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Grade 7/8 Math Circles

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An Introduction to Physics Solutions

Scientific Notation

Exercise: Powers (Exponents) of 10

Calculate the following powers of 10. Look for a pattern in your answers.

a) $10^0 = 1$

b) $10^1 = 10$

c) $10^2 = 100$

d) $10^3 = 1000$

e) $10^6 = 1,000,000$

f) $10^{10} = 10,000,000,000$

g) What is the pattern we can notice working with powers of 10?

The power of 10 is equal to the numbers of zeros in our results. ($10^{\mathbf{2}} = 1\underline{\mathbf{00}}$)

Exercise: Negative Powers of 10

Calculate the following powers of 10. Look for a pattern in your answers. The pattern created by powers of 10 makes them very useful in easily expressing large numbers.

a) $10^{-1} = \frac{1}{10} = 0.1$

b) $10^{-2} = \frac{1}{100} = 0.01$

c) $10^{-3} = \frac{1}{1000} = 0.001$

d) $10^{-5} = \frac{1}{10000} = 0.00001$

e) $10^{-10} = \frac{1}{10000000000} = 0.0000000001$

g) What is the pattern we can notice working with negative powers of 10?

The negative power is equal to the number of zeros before our 1. ($10^{\mathbf{-2}} = \underline{\mathbf{0.0}}1$)

<u>Multiplying by Powers of 10</u>

<u>Exercise:</u>

1. What is the purpose of using scientific notation?
We want to be able to write very large and very small numbers in a shorter form.

2.

Multiplying by positive powers of 10 moves the decimal place to the right

Multiplying by negative powers of 10 moves the decimal place to the left

3. Write out the following numbers in normal form.

a) $1 \times 10^6 = 1,000,000$

b) $3 \times 10^{-2} = 0.03$

c) $10 \times 10^5 = 1,000,000$

d) $1.26 \times 10^3 = 1260$

e) $7.55 \times 10^{-3} = 0.00755$

f) 1.16×10^{10}
$= 11,600,000,000$

Exercise: Express the following numbers in scientific notation.

a) $760 = 7.6 \times 10^2$

b) $36,700 = 3.67 \times 10^4$

c) $564,000,000$
$= 5.64 \times 10^8$

d) $0.034 = 3.4 \times 10^{-2}$

e) $0.00245 = 2.45 \times 10^{-3}$

f) 0.00000679
$= 6.79 \times 10^{-6}$

g) $0.12 = 1.2 \times 10^{-1}$

h) Speed of light:
$299,000,000 m/s = 2.99 \times 10^8$

i) 0.00000423
$= 4.23 \times 10^{-6}$

j) Mass of the Sun: $1,989,000,000,000,000,000,000,000,000,000$ $kilograms = 1.989 \times 10^{30} kg$

k) Mass of the electron: $0.000000000000000000000000000000911$ $kilograms = 9.11 \times 10^{-31} kg$

l) Age of the universe: $13,800,000,000$ $years = 1.38 \times 10^{10} years$

m) Avogadro's Number: 602,252,000,000,000,000,000,000 $= 6.02252 \times 10^{23}$

Physics: Newton's Laws of Motion

Newton's First Law:

1. If I roll a ball along the floor why does it eventually stop?
   There are forces such as friction from the floor acting on the ball causing it to slow down and stop.

2. What happens if I throw a ball in outer space?
   There are no external forces such as friction as space is a vacuum of nothing (not even air!) Thus the ball will continue at the same speed forever at the same speed in the same direction (until it meets a force.)

3. Can you name some of the forces in the universe? *Hint: One is what keeps us on the ground.*
   Gravity, friction, electrogmanetic, etc..

Newton's Second Law:

Practice:

Craig pushes a 5 kg box along the table. It accelerates at a rate of $5m/s^2$ away from him. How much force did Craig apply to the box?

