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Grade 7/8 Math Circles

October 22/23/24 2019

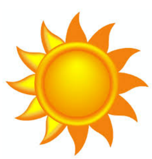
An Introduction to Physics

Scientific Notation

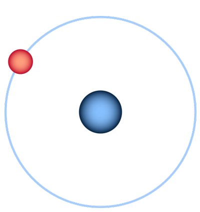
Math and Physics have a closely related relationship. One of the goals in physics is to give us perspective about the things in our universe by analyzing the relationships presented to us by math.

Some of the numbers we come across in all fields of science are extremely large or extremely small. This makes numbers very bothersome to write out.

For example:



The mass of the sun is approximately
1,989,000,000,000,000,000,000,000,000,000,000 kilograms.

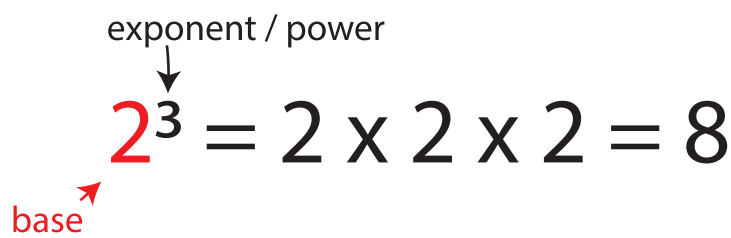


The mass of an electron is approximately
0.000000000000000000000000000000911 kilograms.

Writing these numbers would be very time consuming and make calculations very hard to follow. We use **Scientific Notation** to express numbers that are too big or too small to be conveniently written in “normal form.”

<u>Review:</u> Exponents

Applying an exponent to a number means to multiply the number by itself the amount of times equal to the exponent. For example:



The exponent 3 is applied to the base 2. This means to evaluate 2^3 we calculate the **base number** multiplied by itself **exponent** times.In this case 2 multiplied by itself 3 times or $2 \times 2 \times 2 = 8$

Exercise: Powers (Exponents) of 10

Calculate the following powers of 10. Look for a pattern in your answers.

a) 10^0

b) 10^1

c) 10^2

d) 10^3

e) 10^6

f) 10^{10}

g) What is the pattern we can notice working with powers of 10?

Working with Negative Exponents/Powers:

Negative exponents work very similar to positive exponents. Consider the fact that all rational numbers can be expressed as a fraction.For example, 2 can also be written as $\frac{2}{1}$.

A negative exponent performs the same operation as with a positive exponent, just on the opposite side of the fraction line. For example:

$$\frac{2^{-3}}{1} = \frac{1}{2^3} = \frac{1}{2 \times 2 \times 2} = \frac{1}{8}$$

Exercise: Negative Powers of 10

Calculate the following powers of 10. Look for a pattern in your answers. The pattern created by powers of 10 makes them very useful in easily expressing large numbers.

a) 10^{-1}

b) 10^{-2}

c) 10^{-3}

d) 10^{-5}

e) 10^{-10}

f) What is the pattern we can notice working with negative powers of 10?

Multiplying by Powers of 10

Using the patterns earlier we see something interesting when multiplying numbers by powers of 10. For example:

$$5 \times 10^2 = 5 \times 100 = 500 \qquad\qquad 5 \times 10^{-2} = \frac{5}{100} = 0.05$$

There is a distinct pattern we can notice:

When multiplying by **positive** powers of 10 the decimal place moves to the **right** the same amount of spaces as the power.

When multiplying by **negative** powers of 10 the decimal place moves to the **left** the same amount of spaces as the power.

With this knowledge we can express any number in terms of powers of 10. We can call this **scientific notation**:

Scientific Notation has the form: $N \times 10^n$

N is a number between 1 and 10.

n is a positive or negative integer.

Exercise:

1. What is the purpose of using scientific notation?

2.

Multiplying by positive powers of 10 moves the decimal place to the ____________

Multiplying by negative powers of 10 moves the decimal place to the ____________

3. Write out the following numbers in normal form.

a) 1×10^{6} b) 3×10^{-2} c) 10×10^{5}

d) 1.26×10^{3} e) 7.55×10^{-3} f) 1.16×10^{10}

Converting numbers into Scientific Notation:

Now we want to go the other way and convert numbers from normal form to scientific notation. Let's try to convert the number 323,000 into scientific notation form.

