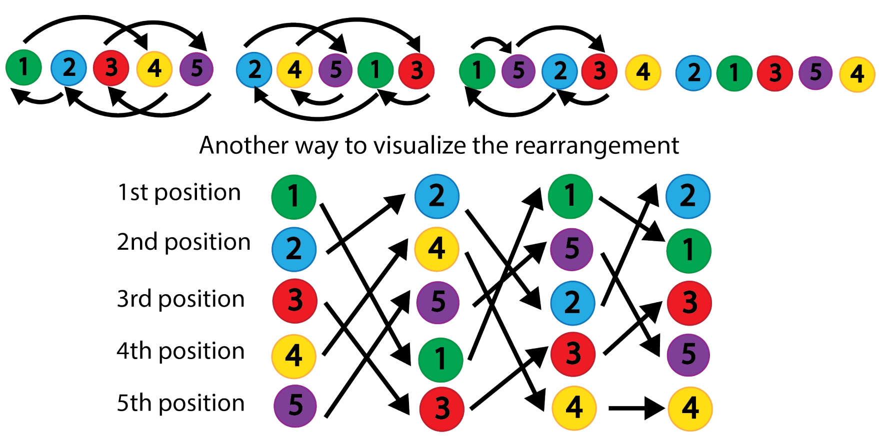
Exercise:

For the following rearrangement actions, determine the equivalent action. and draw the final configurations of where the balls are.

1. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$



2. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 2 & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 5 & 2 & 3 \end{pmatrix}$



Mathematically, we can re-express the three permutations as a single equivalent permutation.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 2 & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 5 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 5 & 4 \end{pmatrix}$$

Undoing the Rearrangement

Suppose we are given the rearrangement rule in the array below, how can return all the balls back to it's original position?

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$$

1. Can you create another rearrangement rule that returns all the balls to their initial position?

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$$

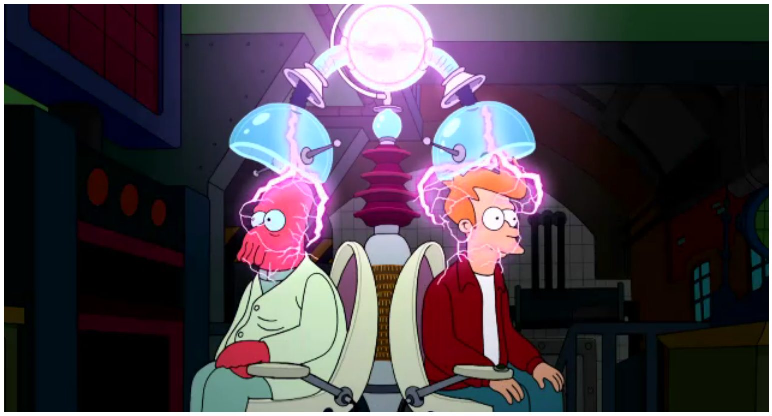
2. Is it possible to keep applying the same rearrangement rule to return all the balls to their initial position?

   Yes, it is possible to keep rearranging the objects in the same manner until we reach the original arrangement. Observe that

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

   In other words, we need to rearrange the objects using the same permutation rule 4 times before we arrive at the same arrangement that we started with.

The Futurama Problem



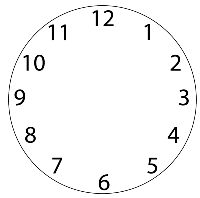
An episode of Futurama, the prisoner of Brenda, received critical acclaim for popularizing math. In this episode, Professor Farnsworth and Amy build a machine that allows them to switch minds. However, the machine can only switch minds between two bodies only once, so they are unable to return to their bodies. In an attempt to return to their original bodies, they can invite other people to switch bodies with them. Is it possible for everybody to return to their original body? If so, how can this be done? How many people do they need to invite?