$Force = mass \times acceleration = ma = 5kg \times 5m/s^2 = 25N$

Craig pushes a 10 kg box along the table with the same force. What is the amount of acceleration of the box now?

$Force = mass \times acceleration = 10\ kg \times a = 25\ N$

Rearranging for a, divide by 25N on both sides of the equation: $\dfrac{25\ N}{10\ kg} = a$

$a = 2.5m/s^2$

Test:

30 N [right] = **-30** N [left]

-12 N [down] = **12 N** N [up]

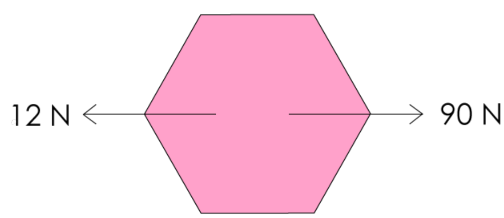
Exercises:

1. Can the acceleration of an object change without changing its speed?
   Yes, acceleration is a vector with direction and size/magnitude. If I maintain my same speed but change direction my acceleration has a new direction and has changed.

2. If Box A has twice the mass of Box B and you push both boxes with the same amount of force, which box will have the greater acceleration?
   Box B is twice the mass so it will have half the acceleration of Box A. This can be seen with the equation $F = ma$.
   I can rearrange the equation by dividing by m on both sides.
   $a = \frac{F}{m}$ From here we can see that if I double my mass m my acceleration a will be cut in half.

3. Use Newton's third law to explain why punching a wall hurts.
   When I apply a force on the wall, it applies the same force back on to me by the third law.

4. Consider the following "Free Body Diagram":



   (a) Find the net force.
   $Force_{net} = 90\ N[\text{right}] + 12\ N[\text{left}]$
   $Force_{net} = 90\ N[\text{right}] - 12\ N[\text{right}]$
   $Force_{net} = 78\ N[\text{right}]$

   (b) If in the diagram above, the hexagon had a mass of 5 kg, how much would it be accelerating and in what direction would it be accelerating?
   We have a mass of 5 kg and a force of 78 N [right]. We can use $F = ma$
   Rearranging the equation for the acceleration by dividing by m on both sides:
   $\frac{F}{m} = a$
   $\frac{78\ N[right]}{5\ kg} = a = 15.6 m/s^2[\text{right}]$ Note: Our acceleration is in the direction of our net force.

(c) With how much force would you have to push to make the hexagonal object accelerate at 10 m/s^2?

We have a mass of 5 kg and an acceleration of $10m/s^2$. We are not concerned with the direction of the acceleration just it's magnitude.

Using $F = ma \to F = 5\ kg \times 10\ m/s^2 = 50\ N$

(d) If I had a new shiny blue hexagon that accelerated at 10 m/s^2 when the net force in part (a) was applied to it, how much mass would my shiny new hexagon have?

We have an acceleration of $10m/s^2$ to the right and our original net force 78 N [right] for our new hexagon.

We use $F = ma$ and rearrange for the mass m of our new hexagon.

Dividing by a on both sides of $F = ma$ we arrive at $\frac{F}{a} = m$

$$\frac{78\ N\ [right]}{10m/s^2\ [right]} = m = 7.8\ kg$$

Proportionality

Exercise: Complete the charts below by finding the area and circumference of each circle.

Remember that the formula for area is $A = \pi r^2$ and the formula for circumference is $C = 2\pi r$.

Radius	***Diameter***	***Circumference of circle***	***Area of Circle***
1	2	2π	π
2	4	4π	4π
3	6	6π	9π
4	8	8π	16π
5	10	10π	25π

Can we identify the variables and constants in our equations for Circumference and Area?

$$A = \pi r^2 \text{ and } C = 2\pi r$$

π is a constant as it a fixed number that can never change. The value of π is always 3.14......... A,C, and r are all variables that are able to change as they do not have fixed values. If I draw two circles of different sizes, the area, circumference, and radius will be different between the circles but the value π is still the same.The ability to change makes A, C, and r variables.

