Modeling the Climate System: an Introduction

Marek Stastna

Goals:

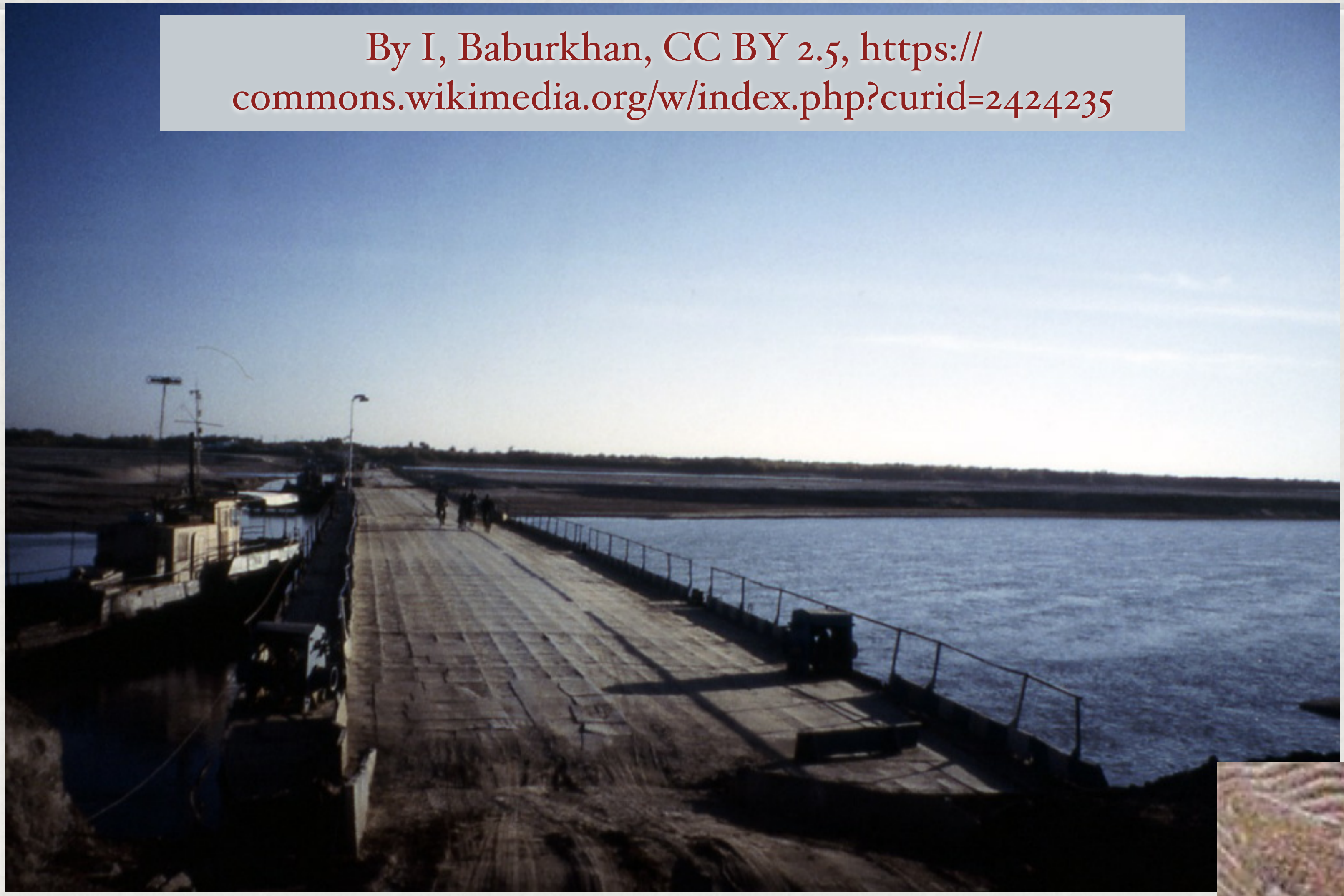
1. To learn about one existing example of modern, man-made climate change involving the sea.
2. To learn about climate change on the ice-age scale (last 20K years).
3. To give examples of scales of motion in the ocean and atmosphere.
4. To give an example of climate issues in urban settings on the global stage.

- In pretty much everything I am going to show you mathematics will be in the background.
- It will be used to analyze models, make pictures from satellite data, and sometimes even formulate equations that govern climate processes.
- There are a lot of mathematical moving parts (vector calculus, numerical linear algebra).
- Moreover the point of view on math is very different from the classroom math you are used to.
- Classroom math generally assumes you're going to do the math because you are told to, and not worry too much about what it is for.
- The math I use is driven by answering questions and how easy or hard it is depends on the question, not on a quest for more math.
- At the end of the day very, very few people get paid to do math contest puzzles. However, many people use math every day...

Part 1: A climactic horror story

The Aral Sea

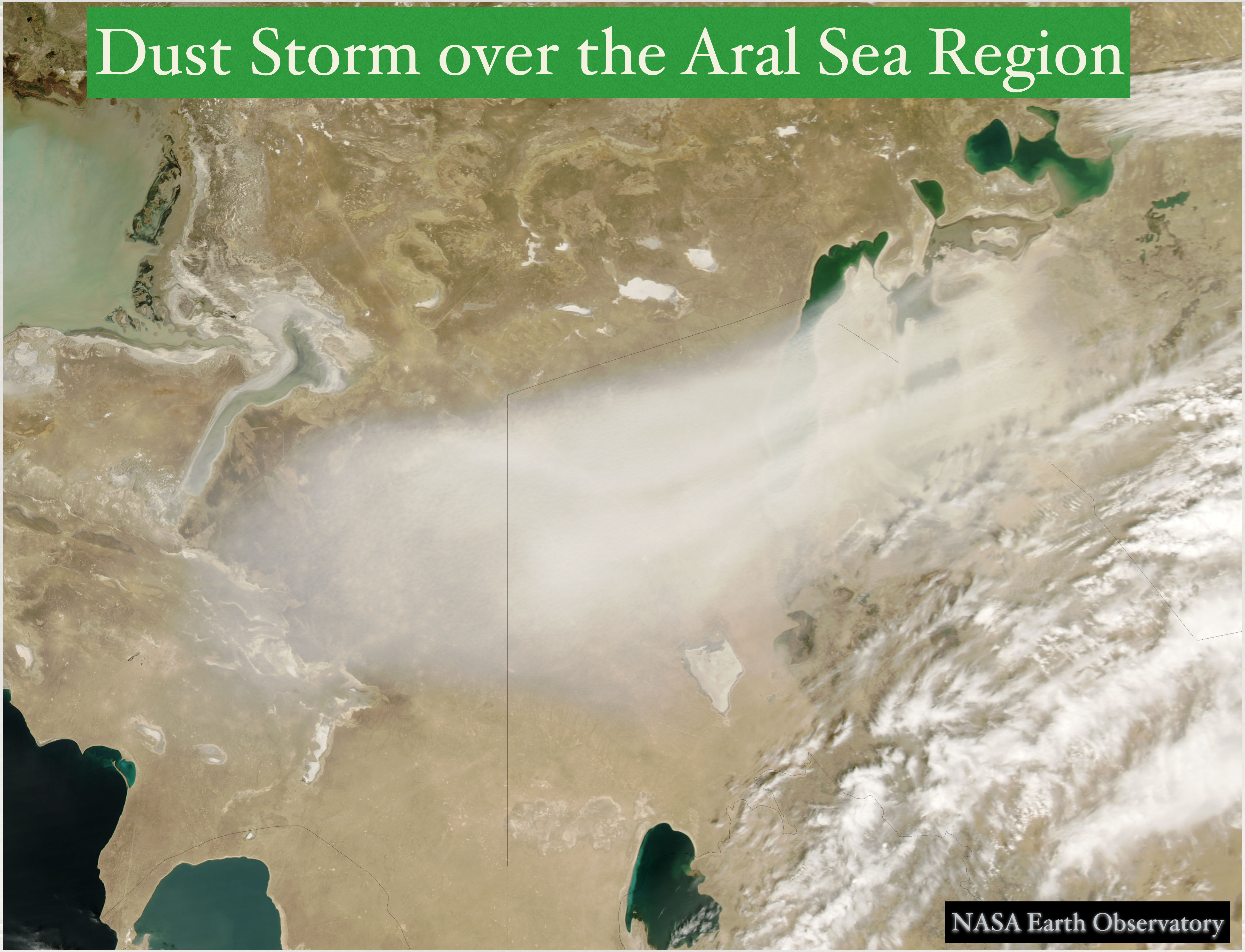
- While the media sometimes treats climate change as “controversial” scientists have no doubt that humans actively modify climate.
- Indeed we have done so for thousands of years, ever since we have begun to practice agriculture in large numbers.
- In more recent times we modify local climate by building cities, paving roads and using water, as the tragic picture of the Aral Sea on the last slide shows.
- The Aral sea was once the jewel of central Asia, but due to poor decisions (growing cotton in a desert and using the rivers that flow into the sea to irrigate the fields) has shrunk to a tiny percentage of its original size.
- The left over salt and fertilizers have created a depressing, dystopian landscape like something out of a (particularly dark) movie.



The Amu Darya (Oxus) in recent times

Historical Account of Alexander the Great crossing the Oxus (Amu Darya): Bessus had tried to prevent the crossing of the Oxus by burning all available ships. However, the Macedonians made rafts. They stuffed animal skins and tents with hay, and five days later, the army was on the other bank in the southeast of what is now called Turkmenistan.

Dust Storm over the Aral Sea Region



- The left over salt and fertilizers have created a depressing, dystopian landscape like something out of a (particularly dark) movie.

- The loss of the Aral sea has enhanced seasonality by 2-6 degrees Centigrade and wiped out many ecosystems.

- Greatly enhanced airborne dust has caused a massive increase in respiratory diseases and the dried up lake bed contains high concentrations of various toxic chemicals (e.g. PCBs) that can now be readily moved into the atmosphere via dust storms.

- The region is an active location for research and some moderately successful restoration efforts are under way:

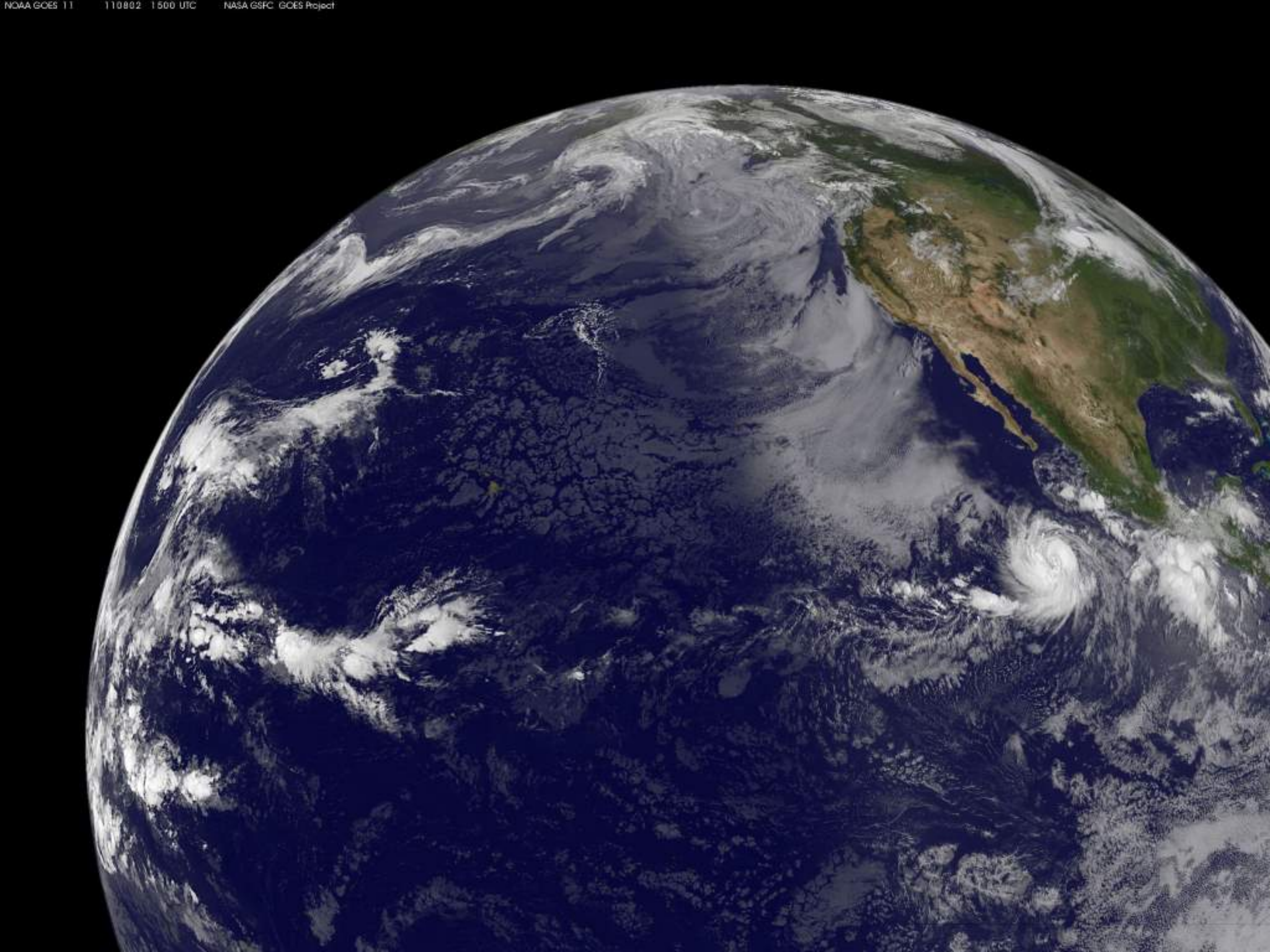
The internal seiche field in the changing South Aral Sea (2006–2013)

Elena Roget[1], Elizaveta Khimchenko[2], Francesc Forcat[1], and Peter Zavialov[2]

[1]Department of Physics, University of Girona, Girona, Catalonia, Spain
[2]Laboratory of land–ocean interactions and the anthropogenic impact,
Shirshov Institute of Oceanology – RAS, Moscow, Russia