We need to invite 2 extra people to return everyone to their bodies. We will use capital letters to denote the body and a lower case letter to indicate that's their mind. For example A_a means that person A's body has person A's mind. Similarly A_b means that person A's body has person B's mind. In the chart below, the underline indicates which 2 people are going to switch next. We shall start with Person A and Person B switching minds first.

$\underline{A_a}$	$\underline{B_b}$	C_c	D_d
$\underline{A_b}$	B_a	C_c	$\underline{D_d}$
A_d	$\underline{B_a}$	$\underline{C_c}$	D_b
$\underline{A_d}$	B_c	$\underline{C_a}$	D_b
A_a	$\underline{B_c}$	C_d	$\underline{D_b}$
A_a	B_b	$\underline{C_d}$	$\underline{D_c}$
A_a	B_b	C_c	D_d

Problem Set

1. **Clock**. The clock is an interesting source of symmetry which naturally makes it of mathematical interest.



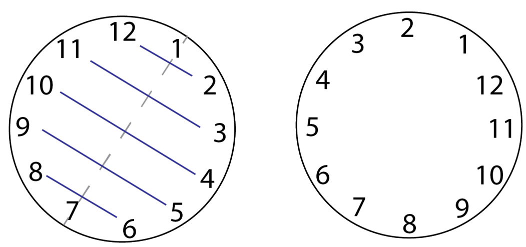
   (a) Suppose we can only rotate the clock by 1 hour. How many possible rotations are there?

   There are 11 possible rotations. There are not 12 rotations because the 12th rotation returns the clock to its initial state.

   (b) How many possible reflections are there? A reflection is done by drawing between two numbers on a clock diametrically opposite away from each other (equal distance away from other). For example 12 and 16 are diametrically opposite as well as 10 and 4. Then all the number reflect across that line.

   There are 6 possible reflections.

   (c) If I combine to reflections together, what is their equivalent action?

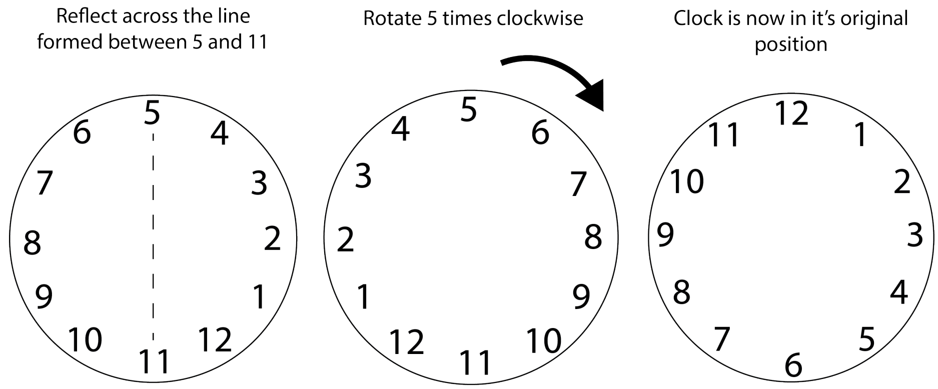


7 and 1 are diametrically opposite so the line from 7 to 1 is the line of reflection
blue line indicates which numbers are swapped

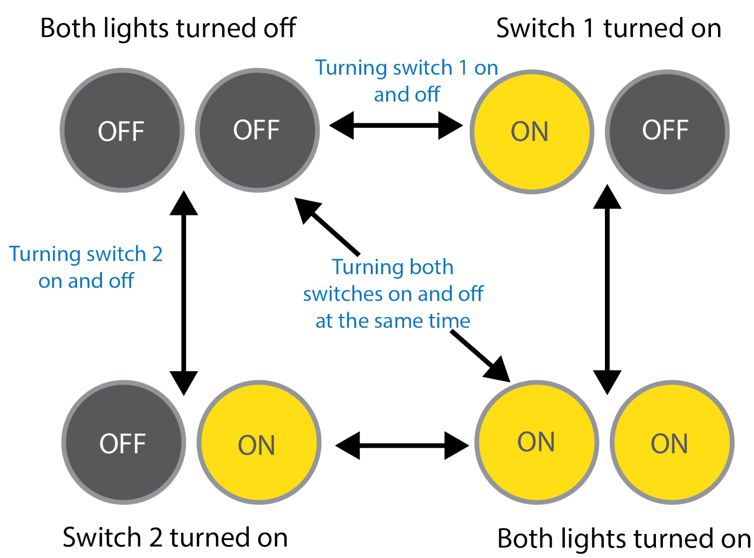
   Two of the same reflection i.e when we reflect across two numbers diametrically opposite each other twice results is the same as not doing anything to the clock. Two different reflections results in a rotation.