Examples:

1. From Newtons second law we get the equation F=ma. What are the variables and constants in this equation? F,m, and a are all variables as force, mass, and acceleration are all values that are able to change. There are no constants in the equation. Consider that I have an object of mass m. I push the object with force F and acceleration a.

   (a) I decide to double the mass and acceleration of my object. What force will I need to do this? Looking at the equation $F = ma$ force is directly proportional to m and directly proportional to a (All variables to the exponent 1.) $F \propto m, F \propto a$. If I double my mass my force would have to double. If I double acceleration my force would have to double. Therefore I would need to double my force twice or quadruple my force.

   Alternatively:
   We could write my required new force as F_2.
   $F_2 = 2m2a = 4ma$ and compare this to my original force $F = ma$.
   I can see that my new force F_2 is 4 times as large as my original force F. Thus I will need a force 4 times as large as my original force or 4F.

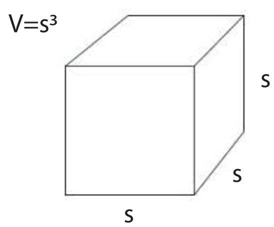
(b) What if I decide to double the original acceleration a of my object and halve my original mass m? What force will I require?

F is directly proportional to m and a. If double acceleration I double my required force. If I halve my mass I halve my required force. If I double and then halve my original force I end up applying the same force.

Alternatively:
My new force F_2 now looks like: $F_2 = \frac{m}{2} \times 2a = ma$ which is equivalent to the original force $F = ma$ Therefore I require the same original force F.

(c) I get tired and push my original object with half the original force F with my original mass m. What happens to the acceleration of my object?

Force and acceleration are directly proportional. Therfore If I halve my force my acceleration will be halved as well.



2. The equation for the volume of a cube is $V = s^3$, where s is the length of each side of the cube and V is the volume of the cube.

(a) If I double the original side-lengths s of my cube what happens to my volume. What if I halve the original side-lengths? $V \propto s^3$
Through the proportionality relationship, if I double the side-lengths of my cube my volume will scale up by 2^3 and $2^3 = 8$ Thus the volume will be eight times greater. If I were to instead halve the side-lengths the volume would be 8 times smaller than the original cube. $(\frac{1}{2})^3 = \frac{1}{8}$

(b) If I make my sides 10 times larger than my original side-lengths s, How much larger is the volume of my cube? Express your answer in scientific notation.
$10^3 = 1000$ times larger or 1×10^3 times larger.

(c) If I want my volume to be $\frac{1}{27}$ of the original volume V of my cube. How should I change my original side-lengths s?
Need volume to be 27 times smaller. Need to pick a new side-length that is smaller. Need to multiply my original side-length by a factor that when cubed is equal to 1/27. Through guess and check I can see that I can multiply my side-length s by $\frac{1}{3}$. Since $(\frac{1}{3})^3 = \frac{1}{27}$.
Thus I should divide my side-lengths by 3.

Problem Set:

1. Express the following numbers in scientific notation:

a) $-12,000 = -1.2 \times 10^4$

b) $32,900 = 3.29 \times 10^4$

c) $3907 = 3.907 \times 10^3$

d) $231,000 = 2.31 \times 10^5$

e) $-667,000,000,000$
$= -6.67 \times 10^{11}$

f) $79,000,000 = 7.9 \times 10^7$

g) $0.320 = 3.2 \times 10^{-1}$

h) Age of Earth:
$4,600,000,000$years
$= 4.6 \times 10^9$years

i) Gravitational Constant:
0.0000000000667
6.67×10^{-11}

2. Why does Newton's first law make seatbelts in vehicles a necessary safety precaution?

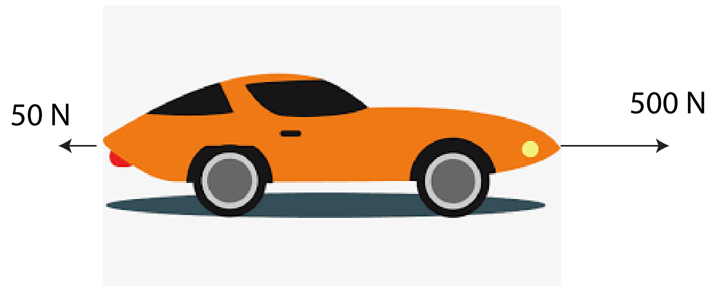
   When travelling in a car your body is in motion with the car. If the car were to brake suddenly then by the first law your body wants to continue in its motion forward. Thus you are pushed forward when the car brakes. The seatbelt applies a force to keep you from being flung forward out of your seat when this happens.

3. A box is resting on a table, what is the net force on the box?
   The box is at rest with no acceleration. By the second law $F = ma$ if $a = 0$ then the net force must also be zero.

4. A car weighs 1.5 tonnes (1500kg) and accelerates at a rate of $12m/s^2$.

   (a) What is the net force on the car in the direction if its motion?
   $F = ma = 1500\ kg \times 12\ m/s^2 = 18{,}000$ N

   (b) Let's say that the car is now at rest. The car's engine then applies a force of 500N to get moving. There is also a force of 50N acting against the motion of the car. Find the net force and acceleration of the car. Include a diagram of the forces on the car.
   Since the direction is not specified, we will assume the car is moving right (could also assume left.)



$Force_{net}$ = 500 N [right] + 50 N [left]
$Force_{net}$ = 500 N [right] - 50 N [right]
$Force_{net}$= 450 N [right]

With the net force of 450N and a mass of 1500kg we can use $Force_{net} = ma$ to find the acceleration:
Rearranging the equation by dividing by m on both sides:

$$\frac{F}{m} = a \implies \frac{450\ N\ [right]}{1500kg} = 0.3m/s^2 \text{ [right]}$$

5. Nasrin decides to push down on the earth with a force of 100 N. If the earth has a mass of $5.972 \times 10^{24}kg$ what is the acceleration of earth from Nasrin's push?
   Using the equation $F = ma$ with a force 100 N and mass $5.972 \times 10^{24}kg$
   Dividing both sides of equation by mass on both sides

$$\frac{F}{m} = a \implies \frac{100\ N\ [down]}{5.972 \times 10^{24}\ kg} = 1.67 \times 10^{-23}\ m/s^2 \text{ [down]}$$

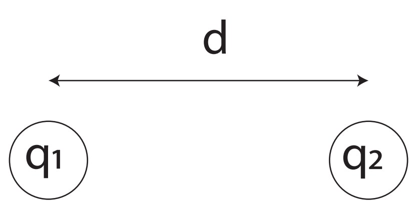
6. You may have heard the saying "opposites attract" before. Well this is true for oppositely charged particles due to the force between them. Each charge applies an equal and opposite charge on the other.

   The force between two charges can be found by the equation:

$$F_{Coulomb} = k \times \frac{q_1 \times q_2}{d^2}$$

Where:

- k is a constant 9×10^9
- q_1 and q_2 are the amount of charge on each particle measured in Coulombs (C)
- d is the distance between the charges in meters.



(a) Say the particle are $7 \times 10^{-5}m$ apart. Their charges are $q_1 = 1.6 \times 10^{-19}$ and $q_2 = -1.6 \times 10^{-19}$. Find the Coulomb Force between them.

$$F = 9 \times 10^9 \times \frac{1.6 \times 10^{-19} \times -1.6 \times 10^{-19}}{(7 \times 10^{-5}\ m)^2} = -4.7 \times 10^{-20}\ \text{N}$$

Try the remaining questions without using a calculator (use proportionality):

(b) If the distance between the particles is doubled what do you expect to happen to the force?