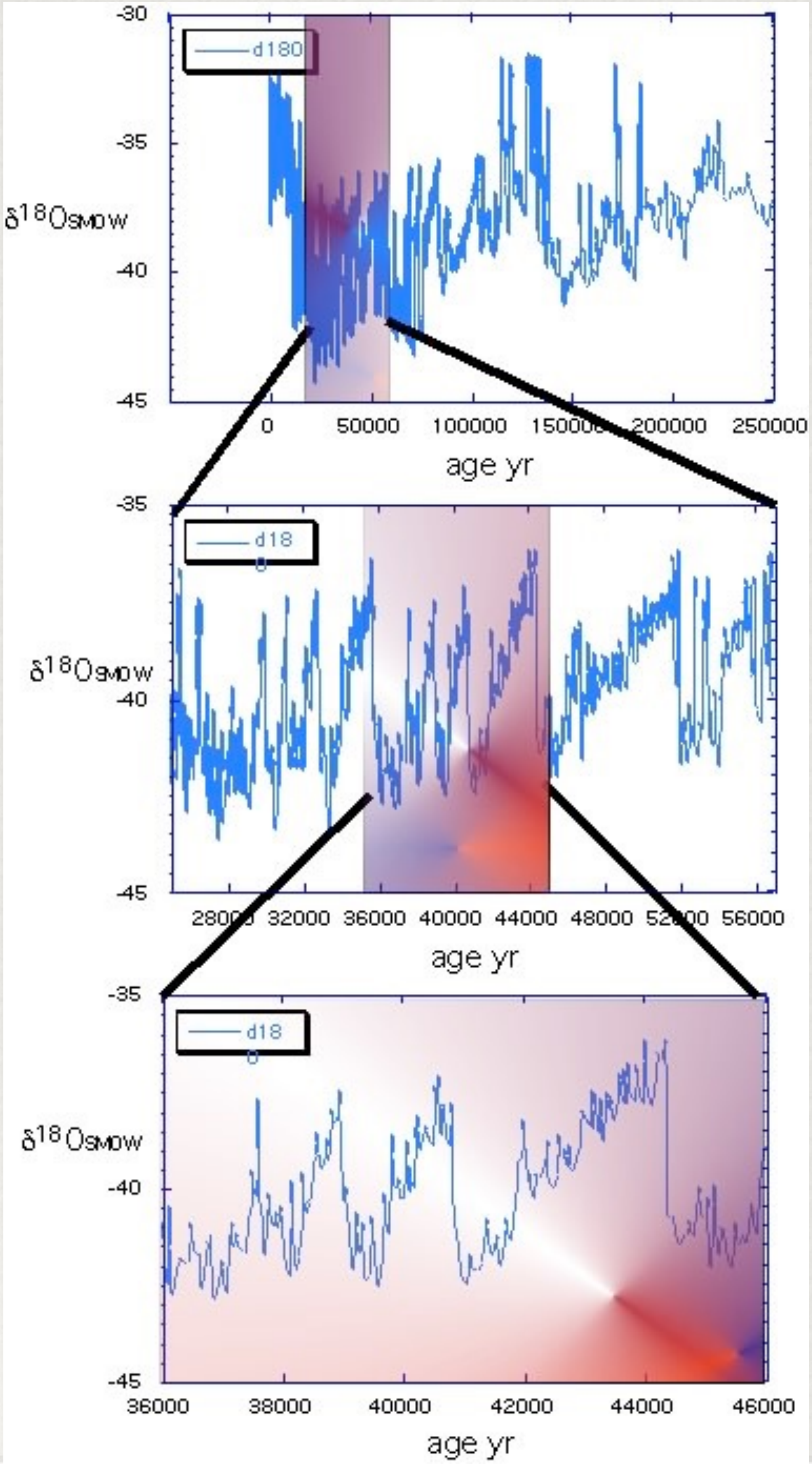
Restoration efforts North Aral Sea:

Part 2: Natural climate change


NOAA GOES 11
110802 1500 UTC
NASA GSFC GOES Project

http://geoinfo.amu.edu.pl/wpk/geos/geo_6/GEO_PLATE_C-24.HTML

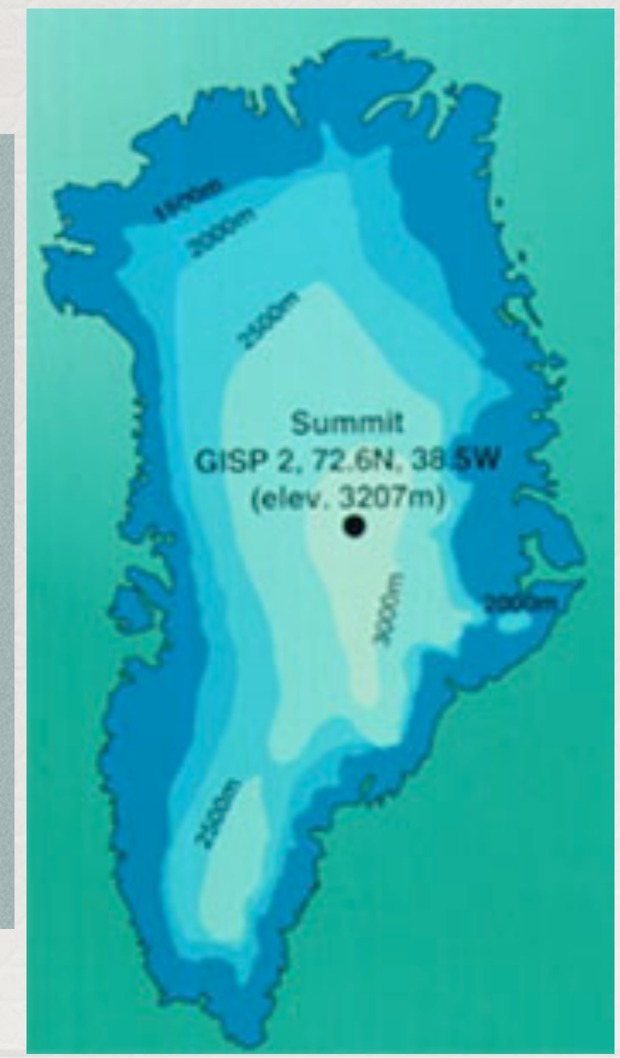
- Of course climate is older than human civilization, and indeed older than human life on Earth.
- Even in more recent times climate has fluctuated wildly due to the fine balance between the solar radiation that comes in from the Sun, and the amazing material water that makes life possible but also makes it difficult when in its frozen form.
- 18,000 years ago, at what is called the Last Glacial Maximum, all of Canada and sizable chunks of the USA were covered by ice. This ice was called the Laurentide ice sheet and in places was several kilometers thick.
- Other parts of the world had smaller, but still large, ice sheets.
- All that ice means that the ocean levels were considerably lower as well (a much bigger change than what is predicted in this century due to carbon dioxide emission caused climate change).
- Also the weight of the ice pushes down the land, and when the ice is gone the land rebounds, like the raised beaches on the last slide.

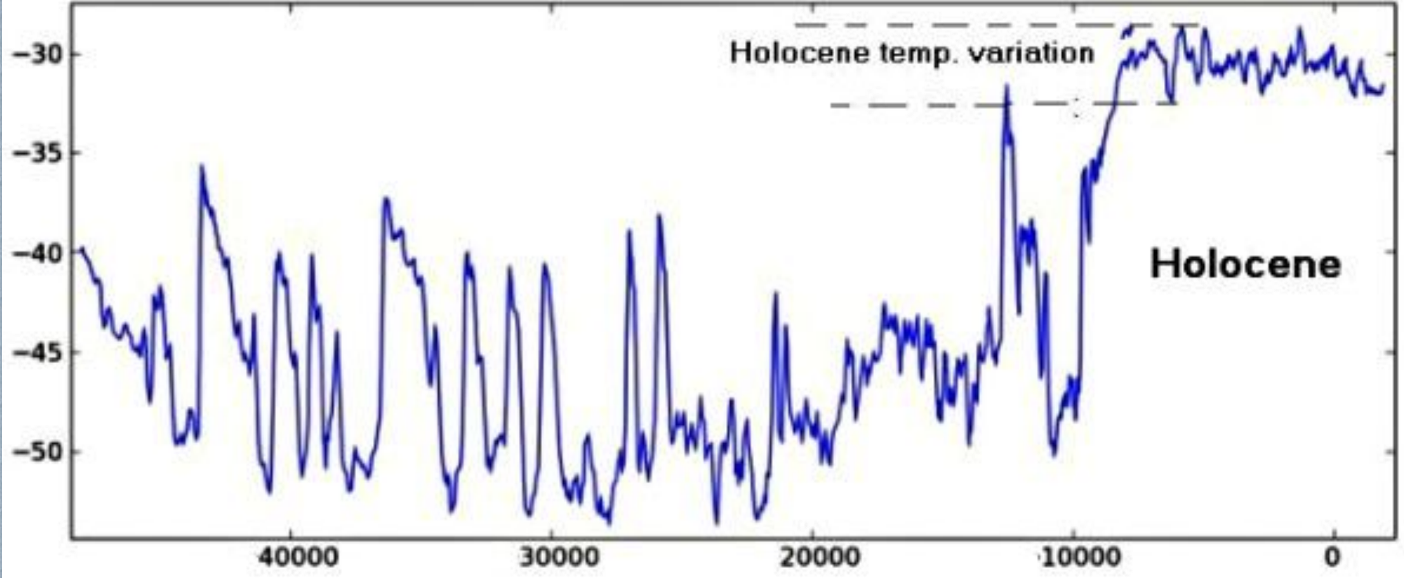


Some ice sheets have not gone away (like the one in Greenland) for many thousands of years and by digging cores and extracting tiny bubbles of Oxygen from these cores we can get information about past climates.
The GRIP and GISP
ice records tell us about variations in the climate
during the glacial period and subsequent deglaciation.

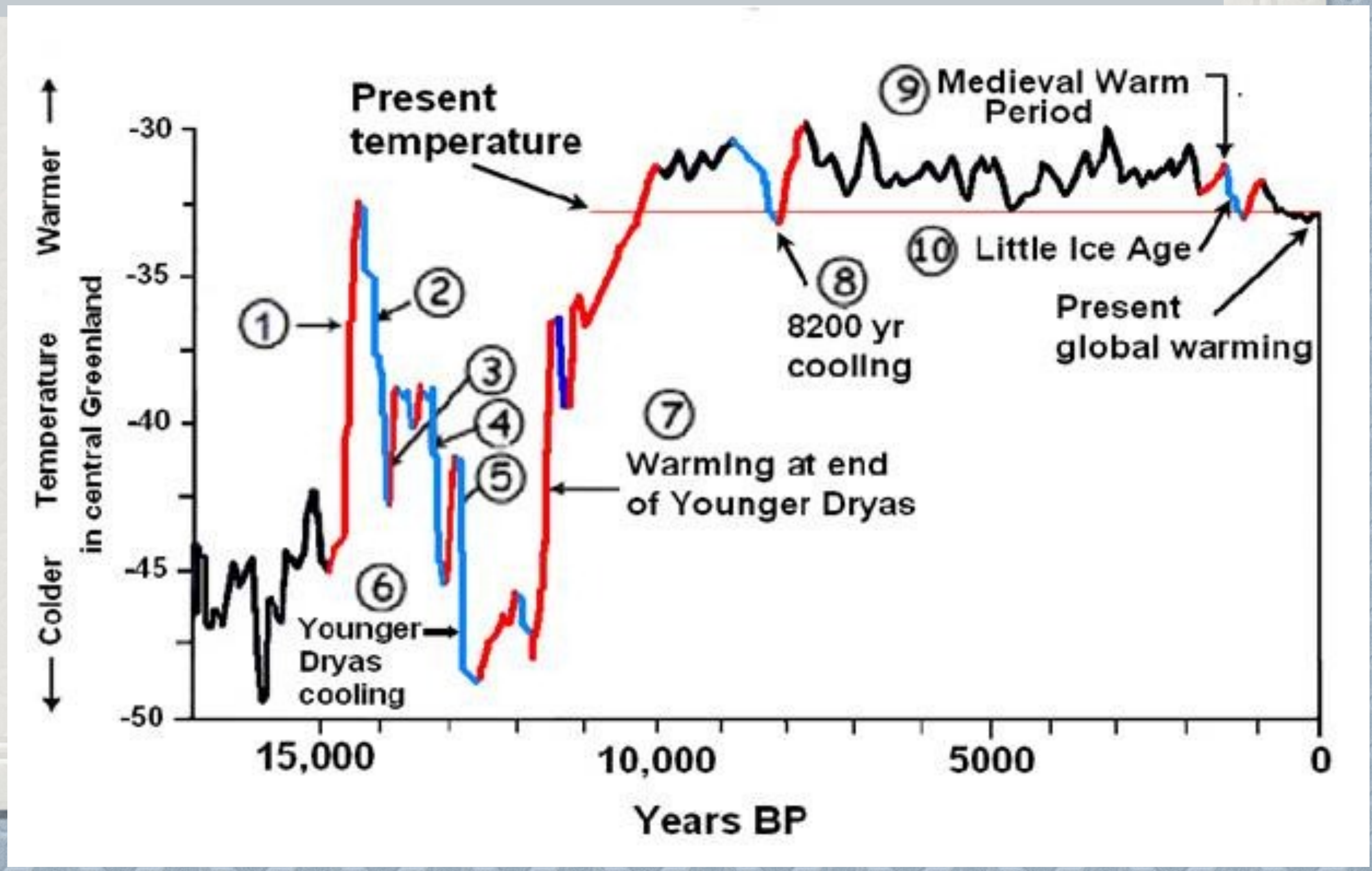
The pictures on the left show the ratio between two isotopes of oxygen over hundreds of thousands of years (top), tens of thousands of years (middle) and thousands of years (bottom).

There are clear changes on all scales.





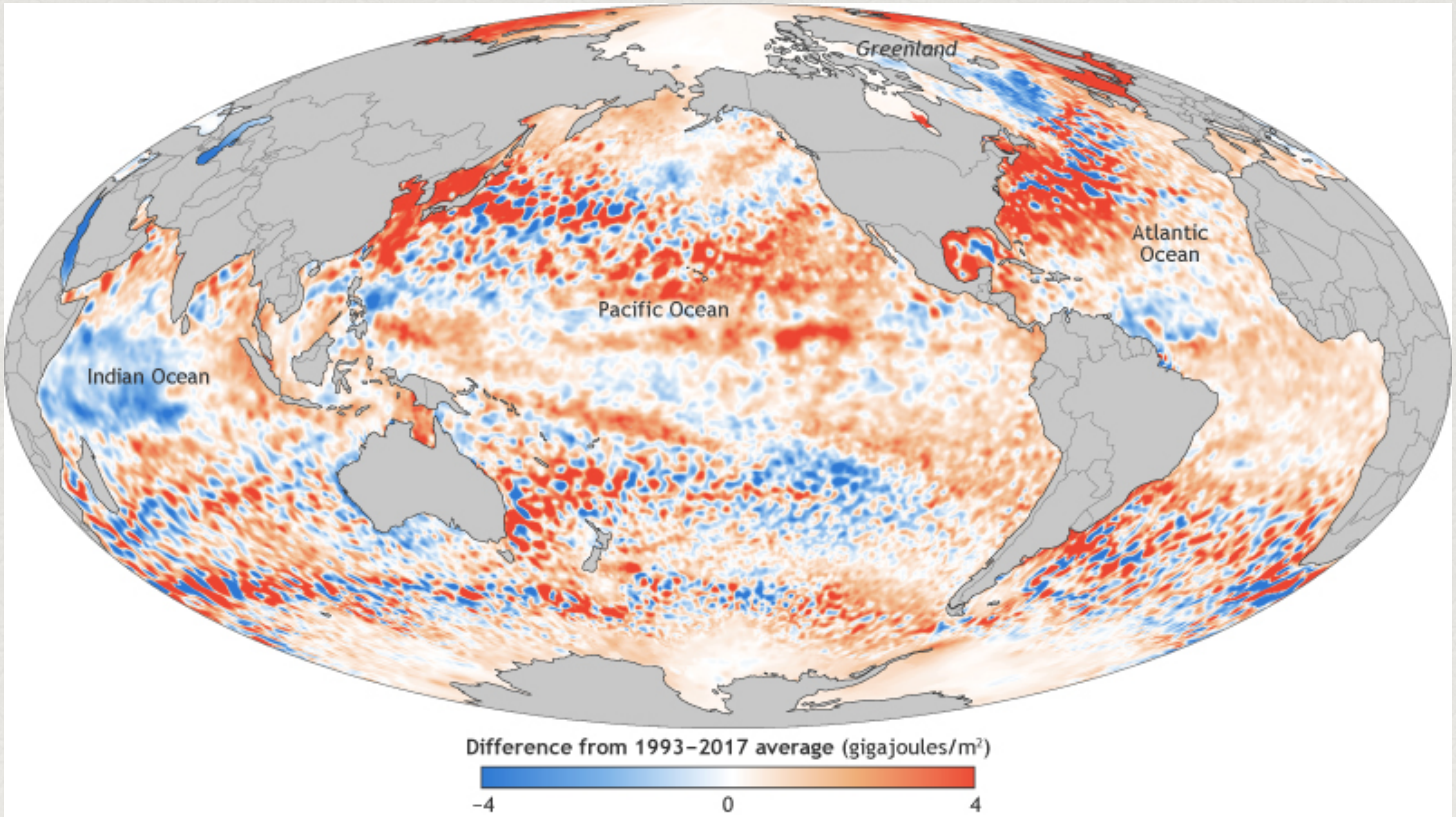
- Climate variability was quite large during the ice age (left side of top left plot).
- The deglaciation did not happen all at once, and indeed there were prominent reversals.



