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Centre for Education in
Mathematics and Computing

Grade 7/8 Math Circles

November $26^{th}/27^{th}/28^{th}$, 2019

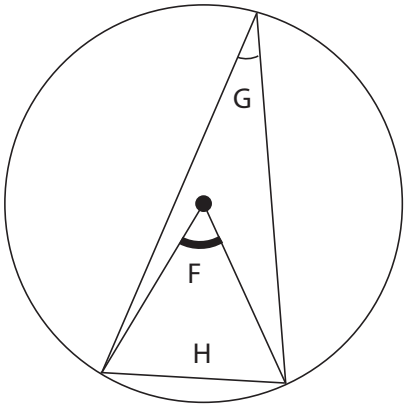
Math Jeopardy Solutions

Introduction

Questions will vary in difficulty with \$100 questions tending to be the easiest, and \$500 questions tending to be the hardest. Do your best, good luck and have fun!

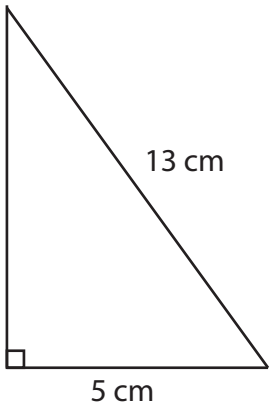
Shapes, Shapes, Shapes

\$100 What does each label represent?



F - Central Angle
G - Inscribed Angle
H - Chord

\$200 What is the area of this triangle?

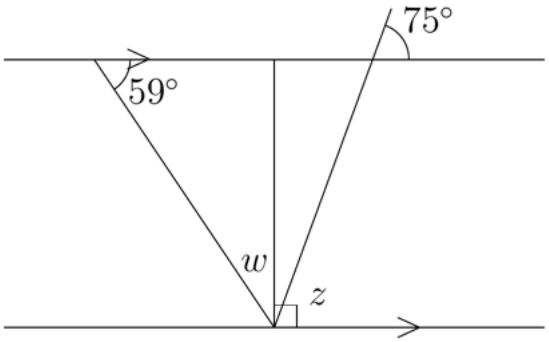


Let x be the missing side length.
Use Pythagorean Theorem to find x:

$$\begin{aligned} 13^2 &= 5^2 + x^2 \\ x^2 &= 169 - 25 \\ x &= 12 \end{aligned}$$

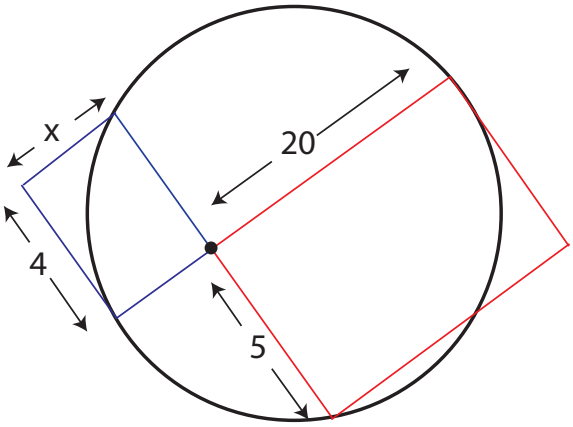
The area is $\dfrac{\text{base} \times \text{height}}{2} = \dfrac{12 \times 5}{2} = 30\text{cm}^2$.

$300 Find the missing angles:



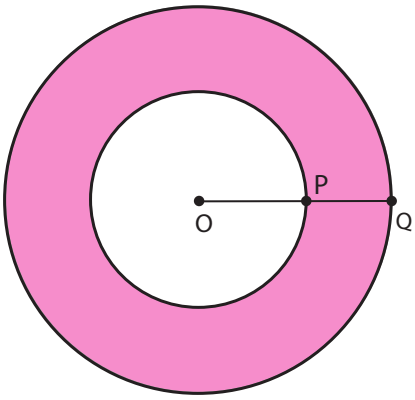
Using the fact that opposite angles are equal and using the Z pattern covered in Triangles, $z = 75°$.
There exists a triangle with angles 59°, w, and 90°. Using the fact that the sum of the angles of a triangle is 180°, $w = 31°$.

$400 Find the ratio between the areas of the two rectangles.



Using the **Crossed Chord Theorem (CCT)** from *Circles*, $20 \times x = 4 \times 5$ so $x = 1$.
Then the areas are 100cm^2 and 4cm^2 so the ratio between the areas is **25:1**.

$500 In the diagram, each of the two circles have centre O. Also, $OP : PQ = 1 : 2$. If the radius of the large circle is 9, what is the area of the shaded region?