Looking at our equation: $F_{Coulomb} = k \times \frac{q_1 \times q_2}{d^2}$

We can say that Force goes as $\frac{1}{d^2}$.

If we double the distance d or force would scale by $\frac{1^2}{2^2} = \frac{1}{4}$

Our force Force would scale 4 times smaller.

(c) If q_1 doubles in charge but q_2 has its charge cut in half what do you expect to happen to the force?

Force is directly proportional to q_1 and also q_2 so the force would double and then be halved. Thus the force would remain the same.

(d) Opposite charges attract but same sign charges repel (two positive charges will repel one another). The Coulomb force should be in one direction if the charges are both positive/negative and in the other if one charge is positive and one charge is negative. Does the equation for Coulomb force reflect this?

Yes if both charges are the same sign the force is positive, if both charges are opposite signs the force is negative. This indicates that the force changes direction (switching signs) for the two different cases of attraction and repulsion.

7. We stay grounded on earth thanks to the help of gravity. We can find the acceleration downwards due to gravity on our planet with the formula:

$$acceleration_{gravity} = \frac{GM}{R^2}$$

Where:

- G is a constant 6.67×10^{-11}
- M is the mass of the earth: $5.972 \times 10^{24} kg$
- R is the radius of the earth: $6.378 \times 10^{6} km$

(a) Find the approximate acceleration due to gravity on earth.Your answer will be in m/s^2.

$9.79\ m/s^2$

(b) Another planet Jupiter has mass approximately 320 times more than Earth and radius 11 times larger than earth. Is the acceleration due to gravity greater or less than on earth? (The above equation holds for all planets.)

New equation with larger radius and mass:

$$acceleration_{gravity} = 6.67 \times 10^{-11} \times \frac{320 \times 5.972 \times 10^{24} kg}{(11 \times 6.378 \times 10^{6} km)^2}$$

$acceleration_{gravity} \approx 25.89 m/s^2$ which is greater than acceleration from gravity on earth.

8. An astronaut is lost in space. He is stationary floating in space when he spots a spaceship in the distance. His only belongings are his spacesuit which he cannot remove and a backpack. What is the only way the astronaut can move towards the spaceship?

Note that he cant just "flail" to the space ship as he needs something to push himself in the direction he wants to go. By Newton's first law he will stay floating still until a force is applied. He needs a force to be applied somehow onto him. Similar to how when we walk on the ground we push on the ground to push ourselves forward using Newton's third law, if the astronaut applies a force and throws his backpack, it will apply a force back on to him. If he throws his pack in the direction opposite of the spaceship, the backpack will apply an opposite force on the astronaut pushing him back toward the spaceship. Since there is no air he continue towards the spaceship until he eventually reaches it.

9. Having weight is a result of gravity acting on mass. We often hear the phrase "the Earth is weighing me down", which is literally true because weight is just the force of gravity. When you feel yourself being pulled toward the Earth by gravity, you are feeling your weight. We can measure any object's weight by finding the force of gravity that acts on them via Newton's second law $F = ma$. On Earth, we say that you are accelerating toward the ground as a result of gravity (you are being pulled toward the ground). Acceleration due to gravity on Earth is 9.8 m/s^2 [down]. So an object with mass 50 kg would weigh $F = (50) \times (9.8) = 490$ N on Earth. Notice that because weight is the *force* of gravity acting on an object, it is measured in Newtons (N).

   (a) How much would an object of mass 42 kg weigh on Earth?

   $$\text{mass} \times 9.8 \text{ m/s}^2 \text{ [down]} = \text{weight}$$
   $$42 \text{ kg} \times 9.8 \text{ m/s}^2 \text{ [down]} = 411.6 \text{ N [down]}$$

   When we talk about how much something weighs, we normally just talk about the *size* of the force of gravity because we know that gravity always pulls down. I will exclude direction when I talk about weight for simplicity. So the object weighs 411.6 N.