1. Find our number N from 1 to 10 by moving the decimal place.

323000. N=3.23

2. Find the power n needed to move the decimal place back to its "original place."

323000. 5 spaces right

To move 5 spaces right we use n=5.

Our number in scientific notation is 3.23×10^{5}.

Note: process is identical for negative numbers

Scientific Notation On a Scientific Calculator:

1. Enter the number N.
2. Press the multiplication button.
3. Enter the number 10.
4. Hit the exponent button ˆ .
5. Enter the power n.

OR

1. Enter the number N.
2. Hit the "EXP" button.
3. Enter the power n.

Exercise: Express the following numbers in scientific notation.

a) 760

b) $36,700$

c) $564,000,000$

d) 0.034

e) 0.00245

f) 0.00000679

g) 0.12

h) Speed of light: 299,000,000 m/s

i) 0.00000423

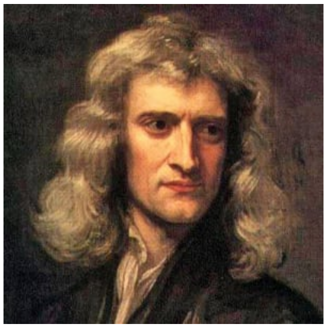
j) Mass of the Sun: $1,989,000,000,000,000,000,000,000,000,000,000$ *kilograms*

k) Mass of the electron: 0.000000000000000000000000000000911 *kilograms*

l) Age of the universe: $13,800,000,000$ *years*

m) Avogadro's Number: 602,252,000,000,000,000,000,000

Physics: Newton's Laws of Motion



Sir Isaac Newton was an English physicist and mathematician who is often credited with "discovering" gravity, but he did much more than that. In fact, Newton invented most the field of physics as we know it today. His three laws of motion are at the core of classical physics. In physics, a **law** is an undisputed truth that describes how the universe behaves under certain conditions.

Newton's First Law:

"An object at rest will remain at rest, and an object in motion will remain in motion unless it is acted upon by an external force."

If an object is still, I must apply a force to cause it to move. To apply a force I can either:

- Push the object (in any direction)
- Pull the object (in any direction)

If an object is moving, it will travel in the same speed in the same direction until a force is applied. I can apply a force to slow down/speed up my object and/or change its direction.

1. If I roll a ball along the floor why does it eventually stop?

2. What happens if I throw a ball in outer space?

3. Can you name some of the forces in the universe? *Hint: One is what keeps us on the ground.*

<u>Newton's Second Law:</u>

"The force acting on an object is equal to the mass of that object times its acceleration."

$$Force = Mass \times Acceleration \rightarrow F = ma$$

In order for us to change an object's motion we must change its acceleration with a force.

Acceleration: The rate at which an object changes its speed and direction.

- A positive acceleration means the object is speeding up in the direction we are measuring.

- A negative acceleration means the object is slowing down in the direction we are measuring.

We describe different quantities with different measures called units. For examples a centimetre is a **unit** of measurement. **Units:**

- Mass is measured in kilograms. Symbol: "**kg**"
- Acceleration is measured in "$\boldsymbol{m/s^2}$"
- Force is measured in Newtons. Symbol: "**N**"

Practice:

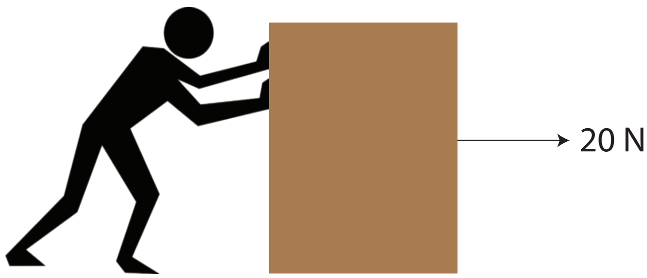
Craig pushes a 5 kg box along the table. It accelerates at a rate of $5m/s^2$ away from him. How much force did Craig apply to the box?