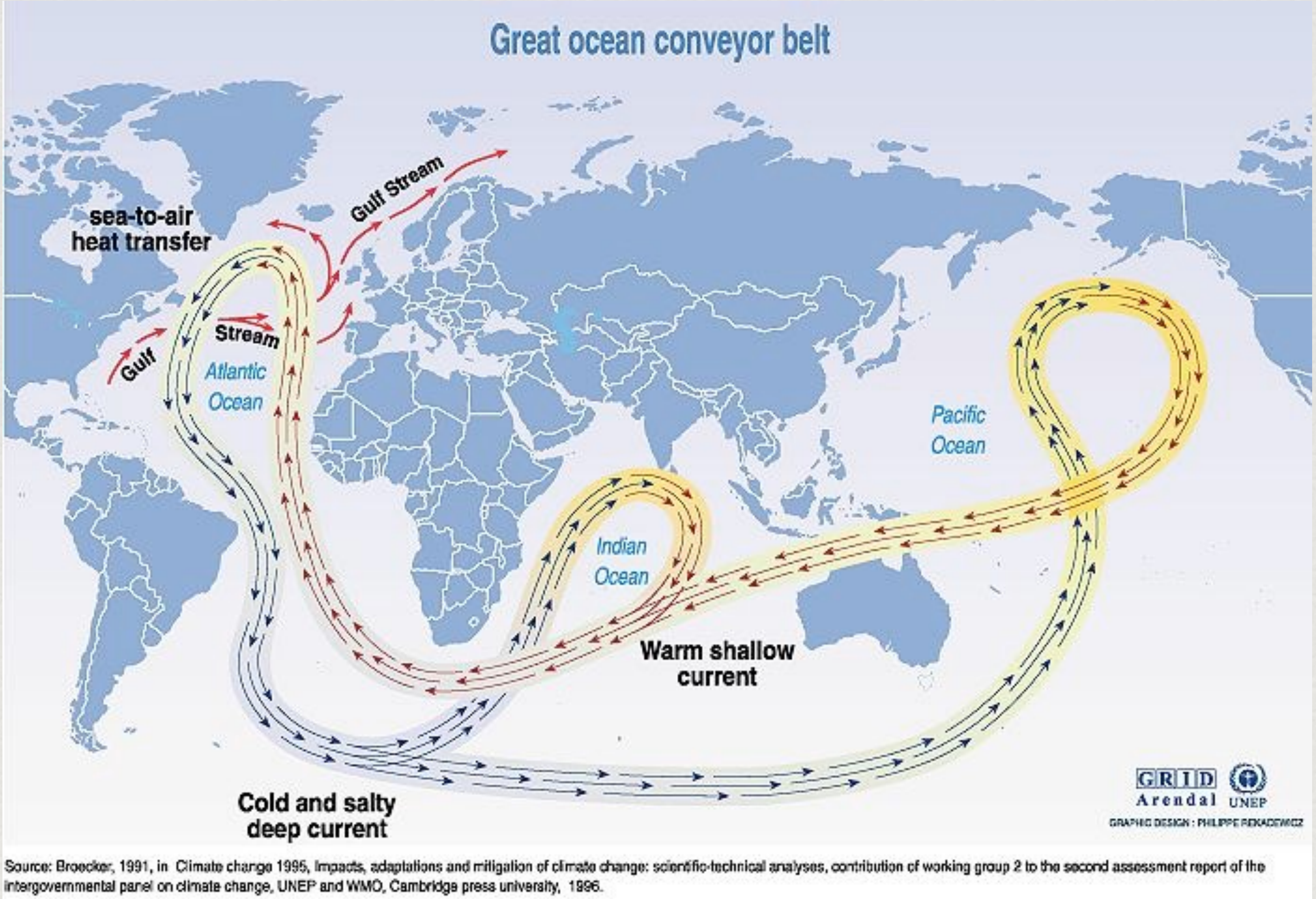
- Let’s take a step back and think about some of the amazing scientific ideas that the last few slides covered.
- First of all, humans had to accept that climate changes: https://history.aip.org/climate/rapid.htm
- To find ways to get information about old (paleo) climates we used various sources, like bubbles trapped in ancient ice.
- We then had to use imagination to extrapolate information from single measurements to global values.
- Often this has been done with the help of climate models called GCMs (global climate models).
- The process improves all the time and like most of science isn’t (and shouldn’t be) static.

Of course floating in space above the Pacific you might point out to me that my focus on land and ice was a little mis- guided

The ocean stores much more heat than the atmosphere and without oceans Earth would not be habitable

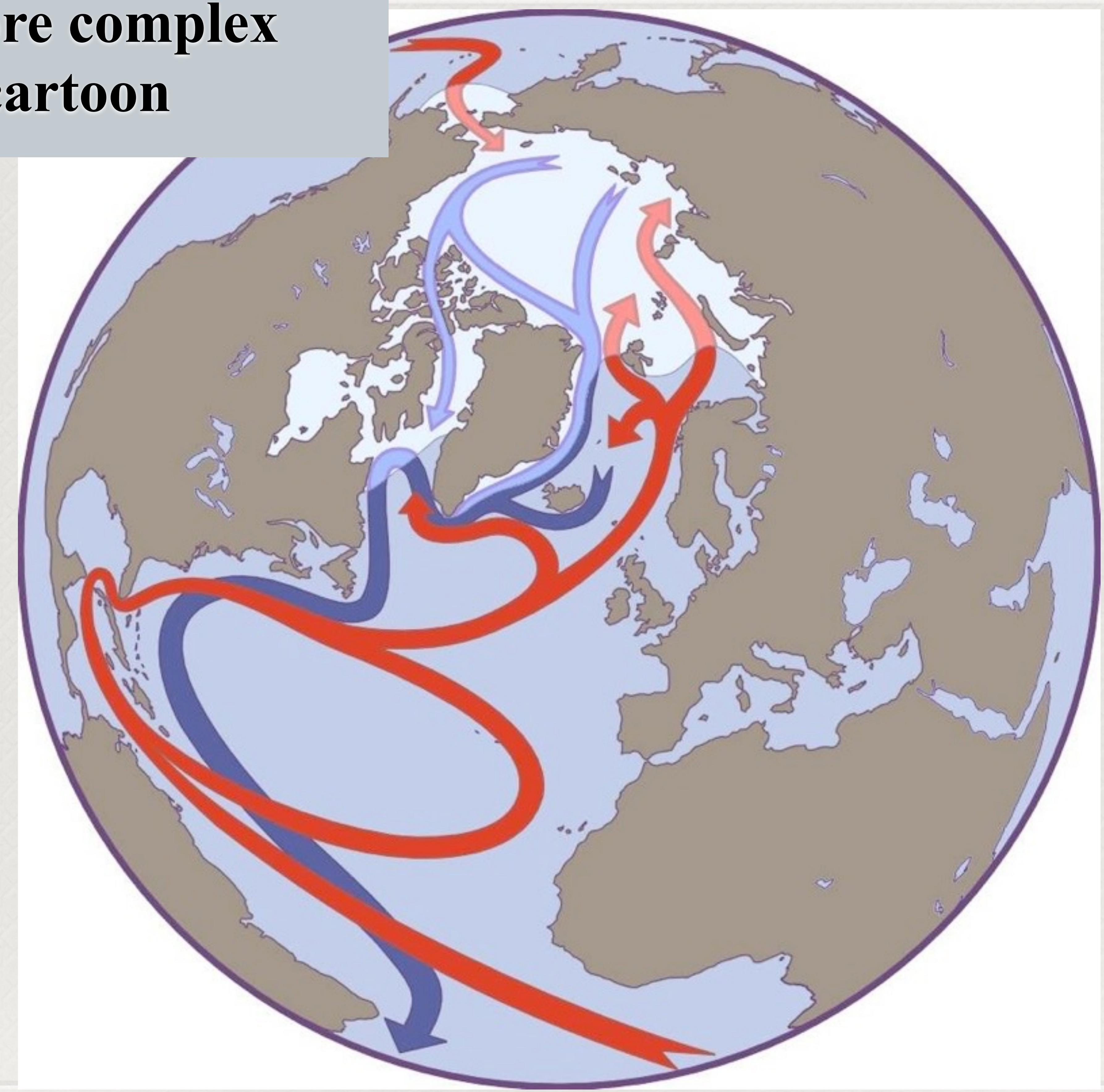


Global map showing where 2017 heat content in the top 700 meters (2,300 feet) of the ocean was higher (orange) or lower (blue) than the 1993–2017 average. NOAA Climate.gov map, adapted from *State of the Climate in 2017*.

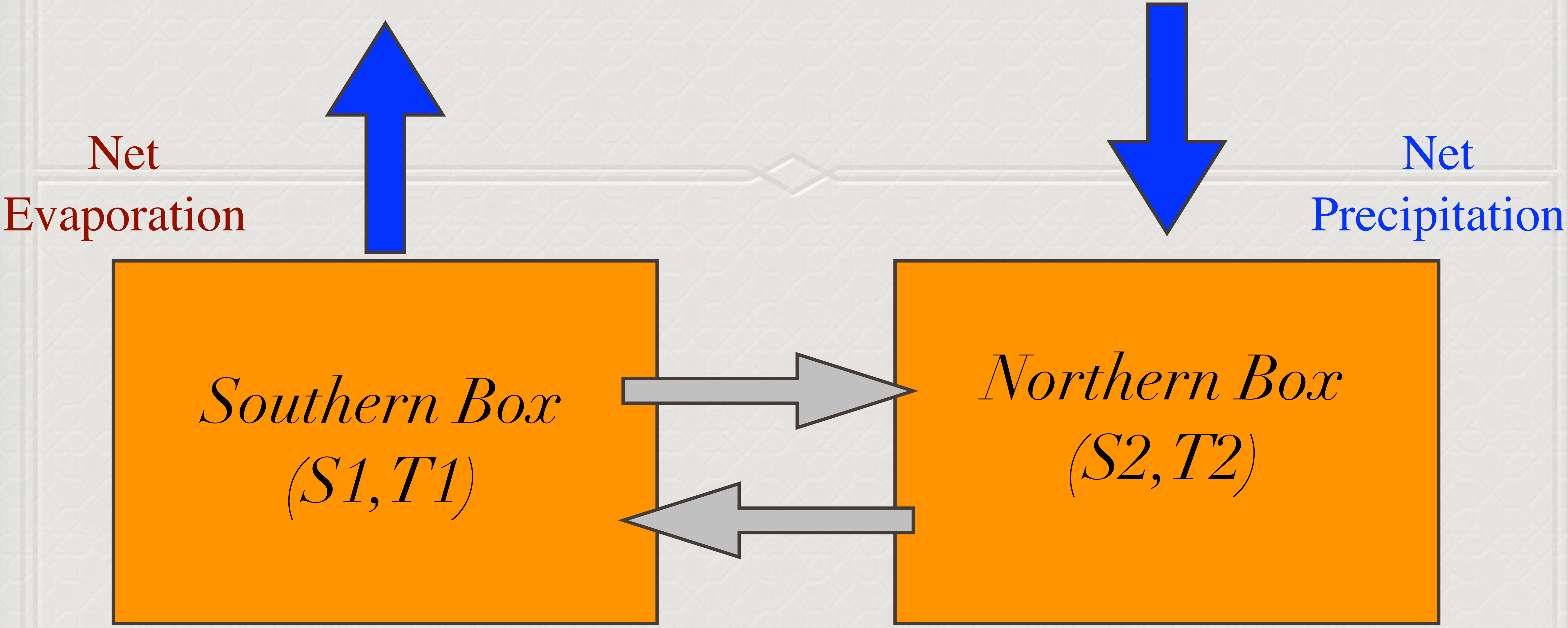


Simplified global cartoon of ocean heat transport: the toy thermohaline circulation

A more complex cartoon

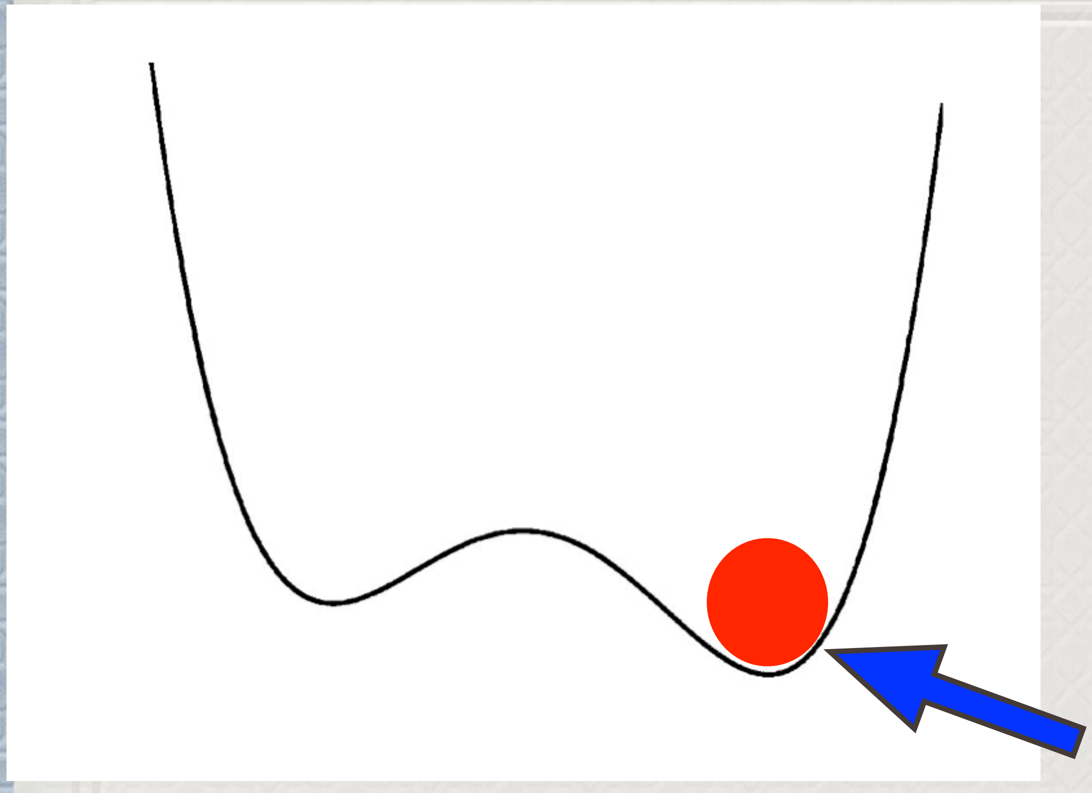


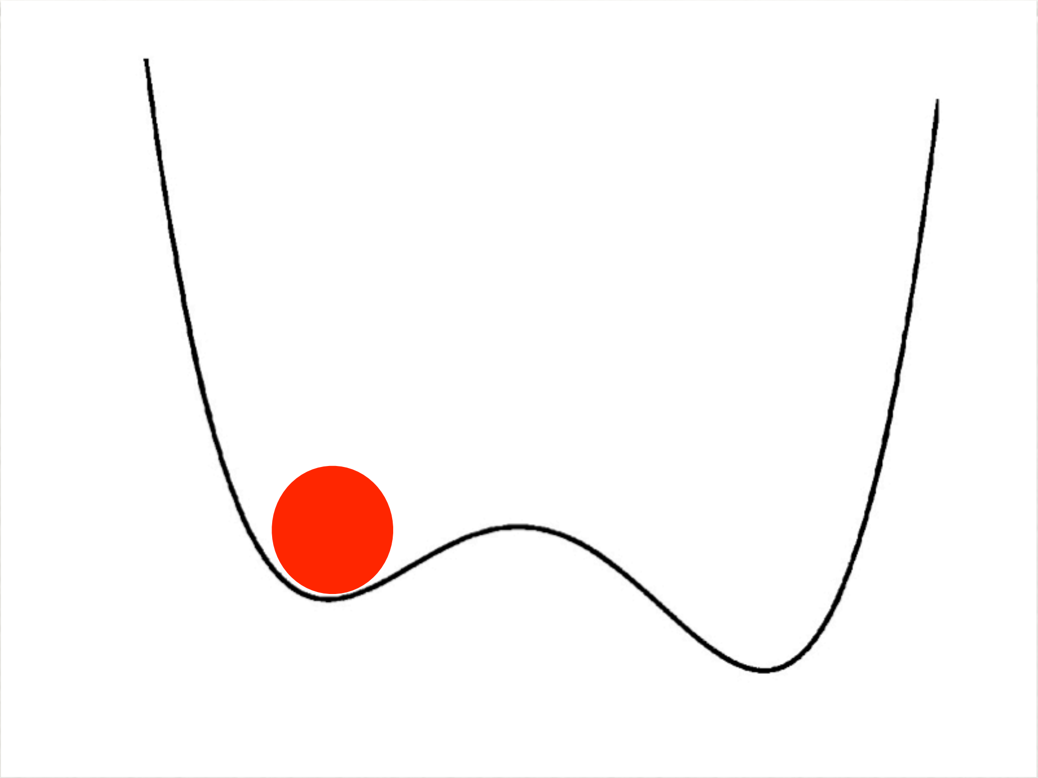
Stommel Two-box model



- The North Atlantic is modeled as two communicating boxes where the net forcing increases (decreases) salinity in the southern (northern) box and varies on many different timescales
- The communication between boxes is "modeled" (i.e. nonlinear ODEs, often with a bistable potential)