   (b) The average African bush elephant weighs 53,900 N on Earth. How much mass does it have? The average African bush elephant weighs 53,900 N on Earth. How much mass does it have?

   $$\text{mass} \times 9.8 \text{ m/s}^2 = \text{weight}$$
   $$\text{mass} = \text{weight} \ / \ 9.8 \text{ m/s}^2$$
   $$\text{mass} = 53{,}900 \text{ N} \ / \ 9.8 \text{ m/s}^2$$
   $$\text{mass} = 5500 \text{ kg (wow!)}$$

   (c) The acceleration due to gravity on the Moon is one sixth of what it is on Earth. How much would an average 12 year old (42 kg) and an average African bush elephant weigh on the moon? Acceleration due to gravity on the moon = $9.8 \div 6$ m/s^2 = 1.63333... m/s^2. You would weigh one sixth of what you do on Earth. Generally:

   $$\text{weight on moon} = \text{mass} \times (9.8 \div 6 \text{ m/s}^2)$$
   $$\text{weight on moon} = \text{weight on Earth} \div 6$$

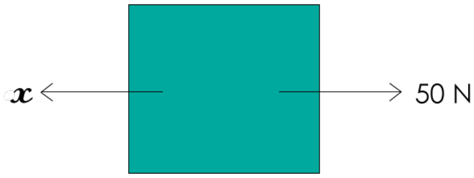
   For the average 12 year old:

   $$\begin{aligned} \text{weight on moon} &= \text{weight on Earth} \div 6 \\ &= 411.6 \text{ N} \div 6 \\ &= 68.6 \text{ N} \end{aligned}$$

   For the average African bush elephant:

   $$\begin{aligned} \text{weight on moon} &= 53,900 \text{ N} \div 6 \\ &= 8983.3 \text{ N} \end{aligned}$$

10. Solve for x in Newtons if the size of acceleration of the block is 10 m/s^2:



What is the new x (call it x') if I securely attach a block of equal mass on top? (Assume the same acceleration and forward force).
Hint: both your answers should have an m in them.

What is x if $m = 5$ kg? What is x'?

The unbalanced force is 50 N [right] - x (since the forces are in the opposite directions, we make sure that every direction faces right. A –5 N force left is the same as a +5 N force right). For a **single block** with mass m, we plug this into Newton's second law, remembering that dividing the force unit N by the acceleration unit m/s^2 results in the mass unit kg:

$$F = ma$$

$$50 \text{ N [right]} - x = m \times (10 \text{ m/s}^2)$$

$$-x = 10m - 50 \text{ N}$$

$$x = (50 - 10m) \text{ N}$$

If I add another box of equal mass on top, then the new mass of the whole system is $m + m = 2m$. Therefore:

$$x' = (50 - (10 \times 2m)) \text{ N} = (50 - 20m) \text{ N}$$

If m =5 kg, Then:

$$\begin{aligned} x &= (50 - 10m) \text{ N} \\ &= (50 - 10(5)) \text{ N} \\ &= (50 - 50) \text{ N} \\ &= 0 \text{ N} \end{aligned}$$

If the box had mass 5 kg and was accelerating at 10 m/s^2, the only force acting on it would be a force of 50 N [right]. This means it would also be accelerating to the right:

$$a = 10 \text{ m/s}^2 \text{ [right]}$$

On the other hand, if we stack two boxes on top of each other:

$$\begin{aligned} x' &= (50 - 20m) \text{ N} \\ &= (50 - (20 \times 5)) \text{ N} \\ &= (50 - 100) \text{ N} \\ &= -50 \text{ N} \end{aligned}$$

So with two boxes, the net force becomes 50 N [right] – (– 50 N) = 100 N [right]. You would have to reverse the direction of the force x so that it was facing the same way as the original force of 50 N. The force x would also have to be 50 N [right] and therefore the total force on the system has to be 100 N [right] in order to get 2 boxes of mass 5 kg each to move with an acceleration of 10 m/s^2 [right].