Craig pushes a 10 kg box along the table with the same force. What is the amount of acceleration of the box now?

If we have forces in different directions we have to sum them up to find the total or **net force.** In order to find the net force we have to take into account the different directions of each force.

It is important to note that **Force and acceleration are what we call vectors. Vectors have a direction and a magnitude.**

- **Magnitude:** The size/amount of a quantity. *Examples: height, mass, length*
- **Direction:** The course that something moves. *Examples: left, right, up, south, west*

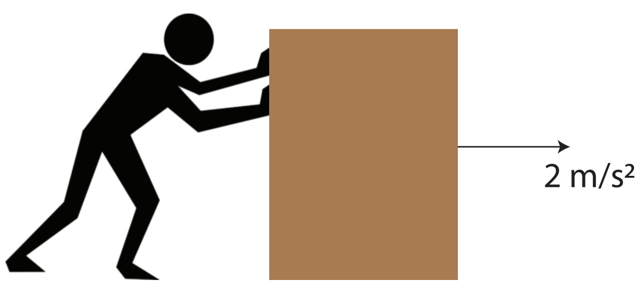


In the diagram we see that this person is pushing this box to the right with a force of 20 Newtons. The **magnitude of our force is 20 N** and the **direction of our force is right.** We write this as:

Magnitude

Force = 20 N [right]

Direction



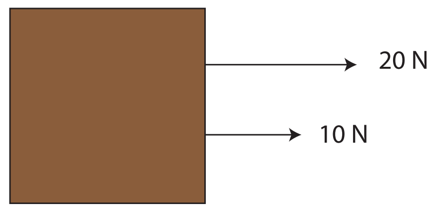
If the box was now accelerating $2m/s^2$ to the right from the forceful push we could write this as: $Acceleration = 2m/s^2$ [right]

Usually objects have multiple forces in different directions acting on them. We need to find a way to add up all of the forces to find the net force. Note: The net force also has magnitude and direction. Once we find net force we can use it as our force in or equation $F = ma$.

$$Force_1 + Force_2 + Force_3 + ... = Net\ Force = ma$$

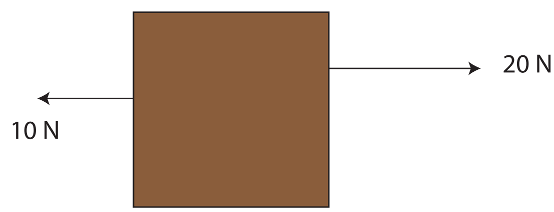
In order to add forces we have to take their directions into account:

- Forces in the same direction add.



Net Force = 20 N [right] + 10 N [right] = 30 N [right]

- For forces in the opposite direction we have to do some more thinking.



Here our equation for net force looks like this:

Net Force = 20 N [right] + 10 N [left]

Our two forces are in opposite directions, one to the right and one to the left. We cannot say we are pushing the box with a force of 30 N to the right anymore.

Consider pushing a box yourself. Your friend then comes and pushes the other side of the box toward you. Clearly it will be harder for you to move the box now. His force is acting against yours and has subtracted from the original force you applied.

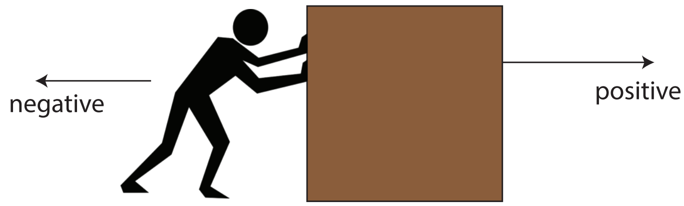
Say you were pushing the box with a force of 20 N and your friend came and pushed the other way with a force of 10 N. He has subtracted 10 N of force from your original force by pushing in the original direction, leaving you with only 10 N of force in the direction you were pushing. This would be your net force.

It would look like this mathematically:

$$Force_{net} = Force_1 + Force_2$$

$$Force_{net} = 20 \text{ N [right]} + 10 \text{ N [left]}$$

Let's say I'm pushing to the right. Forces to the left are then opposite to the direction I'm pushing so I will say they are negative.