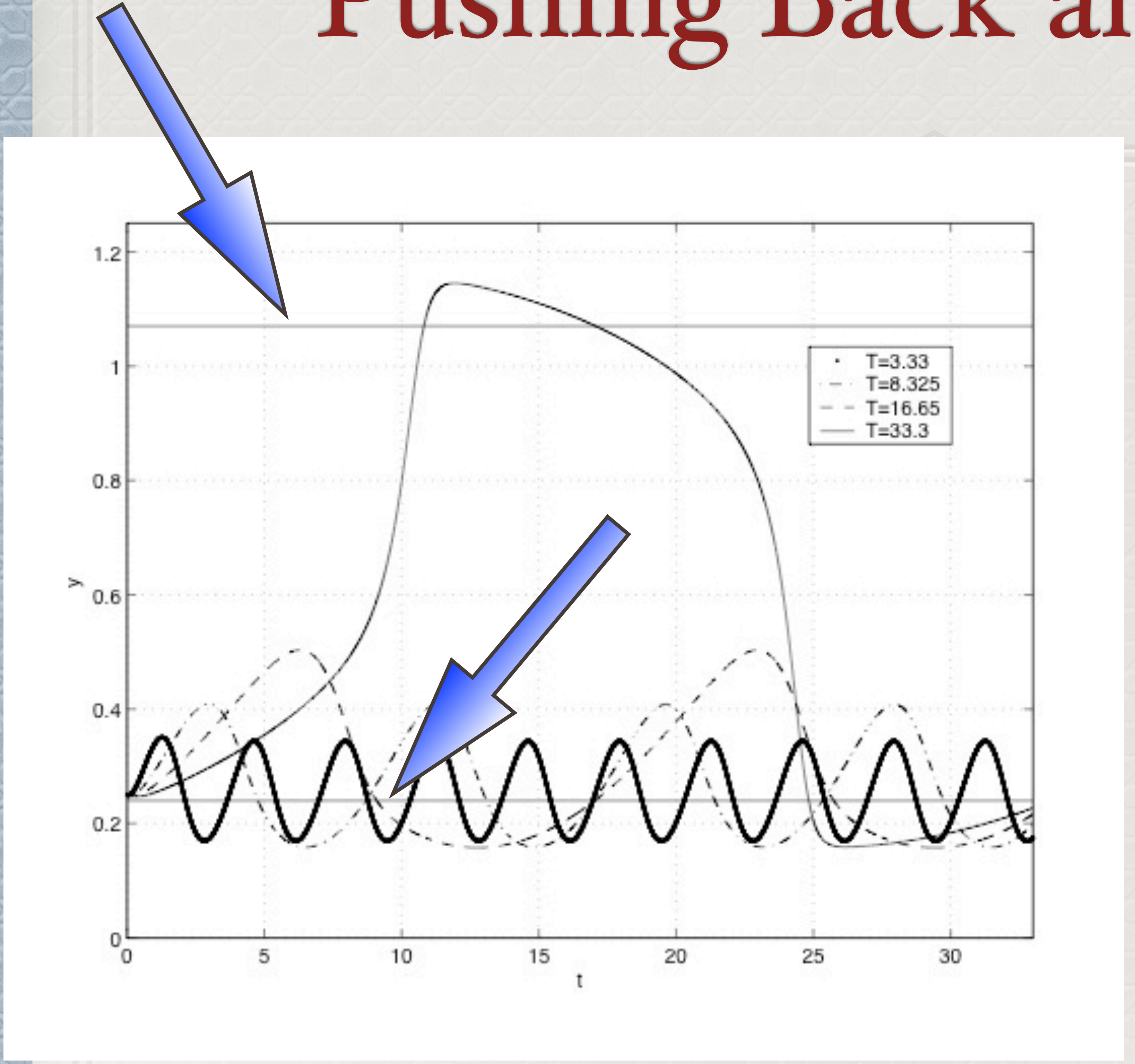
Bistable potential





- Two stable states or holes (the deeper one is called “globally stable”)
- Forcing can lead to transitions from one state to the other
- The recipe would say “roll down hill unless pushed in the opposite direction

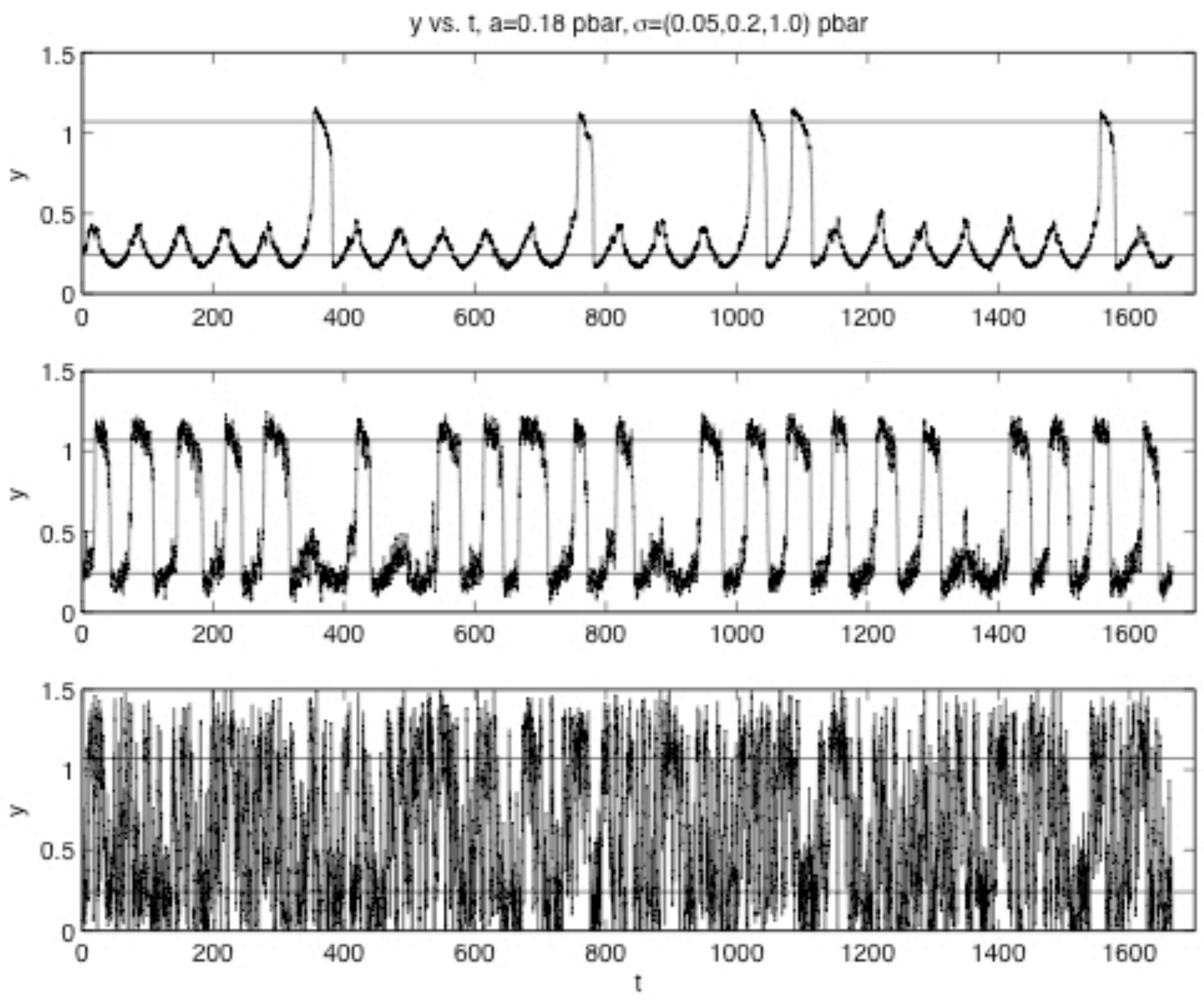
Pushing Back and Forth



- Small pushes mean little wiggles, larger pushes mean go between the two holes.
- Physicists call this multiple equilibria and it means today's world of warm European winters could somehow be disrupted with a big push.

Noisy Pushes

An example of "Stochastic Resonance"

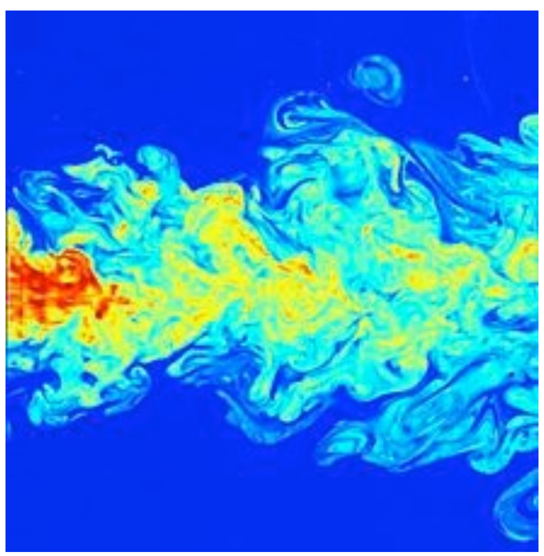


- Imagine starting with small back and forth pushes
- Now add "noise" or random kicks
- If you pick the size of the noise just right you go back and forth between the two holes even for small pushes!

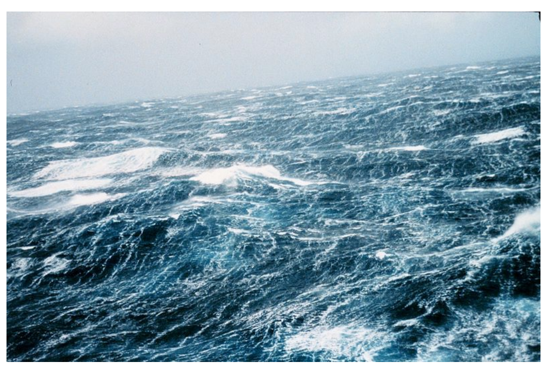
Part 3:
The scales of the ocean and the atmosphere

Scales of Environmental Flows

1 m 10 m 1-10 km 10-100 km >1000 km

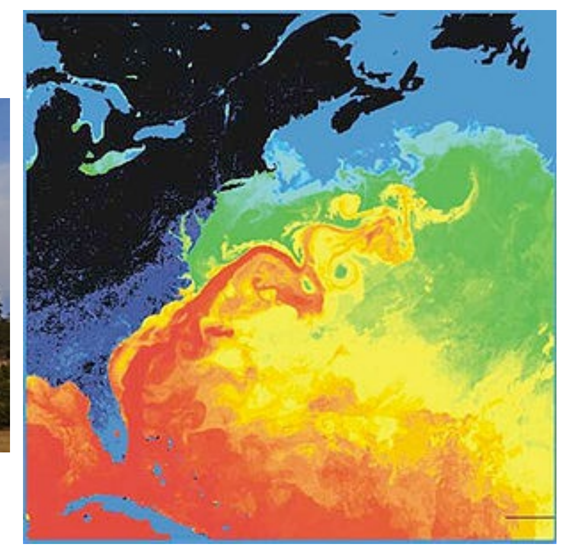


Turbulence



Ocean Waves

Clouds

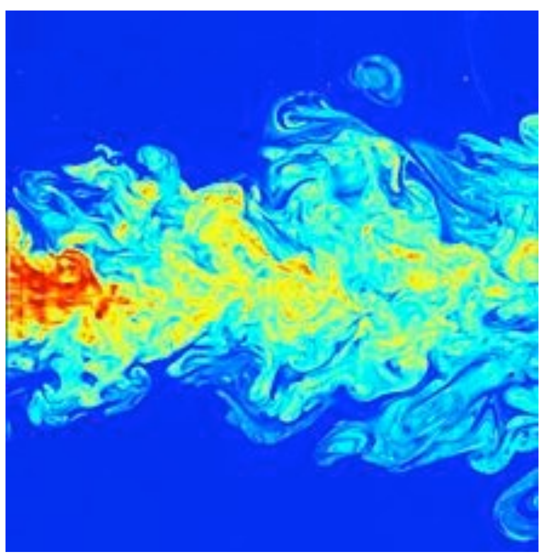
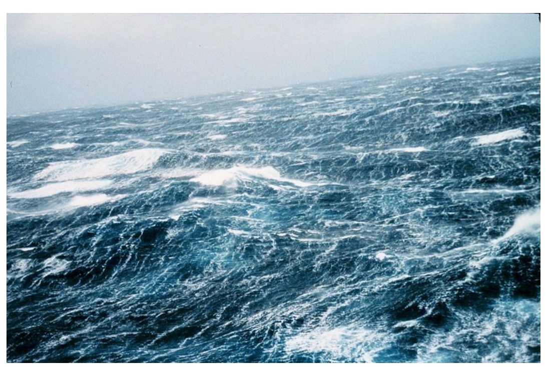