The radius of the large circle, OQ = 9 cm.
Then OP + PQ = 9 cm.
Additionally, PQ = 2×OP.
Using this, OP + 2OP = 9 cm.
This means OP = 3 cm.

The area of the larger circle is $9^2\pi\text{cm}^2$.
The area of the smaller circle is $3^2\pi\text{cm}^2$.
The area of the shaded region is $81\pi - 9\pi = 72\pi\text{cm}^2$.

Physics

$100 Express the following in scientific notation.

$$0.00000327$$

$$3.27 \times 10^{-6}$$

$200 What is Newton's First Law?

"An object at rest will remain in rest, and an object in motion will remain in motion unless acted upon by an external force."

$300 *Proportionality* A circle has area A. If I multiply the diameter of the circle by 3, what is my new area in terms of A?

The area of the circle is $A = \pi \times r^2$ where r is the radius or $r = \frac{d}{2}$ so $A = \pi \times \frac{d}{2}^2$.

Multiplying the diameter by 3 gives $r = \frac{3d}{2}$. The new area is:

$$A_{new} = \pi \times \frac{3d}{2}^2 = \pi \times \frac{9d^2}{4}$$

Comparing this to the previous area gives:

$$A_{new} = 9A$$

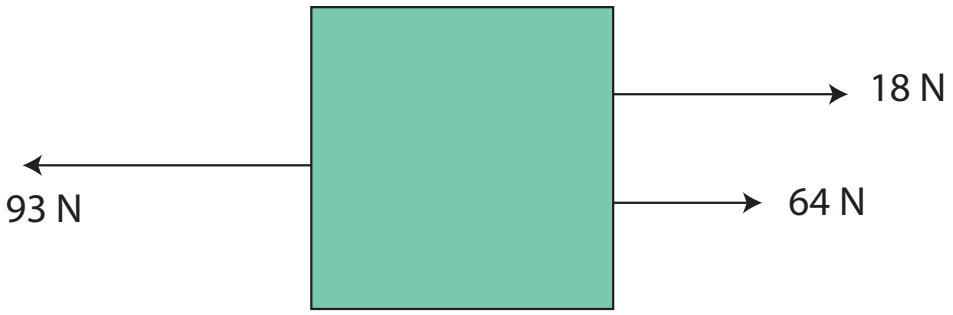
$400 Nicolas pushes open a 4kg door. The door accelerates at a rate of $9\frac{m}{s^2}$ away from him. How much force did Nicolas apply to the box?

We know that Force = Mass × Acceleration. Using this:

$$\begin{aligned} \text{Force} &= 4\text{kg} \times 9\frac{m}{s^2} \\ \text{Force} &= 36N \end{aligned}$$

Nicolas applied a force of **9 N**.

$500 If the following box is accelerating at a rate of $2\frac{m}{s^2}$, what is its mass?



We know Net Force = Mass × Acceleration.
Net Force = 93 [left] + 18 [right] + 64 [right]
Net Force = 93 [left] - 18 [left] - 64 [left]
Net Force = 11 [left]

$$\begin{aligned} \text{Net Force} &= Mass \times 2\frac{m}{s^2} \\ 11N &= Mass \times 2\frac{m}{s^2} \\ \text{Mass} &= 11N \div 2\frac{m}{s^2} \\ \text{Mass} &= 5.5\text{kg} \end{aligned}$$

The box has a mass of **5.5kg**.

It's Probable

$100 How big is the sample space if you roll three 6-sided die?

Let event A be the result of the first die.
Let event B be the result of the second die.
Let event C be the result of the third die.
Each of event A, B, C have 6 possible outcomes.
Using the Product Rule, the total number of possible outcomes is $6 \times 6 \times 6 = 6^3 = 216$.
The sample space has size **216**.

$200 Six balls, numbered 2, 3, 4, 5, 6, 7, are placed in a hat. You select 2 balls without replacement. What is the probability that both balls you choose are prime numbers?

When picking the first ball, there are 6 total choices.
Of the 6, 4 are prime numbers *(2, 3, 5, 7)*.
Suppose you pick the first ball and it is prime.
Now you have 5 total choices and 3 of those will be prime since you've already picked one of the other balls with a prime number.
Using the Product Rule, the probability of this is:

$$\frac{4}{6} \times \frac{3}{5} = \frac{2}{5}$$

$300 The Ministry of Magic is holding a lottery and has sold 2000 tickets. If Hermoine has a $\frac{1}{16}$ chance of winning, how many tickets did she purchase?

Let t be the number of tickets Hermoine has.
Since she has a $\frac{1}{16}$ chance of winning, then:

$$\frac{\text{Number of tickets Hermoine has}}{\text{Total number of tickets}} = \frac{t}{2000} = \frac{1}{16}$$

Using this we can find that Hermoine purchased $\mathbf{t = 125}$ tickets.