I can express all of my forces now in the direction I am looking which is the right. I can do this by saying 10 N [left] is the same as -10 N [right]. If I am facing right the force of 10 N is a negative force in the direction im facing.

$$Force_{net} = 20 \text{ N [right]} + \mathbf{10 \text{ N [left]}}$$

$$Force_{net} = 20 \text{ N [right]} - \mathbf{10 \text{ N [right]}}$$

Now that all of my forces are expressed in one direction I can solve for my net force:

$$Force_{net} = 10 \text{ N [right]}$$

I have now solved that my net force was 10 N to the right. Could I say that my net force was also -10 N [left]?

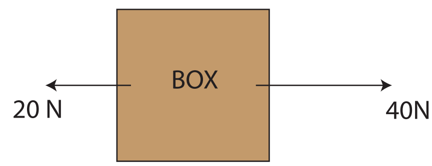
Test:

30 N [right] = _______N [left] -12 N [down] = _______ N [up]

Finding the Net Force

1. Pick a direction to be positive.
2. Forces in that direction are positive, forces in the opposite direction are negative
3. Add all the forces to find the net force.

Example: Push of War



You and your friend are pushing a box weighing 2 kg. If you push to the right with a force of 40 N and they push to the left with a force of 20 N, what is the acceleration of the box (include direction)?

The Net Force is going to be the sum of all the forces acting on the box. We can choose right to be the positive direction. Using Newton's second law, we get:

$$\begin{aligned} F &= ma \\ 40\text{ N [right]} + 20\text{ N [left]} &= 2\text{ kg} \times a \\ 40\text{ N [right]} - 20\text{ N [right]} &= 2\text{ kg} \times a \\ 20\text{ N [right]} &= 2\text{ kg} \times a \\ 20\text{N [right]} \div 2\text{ kg} &= a \\ 10\ m/s^2\text{ [right]} &= a \end{aligned}$$

The box accelerates at a rate of $10m/s^2$ to the right.

Newton's Third Law

"For every action, there is an equal and opposite reaction."

Any force applied by object A onto object B results in a force of *equal size* being applied by object B onto object A in the opposite direction. In other words, if you push the wall with a force of 40 Newtons [right], the wall pushes back on you with a force of 40 Newtons [left]!

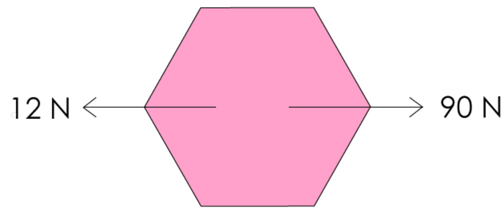
Exercises:

1. Can the acceleration of an object change without changing its speed?

2. If Box A has twice the mass of Box B and you push both boxes with the same amount of force, which box will have the greater acceleration?

3. Use Newton's third law to explain why punching a wall hurts.

4. Consider the following "Free Body Diagram":



(a) Find the net force.

(b) If in the diagram above, the hexagon had a mass of 5 kg, how much would it be accelerating and in what direction would it be accelerating?

(c) With how much force would you have to push to make the hexagonal object accelerate at 10 m/s^2?

(d) If I had a new shiny blue hexagon that accelerated at 10 m/s^2 when the net force in part (a) was applied to it, how much mass would my shiny new hexagon have?

Proportionality

Constants: are exactly as they sound; constant. They have a fixed numerical value which can never change. 2,0, and π are examples of constants.

Variables are values that can change. Essentially values that are not constant. Variables are usually denoted by a letter like x or y.

Proportionality identifies the relationship between two variables in an equation. Lets look at some equations below:

Exercise: Complete the charts below by finding the area and circumference of each circle.

Remember that the formula for area is $A = \pi r^2$ *and the formula for circumference is* $C = 2\pi r$.