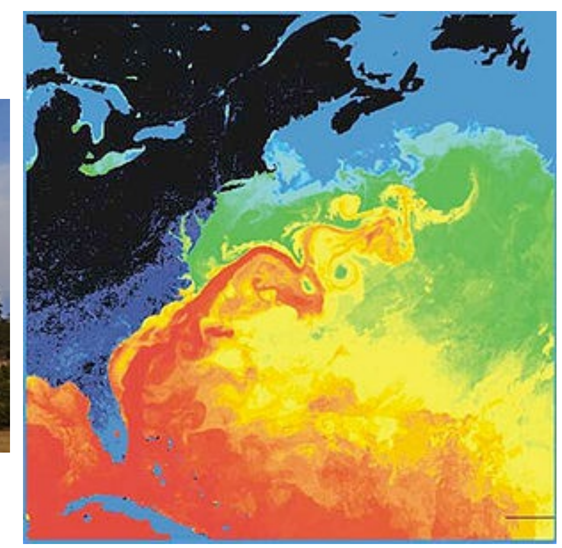
Gulf Stream Eddies

Hurricane

Scales of Environmental Flows

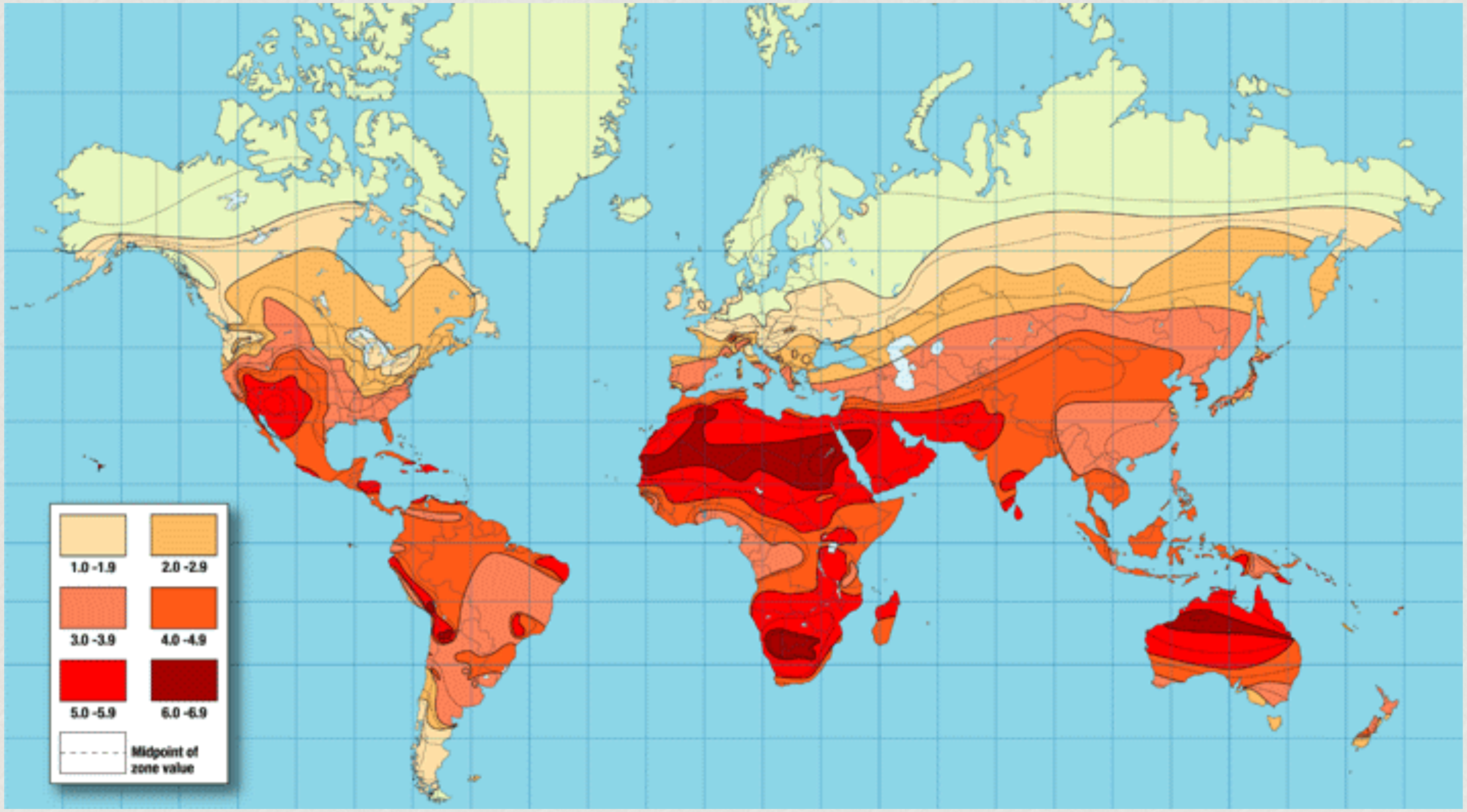
1 m   10 m   1-10 km   10-100 km   >1000 km



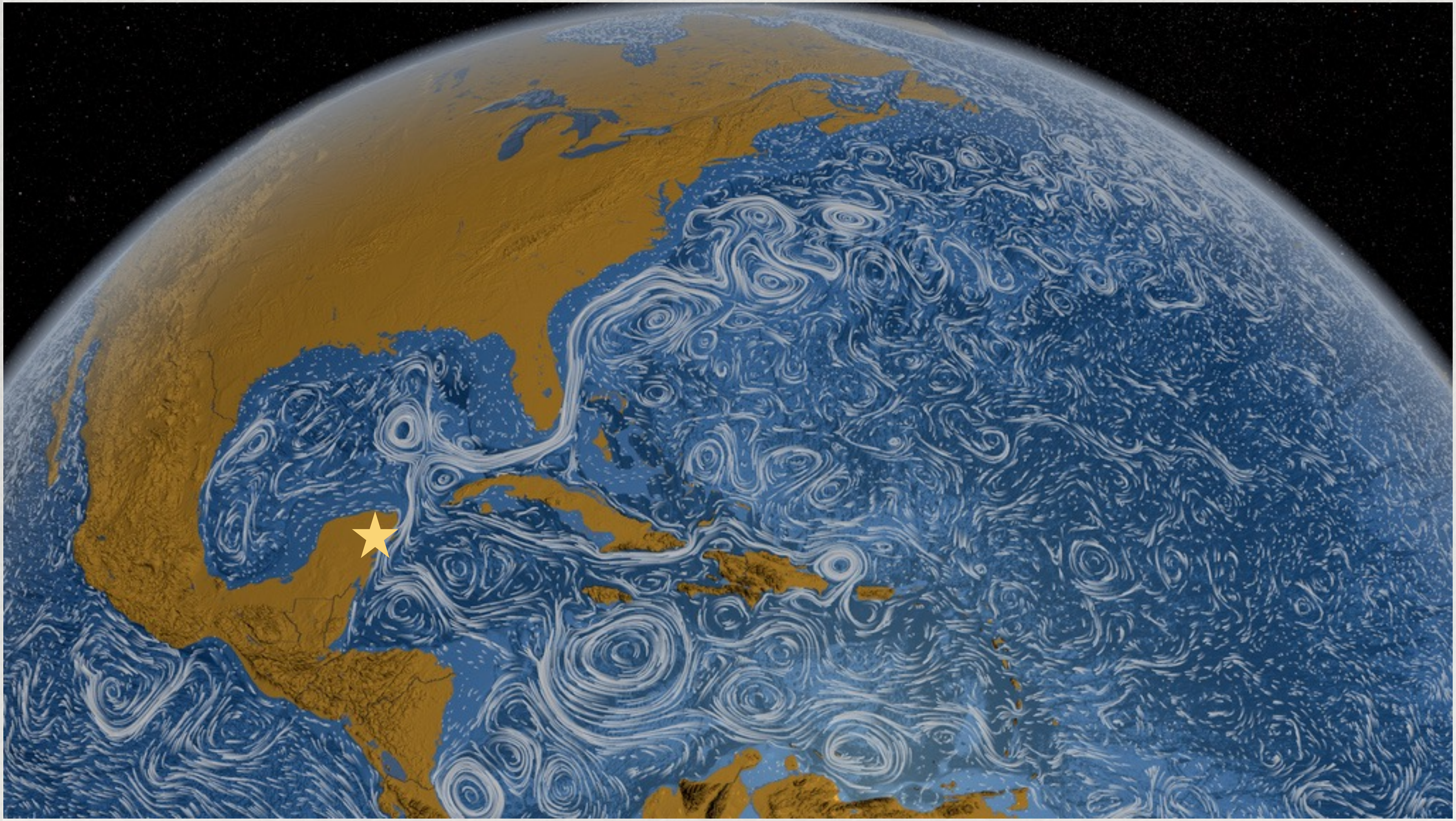

Two of the above are photos one could take, one is a photo from space, and the two remaining are heavily processed, or mathematized images, one from space, the other from a lab.

Solar Insolation map and link to a modern atmosphere simulation



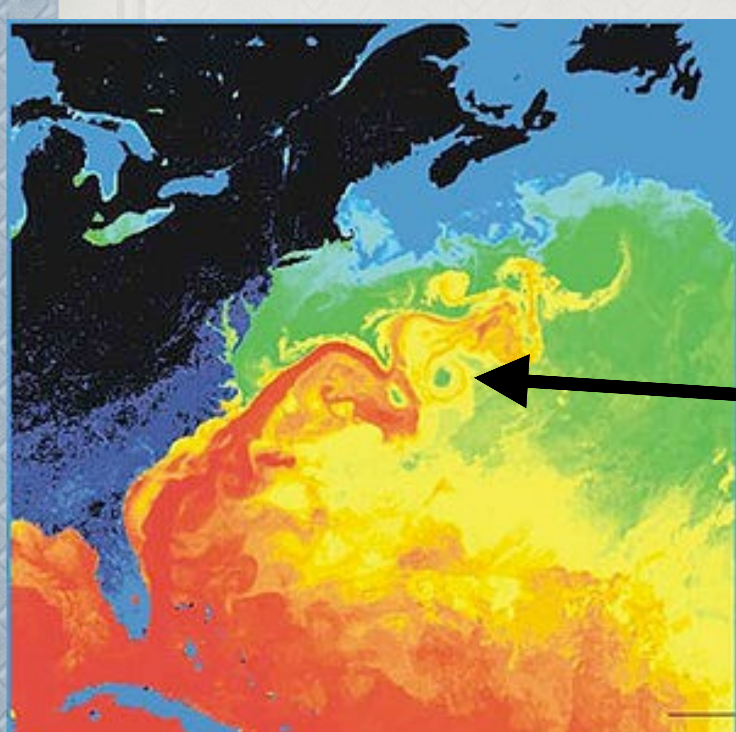
https://www.youtube.com/watch?v=4794mgJLTbU

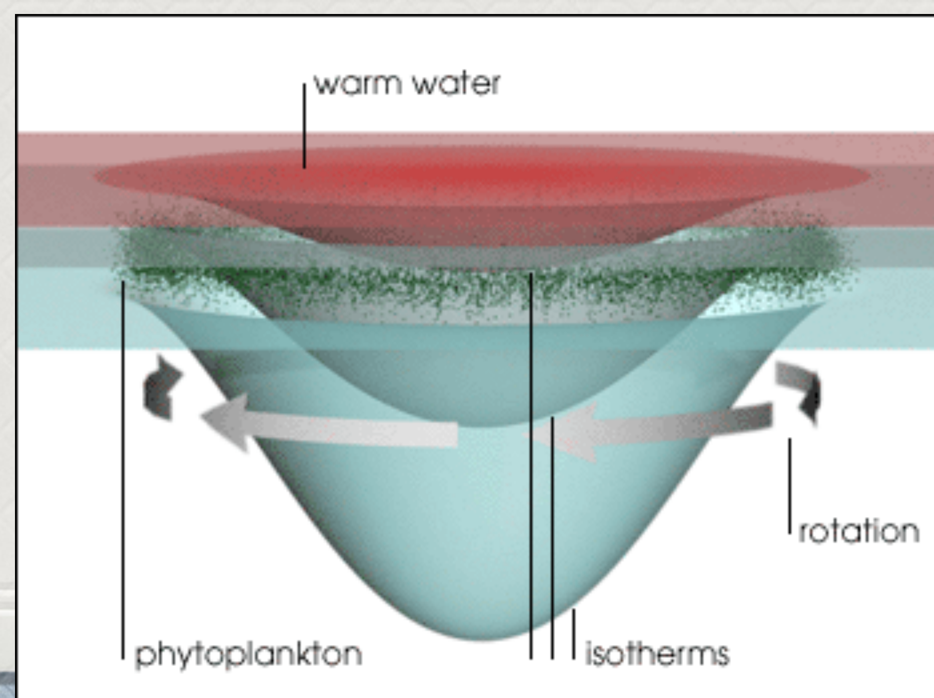
Super high resolution simulation of the ocean showing the dominance of eddies

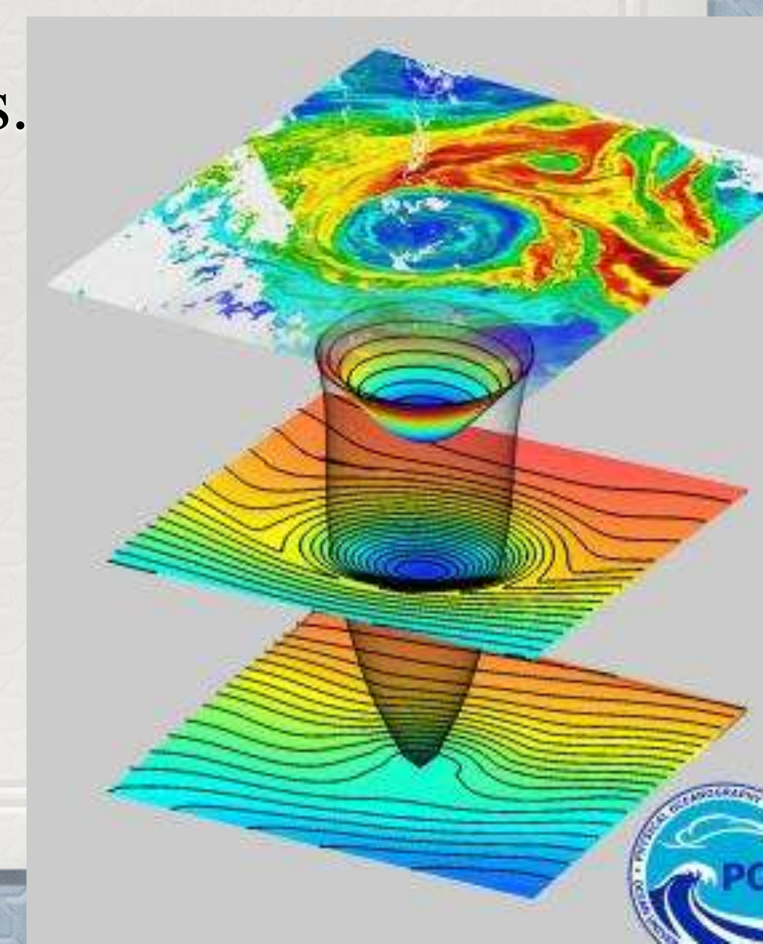


https://www.youtube.com/watch?v=CCmTYoPKGDs

- As the video from the last page shows the ocean is full of swirling motions called eddies, which modulate the slow circulations called gyres (similar to the cartoon circulation shown earlier)
- Prior to the advent of space-based instrumentation oceanographers believed gyres were how the oceans moved.
- The omni-presence of eddies is a paradigm shift in terms of the description of the oceans and requires different mathematical ideas.
- It also suggests eddies should be studied by themselves.







- The large scale motions in the atmosphere and oceans exhibit a great deal of order.