$400 Sam rolls a fair 4-sided die containing 1, 2, 3, 4. Tyler rolls a fair 6-sided die containing 1, 2, 3, 4, 5, 6. What is the probability that Sam rolls a number larger than Tyler?

Let (s, t) be the pair of numbers that Sam and Tyler roll.

Using the Product Rule, there are a total of $4 \times 6 = 24$ possible outcomes.

Of the total possible outcomes, the ones that satisfy the condition are:

$$(2, 1), (3, 2), (3, 1), (4, 3), (4, 2), (4, 1)$$

The probability is:

$$\frac{6}{24} = \frac{1}{6}$$

$500 Two different numbers are randomly selected from the set {-3, -1, 0, 2, 4} and then multiplied together. What is the probability that the product of the two numbers chosen is 0?

Here, we have to pay attention to the fact that in multiplication, $2 \times 4 = 4 \times 2$ and so the order of the numbers doesn't matter.

Using this, there are 10 different multiplications we can do where the two numbers are different.

Of those 10, to get a product of 0, we must have 0 times another number or $(0 \times -3), (0 \times -1), (0 \times 2), (0 \times 4)$.

The probability is:

$$\frac{4}{10} = \frac{2}{5}$$

Sorting Remainders

$100 What are all possible remainders when you divide by 9?
The possible remainders are 0, 1, 2, 3, 4, 5, 6, 7, and 8.

$200 Evaluate the following:

$$63 \equiv \underline{\quad 0 \quad} \mod 9$$
$$42 \equiv \underline{\quad 2 \quad} \mod 5$$
$$765 \equiv \underline{\quad 1 \quad} \mod 4$$

$300 Reduce the expression:

$$(81+26) \times (70+52) \mod 7$$

$$\begin{aligned}(81+26) \times (70+52) \mod 7 &= (4+5) \times (3+0) \mod 7 \\ &= 9 \times 3 \mod 7 \\ &= 27 \mod 7 \\ &= \mathbf{6} \mod 7\end{aligned}$$

$400 Sort the following list of numbers in descending ordering using the insertion method covered in class. How many steps did it take you?

38 4 13 72 96

Recall that each time you move the next number from the unsorted list to the sorted list, that counts as 1 step.
Additionally, after you've moved a number to the sorted list, each comparison you make till the number is *inserted* into the right space is also 1 step.
We first move 38 to the sorted list, that is 1 step.
We then move 4 to the end of the sorted list and the list is in descending order so that is 1 step.
Moving 13 to the end of the sorted list and completing 1 comparison gives 2 steps.
Inserting 72 requires 4 steps and inserting 96 requires 5 steps. In total we have:

$$1+1+2+4+5 = 13 \text{ steps}$$

$500 Reduce the following:

$$2^{82} \mod 3 = (2^2)^{41} \mod 3 = (1)^{41} \mod 3 = 1 \mod 3$$
$$5^{46} \mod 3 = (5^2)^{23} \mod 3 = (1)^{23} \mod 3 = 1 \mod 3$$
$$2^{164} \times 5^{138} \mod 3 = (2^{82})^2 \times (5^{46})^3 \mod 3 = 1^2 \times 1^3 \mod 3 = 1 \mod 3$$

Miscellaneous

$100 Adam and Eve play rock-paper-scissors 10 times. Knowing the following, who won and by how much?

- Eve uses 3 rocks, 6 scissors, 1 paper
- Adam uses 2 rocks, 4 scissors, 4 paper
- There were no ties in all 10 games
- The order of the games is unknown

Note that Eve played 6 scissors. Since there cannot be any ties, then the 2 rocks and 4 papers that Adam played must have been played with the 6 scissors. Of those 6, Eve won 4 with scissors against paper and Adam won 2 with rock against scissors.
In the remaining games, Adam played 4 scissors and Eve played 3 rocks and 1 paper. Of the 4 games, Even won 3 with rock against scissors and Adam won 1 with paper against rock.
In total, Eve won 7 games and Adam won 3.

$200 Given the following equivalences, what's the missing number?