Radius	***Diameter***	***Circumference of circle***	***Area of Circle***
1			
2			
3			
4			
5			

Can we identify the variables and constants in our equations for Circumference and Area?

$$A = \pi r^2 \text{ and } C = 2\pi r$$

Diameter and Circumference:

Let's look at the relationship between diameter and Circumference. We can notice that the Circumference is equal to the diameter multiplied by pi at every stage. Further we can see that the ratio of Circumference and diameter is constant as a result. $\frac{C}{d} = \pi \approx 3.14$
If we double or triple diameter, our circumference will also be scaled by the same amounts.

We describe this relationship as **directly proportional**. If we were to scale (multiply) the diameter by some amount then the circumference would scale by that same amount. For example:

$$3 \times diameter \rightarrow 3 \times circumference \text{ or } \frac{1}{2} \times diameter \rightarrow \frac{1}{2} \times circumference$$

We show proportionality with the symbols: $\propto$ or $\sim$. For this example we could say:

$$\text{C} \propto \text{d or C} \sim \text{d}$$

We call the value π the **constant of proportionality** that "connects" circumference and diameter.

We could say the same with the relationships between **radius** $\leftrightarrow$ **diameter** and **diameter** $\leftrightarrow$ **circumference.** Scaling one variable by an amount scales the other variable by the same amount.

Radius and Area:

Let's look at the relationship between area and radius below:

Radius	Area(πr^2)
0	0
1	π
2	4π
3	9π
4	16π

We can notice that when the radius doubles from 1 to 2 the radius quadruples from π to 4 π. Thus when we double the radius the area does not double, it quadruples. Therefore radius and area are not directly proportional.

This can be seen from the equation for area for a circle. The exponent on the radius is what makes this relationship not directly proportional.

From the equation we can see that when we scale the radius, the area will scale by the square of that amount. The result is also shown in the table on the last page. For example if we double radius ($\times 2$) then the area will be multiplied by $2^2 = 2 * 2 = 4$
Tripling the radius ($\times 3$) will result in the area being $3^2 = 9$ times larger.

$$A = \pi r^2$$

Mathematically we say: A $\propto r^2$ or A $\sim r^2$. We could also say "A goes as r^2."

In the equation for circumference we only have an exponent of 1 ("no exponent") on the diameter which is why we have a directly proportional relationship between circumference and diameter.

$$C = \pi d$$

Proportionality allows us to quickly identify expected results in science.

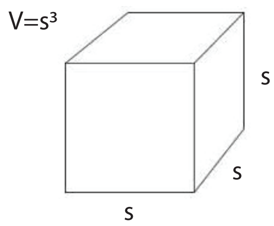
Examples:

1. From Newtons second law we get the equation F=ma. What are the variables and constants in this equation? Consider that I have an object of mass m. I push the object with force F and acceleration a.

   (a) I decide to double the mass and acceleration of my object. What force will I need to do this?

   (b) What if I decide to double the original acceleration a of my object and halve my original mass m? What force will I require?

   (c) I get tired and push my original object with half the original force F with my original mass m. What happens to the acceleration of my object?



2. The equation for the volume of a cube is $V = s^3$, where s is the length of each side of the cube and V is the volume of the cube.

(a) If I double the original side-lengths s of my cube what happens to my volume. What if I halve the original side-lengths?

(b) If i make my sides 10 times larger than my original side-lengths s, How much larger is the volume of my cube? Express your answer in scientific notation.

(c) If I want my volume to be $\frac{1}{27}$ of the original volume V of my cube. How should I change my original side-lengths s?

Problem Set:

1. Express the following numbers in scientific notation:

a) $-12,000$

b) $32,900$

c) 3907

d) $231,000$

e) $-667,000,000,000$

f) $79,000,000$

g) 0.320

h) Age of Earth: 4,600,000,000 years

i) Gravitational Constant: 0.0000000000667

2. Why does Newton's first law make seatbelts in vehicles a necessary safety precaution?

3. A box is resting on a table, what is the net force on the box?

4. A car weighs 1.5 tonnes (1500kg) and accelerates at a rate of $12m/s^2$.

   (a) What is the net force on the car in the direction if its motion?

   (b) Let's say that the car is now at rest. The car's engine then applies a force of 500N to get moving. There is also a force of 50N acting against the motion of the car. Find the net force and acceleration of the car. Include a diagram of the forces on the car.