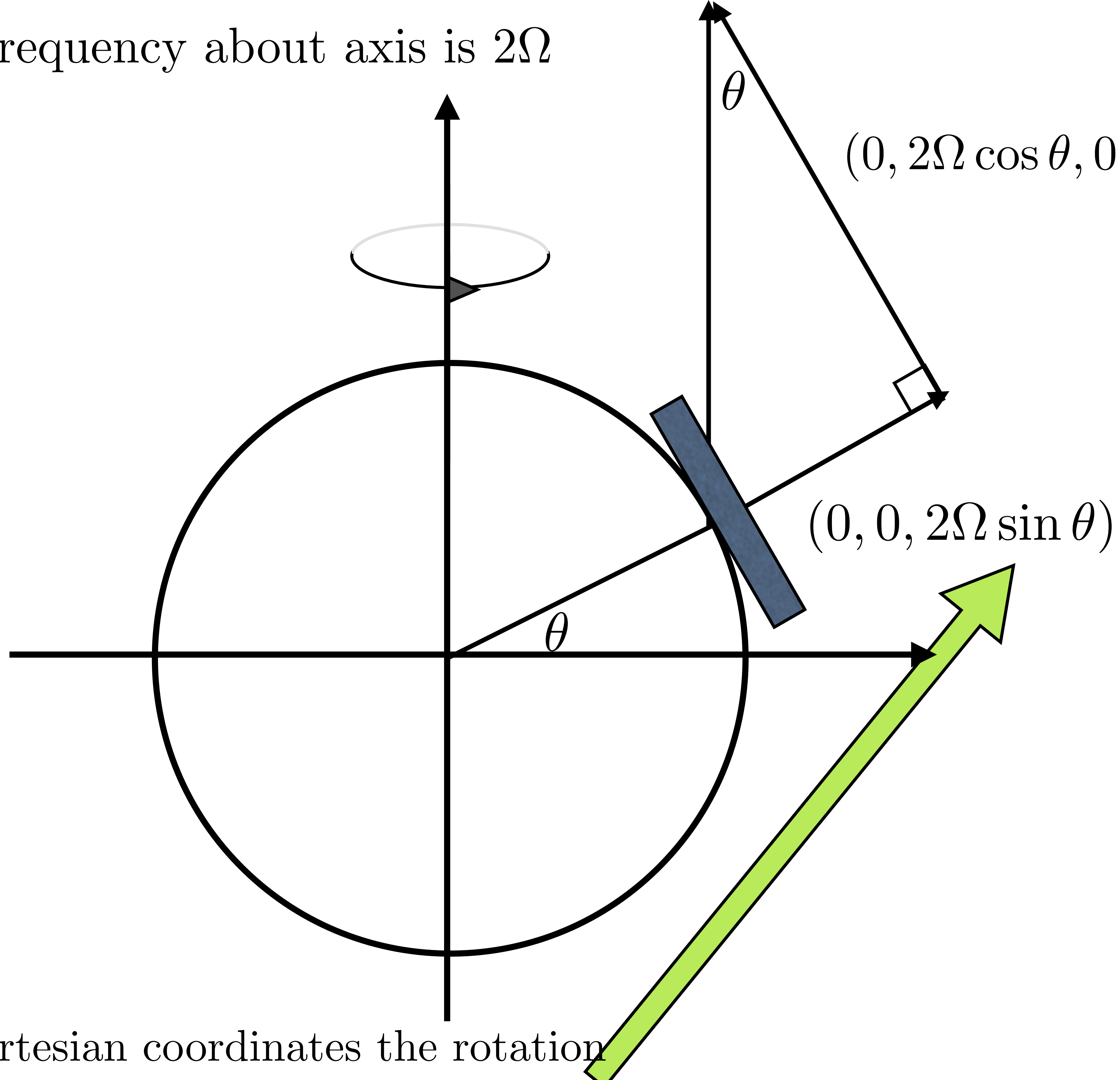
- Some of this has been known for thousands of years (e.g. the trade winds), and some is quite new.

- For latitudes away from the equator by about 10 degrees or more, the Earth's rotation plays a fundamental role in the dynamics.

- Because the vertical length scale of the atmosphere and ocean is measured in kilometers while their horizontal extent is measured in thousands of kilometers, large scale motions can be treated in a simplified, nearly two dimensional manner.

- In the next few (blue) slides I give you an introduction to this theory using the so called shallow water equations.

Rotation frequency about axis is 2Ω



In local Cartesian coordinates the rotation vector has two nonzero components: $(0, f^*, f)$

Vortices in the late 1400s and the early 2000s

Vorticity

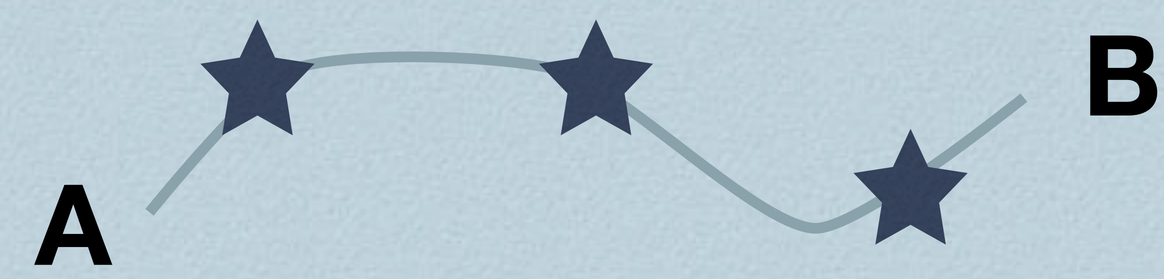
$$\vec{\nabla}\vec{u} = e_{ij} + r_{ij}$$

Nasty tensor stuff from physics

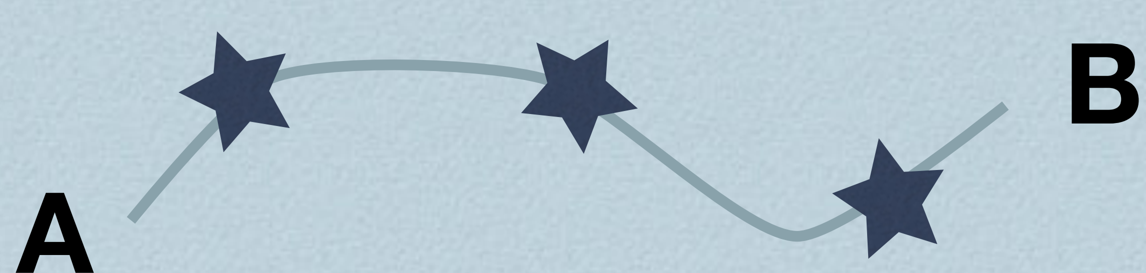
In continuum mechanics we learned that the gradient of velocity can be decomposed into the rate of strain tensor plus the rate of rotation tensor. The non-zero entries of the rotation tensor are the entries of the vorticity vector which is given more simply as the curl of the velocity.

$$\vec{\omega} = \vec{\nabla} \times \vec{u}$$

Less nasty vector stuff from vector calculus

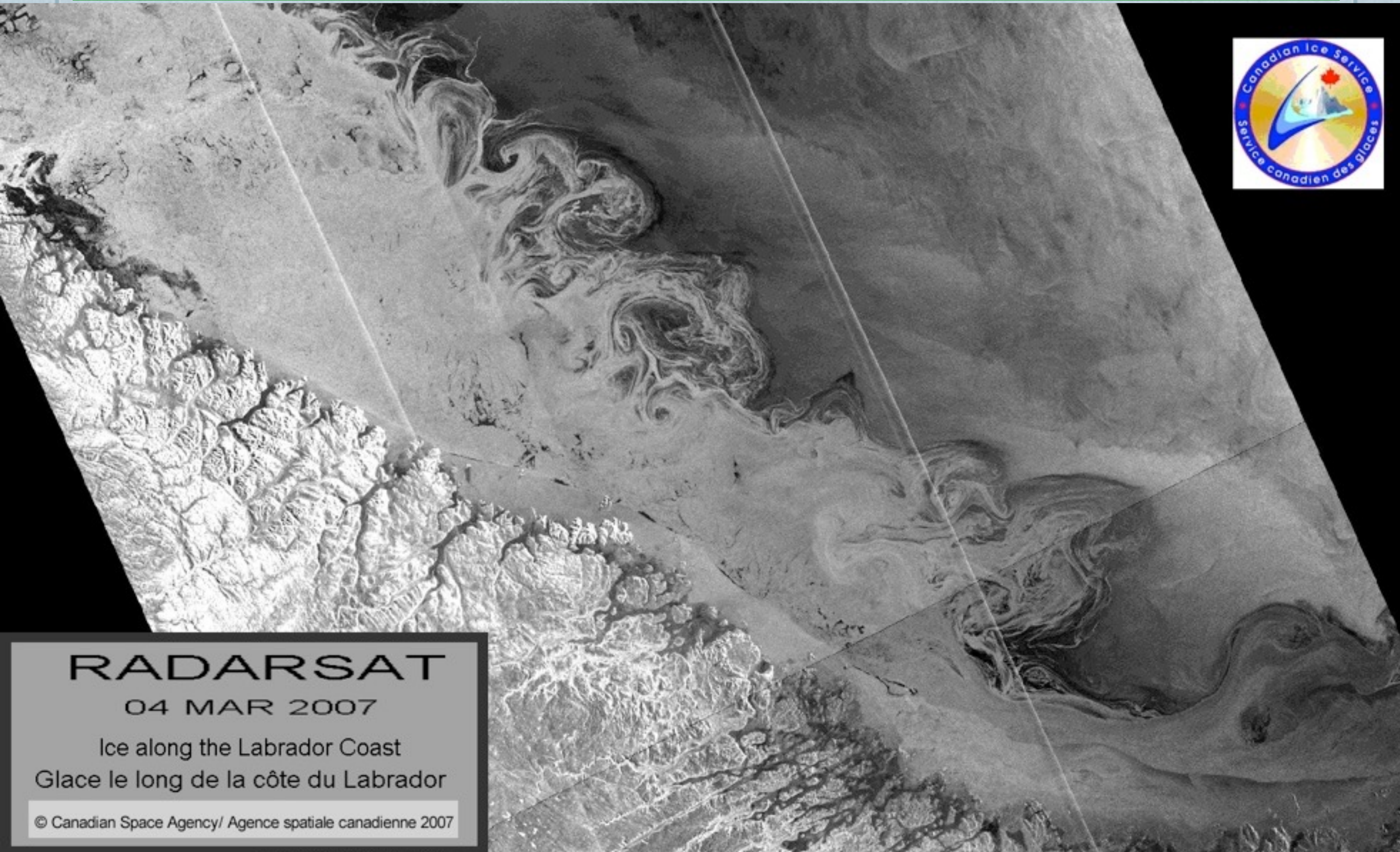


For the upper path the vorticity is zero while for the lower it is non-zero.



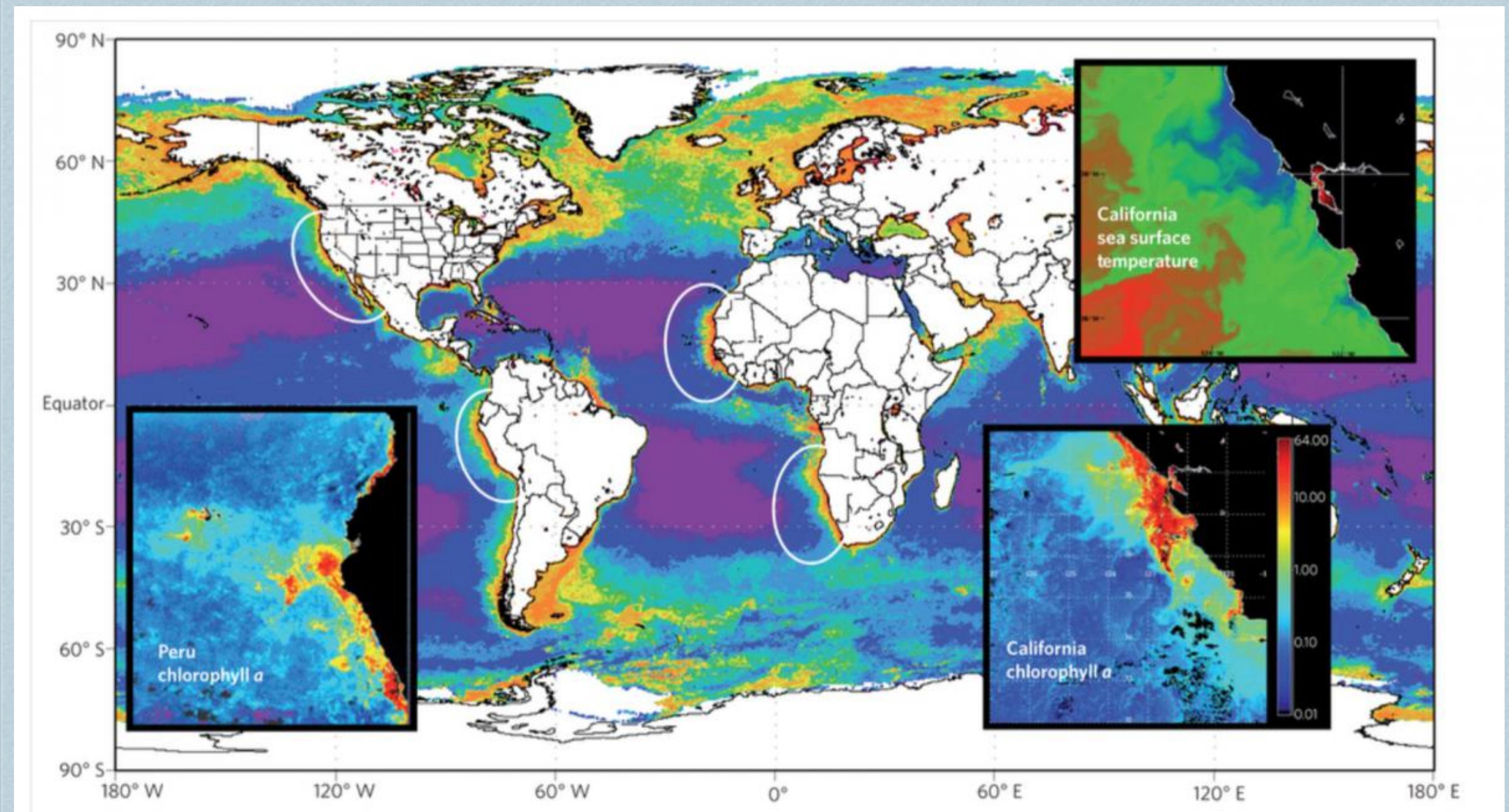
A visual way to understand the same thing