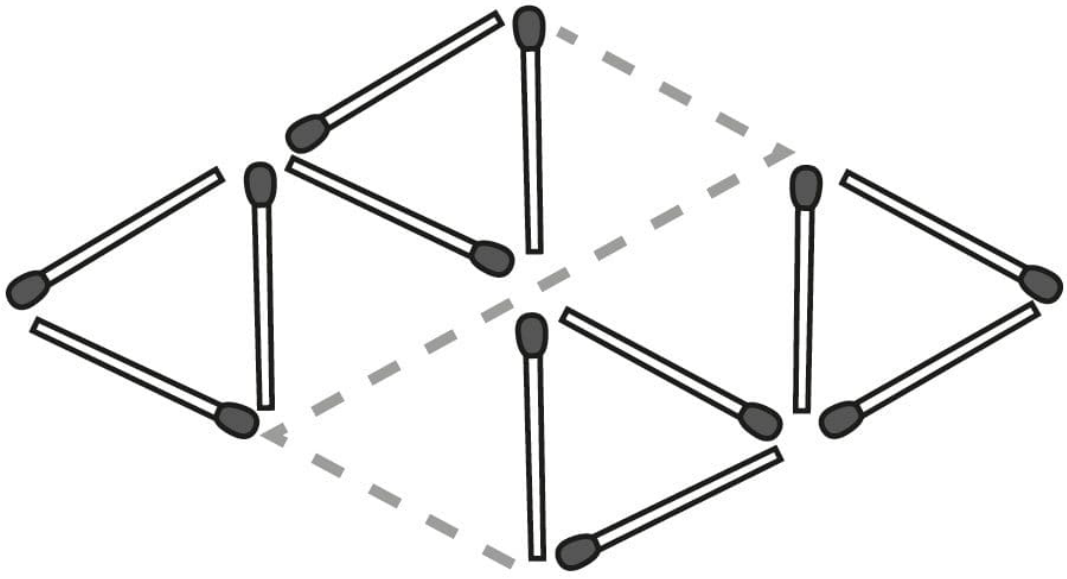
$$12 = 6$$

$$6 = 3$$

$$5 = \underline{\quad 4 \quad}$$

Twelve has 6 letters. Six has 3 letters. Five has 4 letters.

$300 The following 16 matches form 8 equilateral triangles. Remove **4** matches to leave exactly equilateral triangles, leaving no loose ends or unused matches.



$400 Mr. and Mrs. Tan have 4 children - 3 boys and 1 girl who each like one of the colours blue, red, green, yellow and the letters P, Q, R, S. Based on the following facts, which child is Darius?

- The oldest child likes the letter Q.
- The youngest child likes green.
- Alfred likes the letter S.
- Brenda has an older brother who likes R.
- The one who likes blue isn't the oldest.
- The one who likes red likes the letter P.
- Charles like yellow.

This is similar to the Einstein riddle. Solving this, you will get that Darius is the **second** oldest child.

$500 Solve the following Sudoku puzzle. *Each row, column and* 3×3 *square can contain the numbers 1-9 only* ***once****.*

1	5	9	3	4	2	7	8	6
2	7	4	5	6	8	3	1	9
8	3	6	1	9	7	4	5	2
7	1	8	9	2	6	5	4	3
4	9	3	8	5	1	6	2	7
5	6	2	4	7	3	1	9	8
3	2	1	6	8	5	9	7	4
9	8	5	7	3	4	2	6	1
6	4	7	2	1	9	8	3	5

Gauss Contest

Solutions for the questions below can be found on the CEMC website under ***Past Contests****.*

$100 If x is a number between 0 and 1, which of the following represents the smallest value?
(Source: 2011 Gauss (Grade 8), #17)

(A) x (B) x^2 (C) $2x$ (D) $\sqrt{x}$ (E) $\frac{1}{x}$

$200 A fraction is equivalent to $\frac{5}{8}$. Its denominator and numerator add up to 91. What is the difference between the denominator and numerator of this fraction?
(Source: 2006 Gauss (Grade 7), #16)

The fraction is $\frac{35}{56}$ and so the difference is 21.

$300 If each of the fours numbers 3, 4, 6, and 7 replaces a $\square$, what is the largest possible sum of the fractions shown? $\frac{\square}{\square} + \frac{\square}{\square}$
(Source: 2010 Gauss (Grade 7), #19)

$\frac{7}{3} + \frac{6}{4} = \frac{23}{6}$

$400 Lorri took a 240 km trip to Waterloo. On her way there, her average speed was 120 km/h. She was stopped for speeding, so on her way home her average speed was 80 km/h. What was her average speed, in km/h, for the entire round-trip?
(Source: 2007 Gauss (Grade 8), #20)

96 km/h

$500 Five students wrote a quiz with a maximum score of 50. The scores of four of the students were 42, 43, 46 and 49. The score of the fifth student was N. The average (mean) of the five students' scores was the same as the median of the five students' scores. The number of values of N which are possible is?
(Source: 2006 Gauss (Grade 7), #25)

3 possible values for N: 35, 50, 45

Final Jeopardy

How many different pairs (m, n) can be formed using numbers from the list of integers {1, 2, 3, ..., 20} such that $m < n$ and $m + n$ is even?
(Source: 2010 Gauss (Grade 7), #24)

90 different pairs