   (d) The clock below is scrambled. Can you using just rotations and reflections, return the clock back to it's normal face? How many actions do you require? Can you come up with multiple ways?

Solutions may vary, but one possible way is shown below.

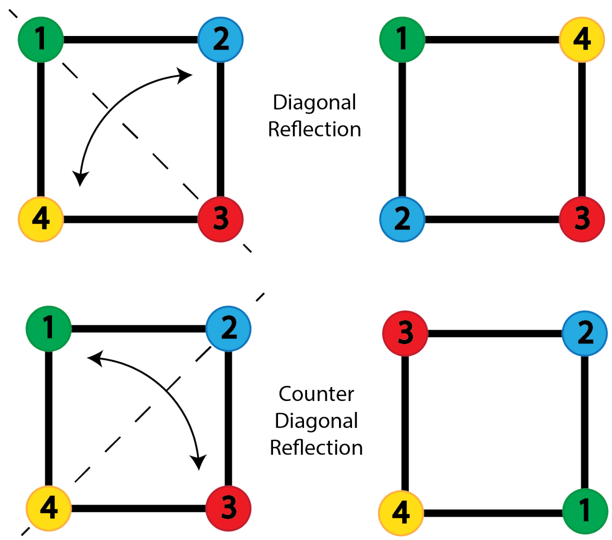


2. **The Light Switch**. Suppose we have two light switches one next to the other. You have the following actions - flipping the first switch, flipping the second switch, switching both switches, and as usual doing nothing. Draw all the possible configurations. Is this a group?



From the diagram above, we can see that every action is indeed reversible and any two combination of two actions will result in an action we allowed. Hence the two light switches form a group.

3. Using the square below (the same as the class example), but now we add a reflection diagonally



With the addition of these two actions (reflection diagonally) F_d and a reflection counter diagonally F_c, along side the actions we did in class i.e. rotate by 90° $R_{90°}$, rotate by 180° $R_{180°}$, rotate by 270° $R_{270°}$, horizontal reflection F_h, and vertical reflection F_v. Draw out the Cayley Table. After seeing the Cayley Table, determine if this is a group.

Action	**I**	$\mathbf{R_{90°}}$	$\mathbf{R_{180°}}$	$\mathbf{R_{270°}}$	$\mathbf{F_v}$	$\mathbf{F_h}$	$\mathbf{F_d}$	$\mathbf{F_c}$
I	I	$R_{90°}$	$R_{180°}$	$R_{270°}$	F_v	F_h	F_d	F_c
$\mathbf{R_{90°}}$	$R_{90°}$	$R_{180°}$	$R_{270°}$	I	F_c	F_d	F_v	F_h
$\mathbf{R_{180°}}$	$R_{180°}$	$R_{270°}$	I	$R_{90°}$	F_h	F_v	F_c	F_d
$\mathbf{R_{270°}}$	$R_{270°}$	I	$R_{90°}$	$R_{180°}$	F_d	F_c	F_h	F_v
$\mathbf{F_v}$	F_v	F_d	F_h	F_c	I	$R_{180°}$	$R_{90°}$	$R_{270°}$
$\mathbf{F_h}$	F_h	F_c	F_v	F_d	$R_{180°}$	I	$R_{270°}$	$R_{90°}$
$\mathbf{F_d}$	F_d	F_h	F_c	F_v	$R_{90°}$	$R_{270°}$	I	$R_{180°}$
$\mathbf{F_c}$	F_c	F_v	F_d	F_h	$R_{90°}$	$R_{270°}$	$R_{180°}$	I