Vorticity is vital in quantitatively understanding large scale motions



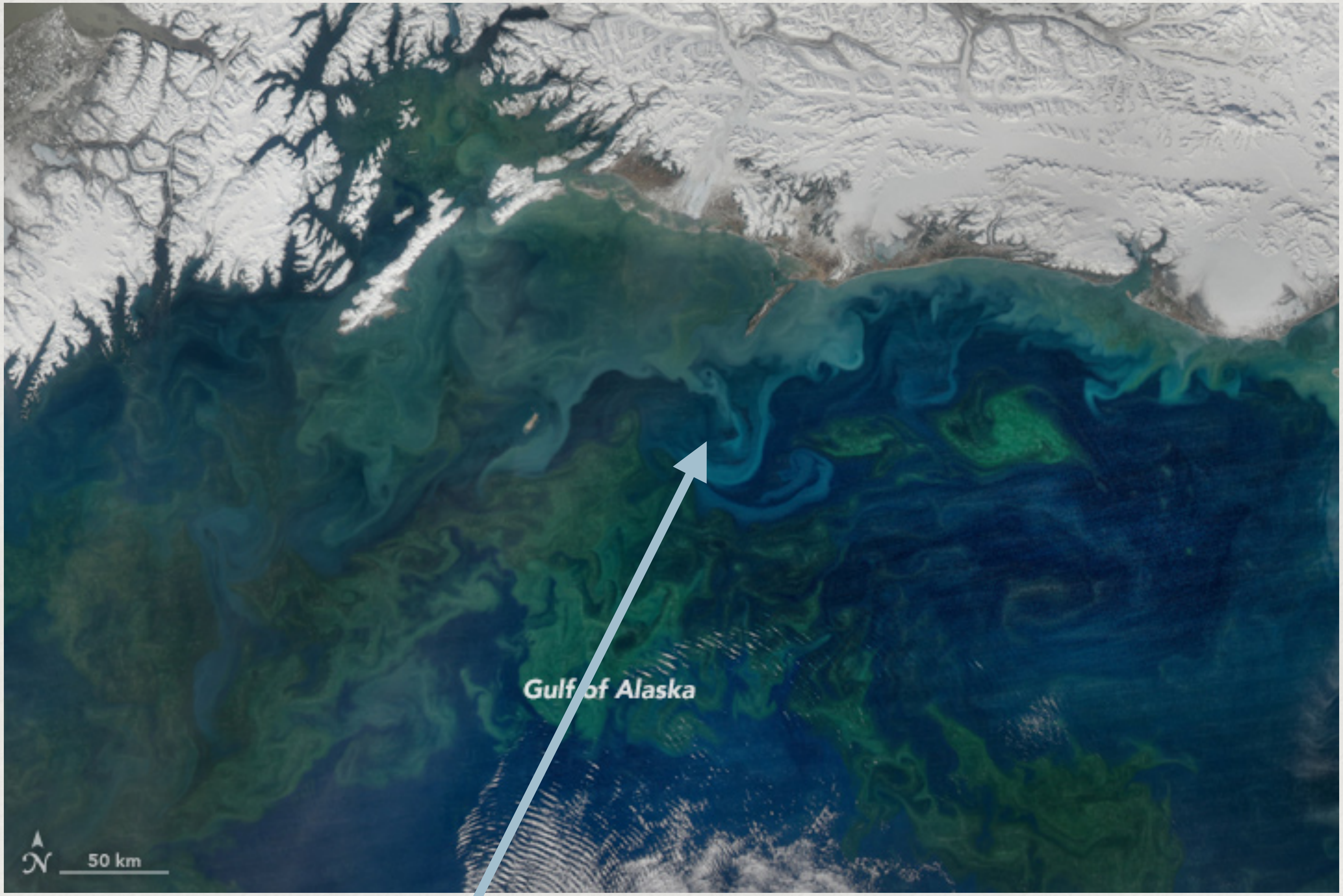
This is evident without any mathematics in the above

Coastal upwelling from space



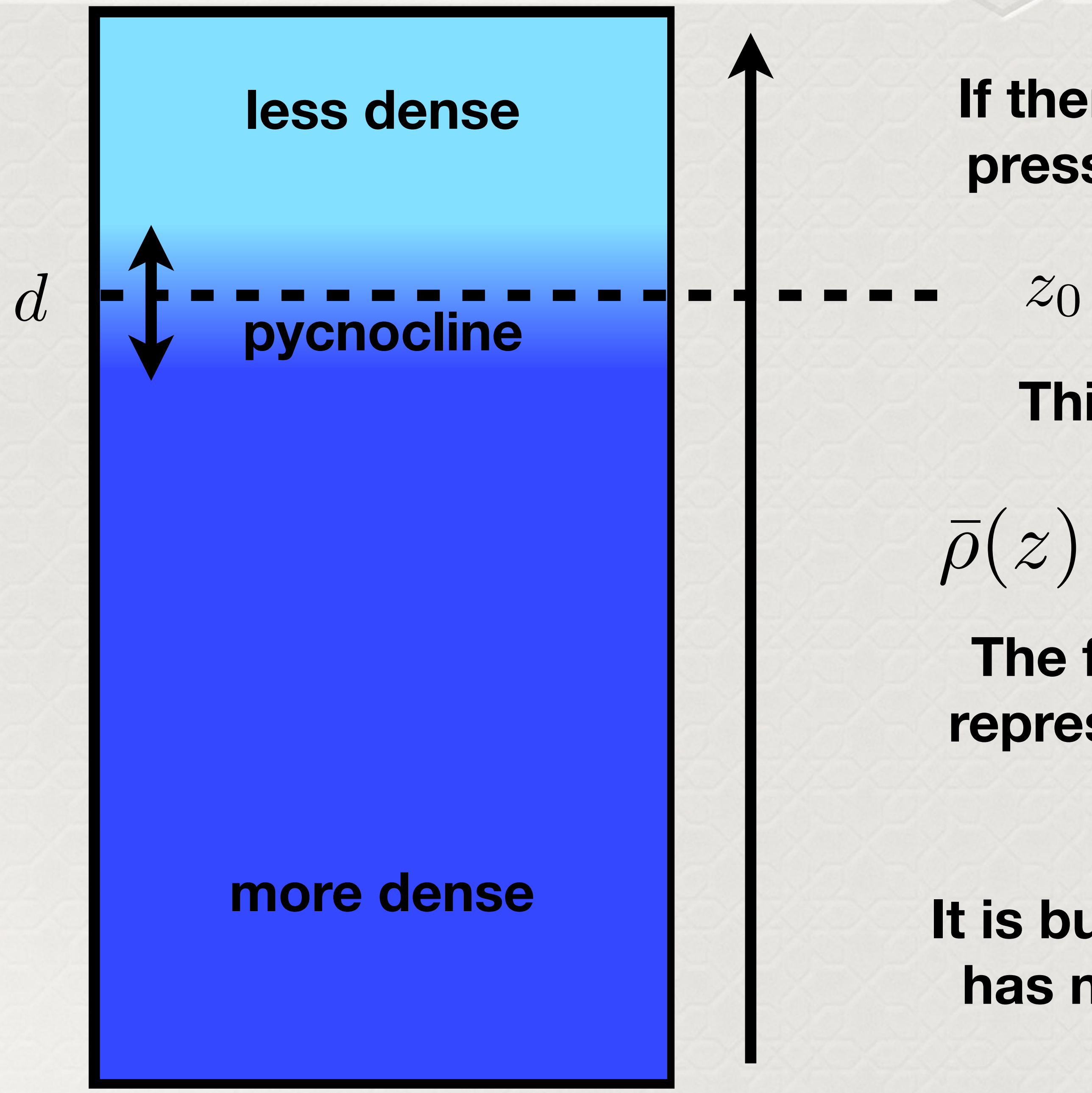
http://www.clivar.org/research-foci/upwelling

Modern research is probing even smaller scales



sub-mesoscale filaments off Alaska

Vertical Structure of the Oceans: Stratification



If there is no motion at each depth the pressure is balanced by the weight of the overlying fluid

This is called *hydrostatic balance*

$$\bar{\rho}(z) = 1 - a\tanh\left[(z - z_0)/d\right]$$

The formula above is a mathematical representation of how we think density varies with depth.

It is built out of exponential functions so has nice mathematical properties and closed form derivatives.

Remote sensing of internal waves and the sediment they kick up off the coast of Portugal

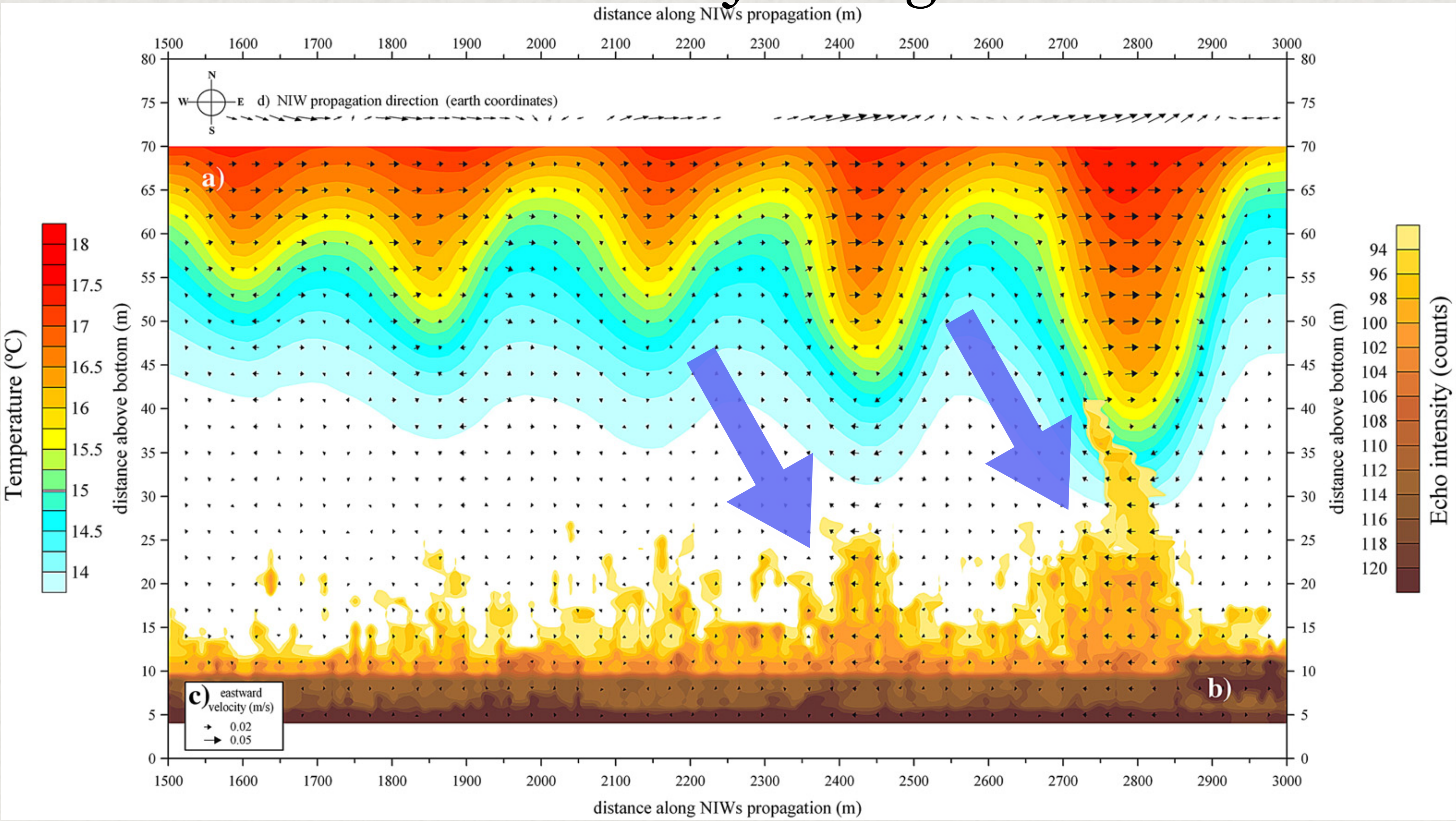
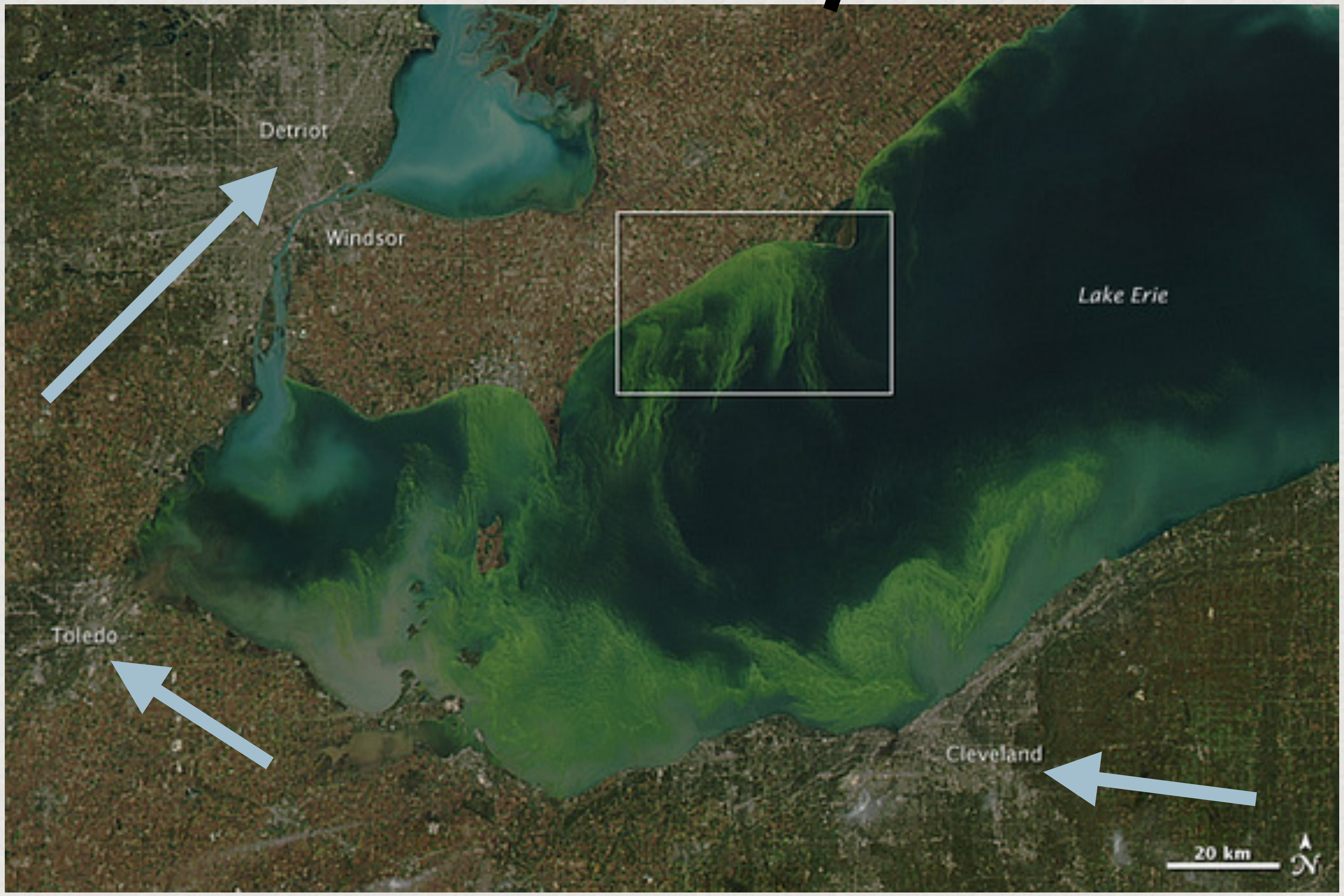


Fig. 15. *In-situ* NIW observations over *Station B* (2004 experiment). The figure shows the propagation of the first five NIWs represented in Fig. 9 (corresponding to the strongest NIW packet observed during the experiment). It is composed as an overlay of the following recorded series: a) temperature profile (°C), obtained by the thermistors chains; b) 300 kHz ADCP echo intensity (*in* counts); c) 300 kHz ADCP eastward velocity (as vectors). d) Representation of the short-period current velocity vectors (observed near the surface) showing the propagation direction of each soliton (see Section 6.4). All presented time series are converted in spatial series (along eastward NIW propagation), assuming a constant phase velocity of 0.34 m s^{-1} (estimated *in* Quaresma (2006)).