5. Nasrin decides to push down on the earth with a force of 100 N. If the earth has a mass of $5.972 \times 10^{24} kg$ what is the acceleration of earth from Nasrin's push?

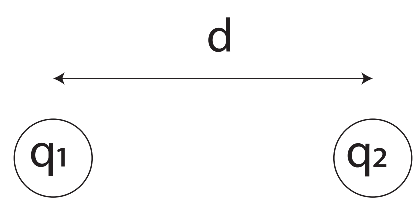
6. You may have heard the saying "opposites attract" before. Well this is true for oppositely charged particles due to the force between them. Each charge applies an equal and opposite charge on the other.

   The force between two charges can be found by the equation:

$$F_{Coulomb} = k \times \frac{q_1 \times q_2}{d^2}$$

   Where:

   - k is a constant 9×10^9
   - q_1 and q_2 are the amount of charge on each particle measured in Coulombs (C)
   - d is the distance between the charges in meters.



   (a) Say the particle are $7 \times 10^{-5} m$ apart. Their charges are $q_1 = 1.6 \times 10^{-19}$ and $q_2 = -1.6 \times 10^{-19}$. Find the Coulomb Force between them.

   Try the remaining questions without using a calculator (use proportionality):

   (b) If the distance between the particles is doubled what do you expect to happen to the force?

   (c) If q_1 doubles in charge but q_2 has its charge cut in half what do you expect to happen to the force?

(d) Opposite charges attract but same sign charges repel (Two positive charges will repel one another.) The Coulomb force should be in one direction if the charges are both positive/negative and in the other if one charge is positive and one charge is negative. Does the equation for Coulomb force reflect this?

7. We stay grounded on earth thanks to the help of gravity. We can find the acceleration downwards due to gravity on our planet with the formula:

$$acceleration_{gravity} = \frac{GM}{R^2}$$

Where:

- G is a constant 6.67×10^{-11}
- M is the mass of the earth: $5.972 \times 10^{24} kg$
- R is the radius of the earth: $6.378 \times 10^{6} km$

(a) Find the approximate acceleration due to gravity on earth.Your answer will be in m/s^2.

(b) Another planet Jupiter has mass approximately 320 times more than Earth and radius 11 times larger than earth. Is the acceleration due to gravity greater or less than on earth? (The above equation holds for all planets.)

8. An astronaut is lost in space. He is stationary floating in space when he spots a spaceship in the distance. His only belongings are his spacesuit which he cannot remove and a backpack. What is the only way the astronaut can move towards the spaceship?

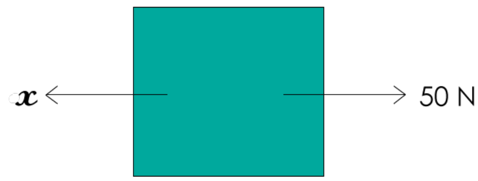
9. Having weight is a result of gravity acting on mass. We often hear the phrase ̈the Earth is weighing me down", which is literally true because weight is just the force of gravity. When you feel yourself being pulled toward the Earth by gravity, you are feeling your weight. We can measure any object's weight by finding the force of gravity that acts on them via Newton's second law $F = ma$. On Earth, we say that you are accelerating toward the ground as a result of gravity (you are being pulled toward the ground). Acceleration due to gravity on Earth is 9.8 m/s^2 [down]. So an object with mass 50 kg would weigh $F = (50) \times (9.8) = 490$ N on Earth. Notice that because weight is the *force* of gravity acting on an object, it is measured in Newtons (N).

   (a) How much would an object of mass 42 kg weigh on Earth?

   (b) The average African bush elephant weighs 53,900 N on Earth. How much mass does it have?

   (c) The acceleration due to gravity on the Moon is one sixth of what it is on Earth. How much would an average 12 year old (42 kg) and an average African bush elephant weigh on the moon?

10. Solve for x in Newtons if the size of acceleration of the block is 10 m/s^2:



What is the <u>new</u> x (call it x') if I securely attach a block of equal mass on top? (Assume the same acceleration and forward force).
Hint: both your answers should have an m in them.

What is x if $m = 5$ kg? What is x'?