4. Simplify multiple permutation actions as one equivalent permutation action and draw out the final configuration.

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$$

5. Let $\mathbb{Q}$ be the set of all rational numbers with the action of addition. Prove that this is a group.

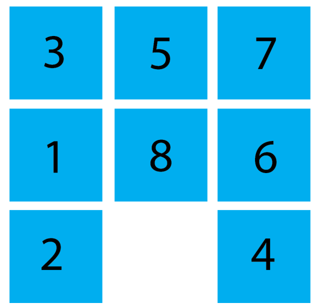
   - *Inverse Element*: We must first show that every action is reversible by another action. Whenever we add a rational number $\frac{a}{b}$, we can reverse the action by subtracting $\frac{a}{b}$ or adding $\frac{-a}{b}$.
   - *Identity*: Clearly if we add 0 to any rational number, we are not changing anything so adding 0 is the identity action and 0 is the **identity element**.
   - *Closure*: Adding any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ results in:

   $$\frac{a}{b} + \frac{c}{d} = \frac{a \times d}{b \times d} + \frac{c \times b}{d \times b} = \frac{a \times d + c \times b}{b \times d}$$

   Since a, b, c, d are integers, then $a \times d$, $c \times b$ and $b \times d$ are integers and so $a \times d + c \times b$ is an integer which makes $\frac{a \times d + c \times b}{b \times d}$ a rational number which is an element in our set so closure exists.

   All required conditions are met so the set of rational numbers with the action of addition is a **group**.

6. **Sliding Puzzle** In the sliding puzzle, there is vacant spot, you may move any block adjacent to the vacant space (either horizontally or vertically in the vacant spot). Is it possible to keep moving these places around given one space to arrange the 3×3 block into block that puts all the number in order?



Yes it is possible. Solutions may vary.

7. **Three Cups Problem** We are given three cups. One cup is upside down, and the other two is right-side up. The objective is to turn all cups right-side up in no more than six moves. Each time, you must turn over exactly two cups per move. Is this possible?

Suppose we start with 2 right cups and 1 wrong cup. By changing 1 right and 1 wrong, situation remains the same. By changing 2 rights, we land up at 3 wrongs. Next move takes us back to the original position of 1 wrong. Thus, any number of moves leaves us either with 3 wrongs or with 1 wrong, and never with 0 wrongs. More generally, this argument shows that for any number of cups, we cannot reduce the number of wrongs to 0 if we initially start with an odd number of cups.

8. **The $100 Prize** The principal of a school offers 100 students, who are numbered from 1 to 100, a chance to win $100 each. A room contains a cupboard with 100 drawers. The principal randomly puts one student's number in each closed drawer. The students enter the room, one after another. Each student may open and look into 50 drawers in any order. The drawers are closed again afterwards. If, during this search, every student finds his number in one of the drawers, all students win the $100 prize each. If just one student does not find his number, no student wins. Before the first student

enters the room, the students may discuss strategy - but may not communicate once the first student enters to look in the drawers. What is the students' best strategy?

Surprisingly, there is a strategy that provides a winning probability of more than 30%. The key to success is that the students do not have to decide beforehand which drawers to open. Each student can use the information gained from the contents of previously opened drawers to help decide which drawer to open next. Another important observation is that this way the success of one student is not independent of the success of the other students.

To describe the strategy, not only the students, but also the drawers are numbered from 1 to 100, for example row by row starting with the top left drawer. The strategy is now as follows:

- Each student first opens the drawer with their own number.
- If this drawer contains their number, they are done and they were successful.
- Otherwise, the drawer contains the number of another student and the student next opens the drawer with this number.
- The student repeats steps 2 and 3 until they find their own number or has opened 50 drawers.

This approach ensures that every time a student opens a drawer, they either find their own number or the number of another student they have not yet encountered.