Part 4: Coastal Urban Centers

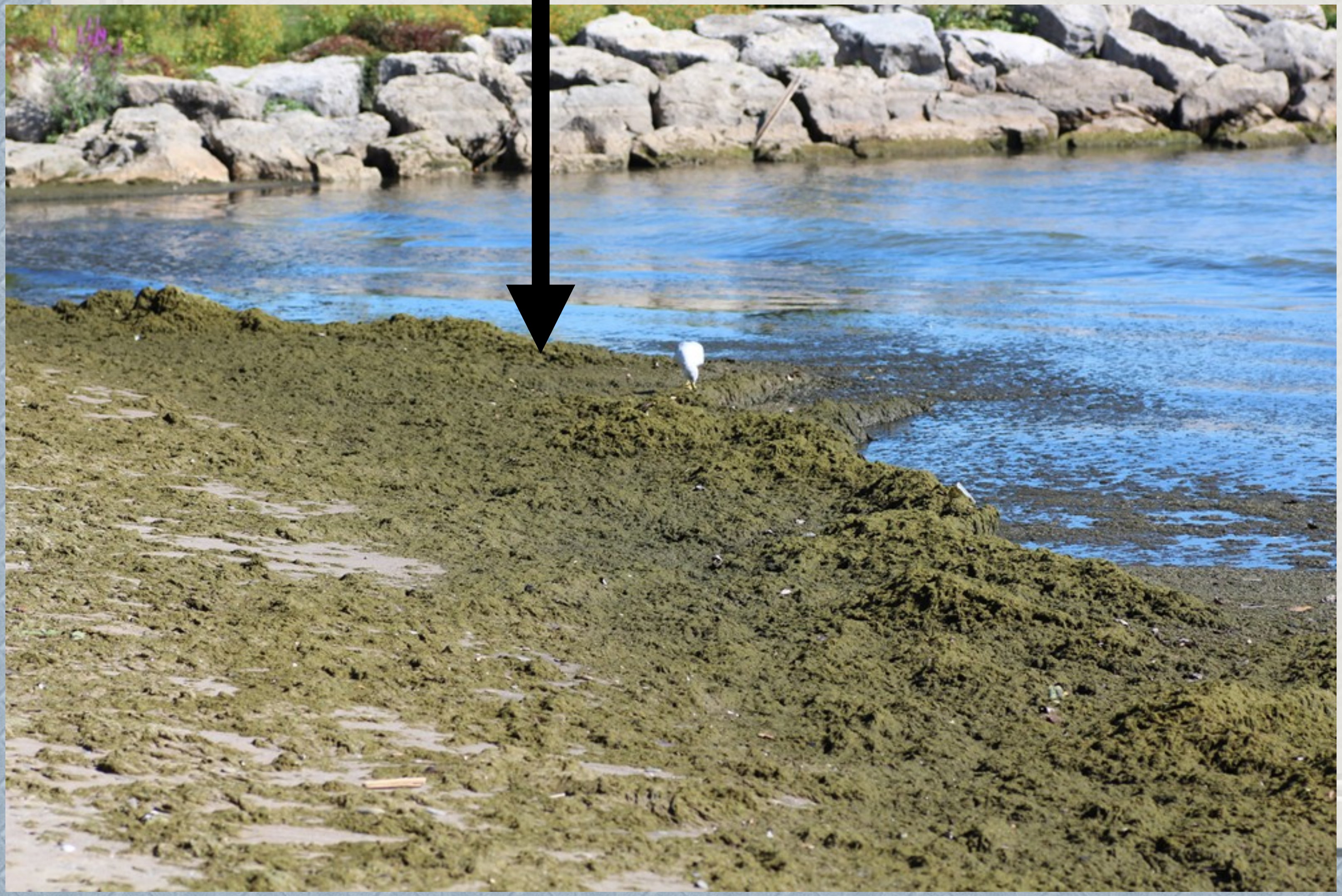
- We have done an express tour through modern oceanography.
- This is interesting, but it becomes more pressing when one considers just how much of the world's population lives on the coast (634 million within 10m of elevation).
- Large urban centers in particular often use the ocean as a place to dump waste.
- In the final section I want to take a quick visual tour of the effects of some of the motions I have shown on urban centers.
- We begin with a local example, namely the Laurentian Great Lakes.



- Plankton populations are seasonal, with prominent blooms, like the Spring bloom in the shallow Western Basin of Lake Erie shown above.
- Blooms typically involve many different species.

Lake Erie is the shallowest, and most biologically productive, but Lake Ontario has similar blooms

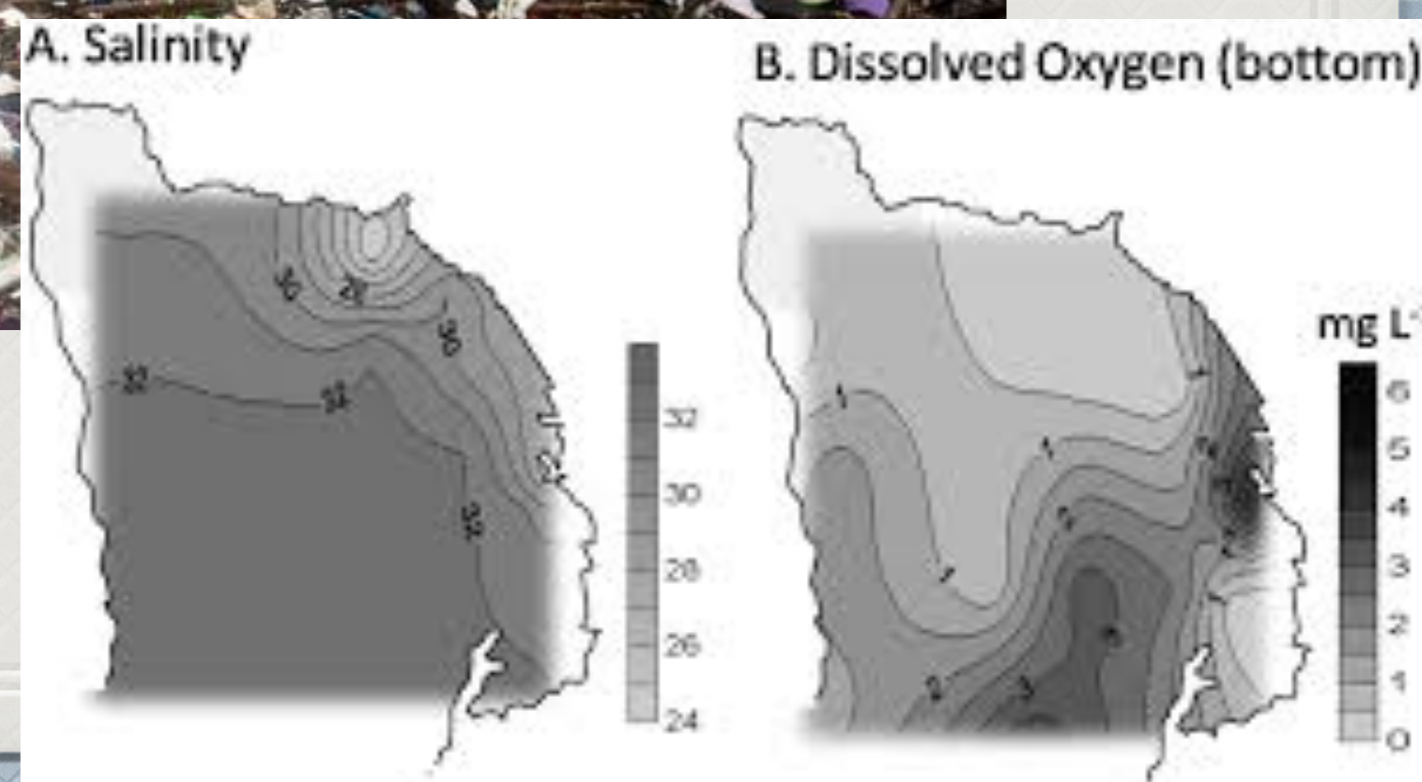
The implications of biological activity can involve nuisances like the cladophra below.



Or outright danger like the bacterial contamination above.

- Urban centers like Toronto that border the Great Lakes have significant challenges, but they also have significant resources to tackle the problem.
- In the developing world the population pressures are just as great, but the resources that can be brought to bear can be limited.
- Manila is the most densely populated city in the world. Around 15 million live around Manila Bay.
- The tropical climate means that torrential rainfall can occur leading to a significant garbage influx into Manila Bay.
- Along with physical garbage various chemical species as well as sewage are washed in as well.
- Environmental problems include eutrophication, population pressure, loss of biodiversity and over fishing.

Tsunami modelling of Manila Bay (the Philippines)



A. Salinity
B. Dissolved Oxygen (bottom)
mg L-1
32
30
28
26
24
6
5
4
3
2
1
0

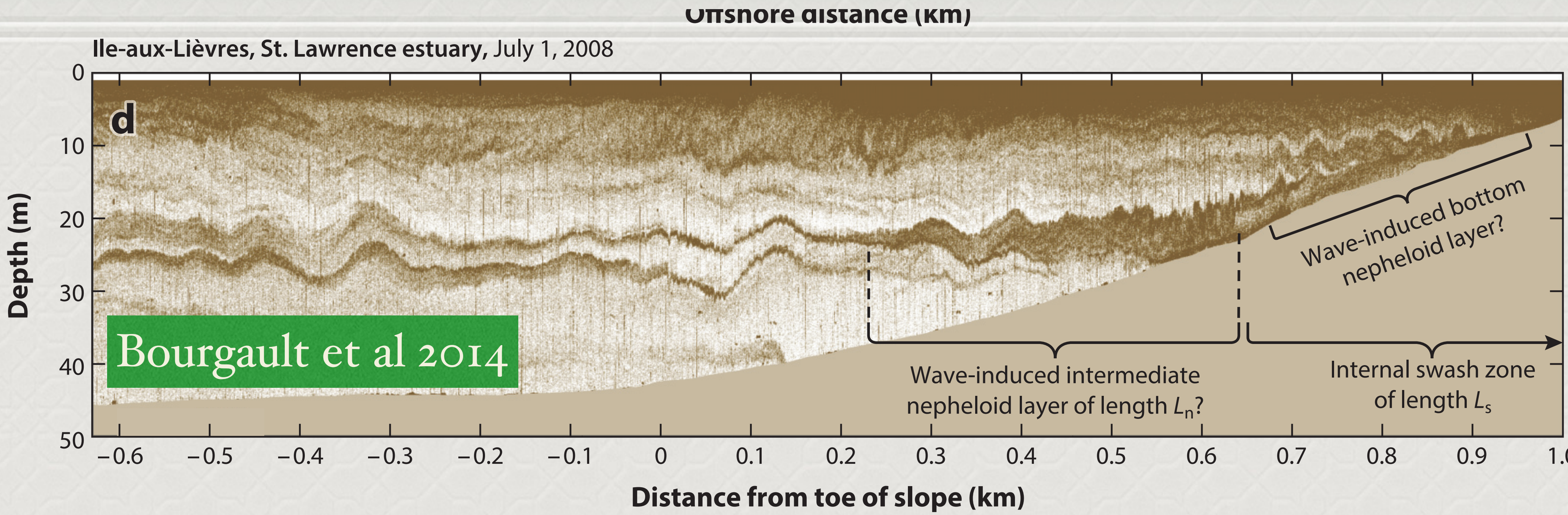
- Awareness of the environmental problem has increased significantly, however solutions are a fair way off.
- Shifts in ocean currents would change the rate, and possibly the manner, in which a large bay like Manila bay is flushed.
- The rate of flushing is a vital variable for any quantitatively based remediation effort.
- Successful numerical modeling efforts would be at the cutting edge of the research world, and hence possibly beyond the reach of NGOs, or local governments.
- A more basic issue may be politicians who treat environmental problems as something that can be plausibly denied.
- In many ways, the oceans are the ultimate tragedy of the commons...

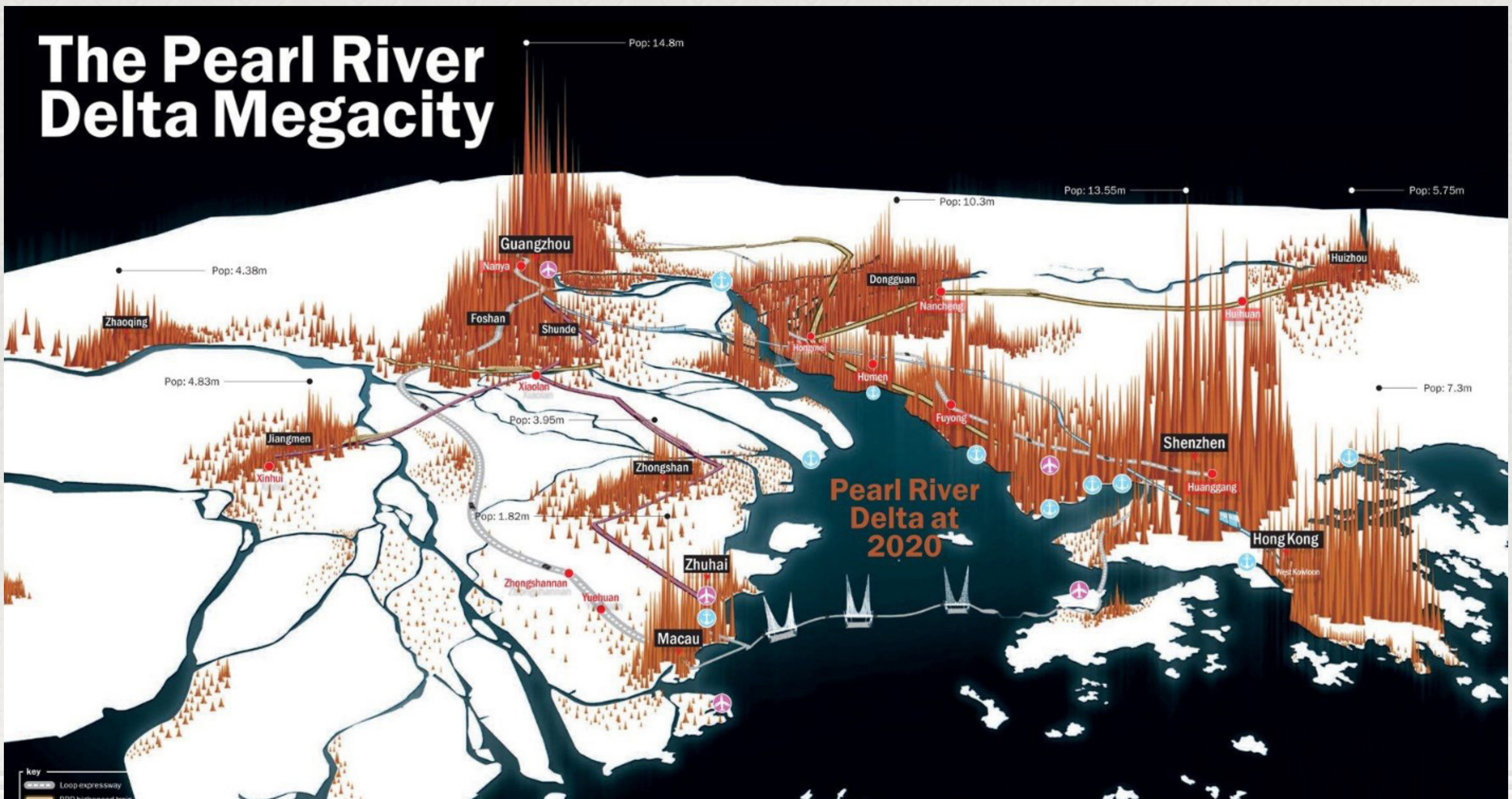
Summary 1

- We saw a rather nasty examples of man induced climate change.
- The Aral Sea disaster is muted due to the lack of major cities nearby.
- We then saw an example of natural climactic changes that coincided with the rise of civilization.
- Is climate stability a pre-requisite for civilization? Perhaps we can hope that once a civilization is advanced instability can be mitigated.

Summary 2

- I then catalogued some of the changes in perception on motions in the ocean that have occurred in the last 50 years.
- Natural science flies a bit below the public radar, but in my opinion this has been on par with scientific revolutions like quantum mechanics.
- I then showed an example of a coastal-urban environmental issue.
- A quantitative approach to this issue would require successful modeling on scales even smaller than the sub-mesoscale.


Offshore distance (km)
Ile-aux-Lièvres, St. Lawrence estuary, July 1, 2008
d
Depth (m)
0
10
20
30
40
50
Bourgault et al 2014
Wave-induced bottom nepheloid layer?
Wave-induced intermediate nepheloid layer of length L_n?
Internal swash zone of length L_s
−0.6
−0.5
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Distance from toe of slope (km)


The Pearl River Delta Megacity
Pop: 14.8m
Pop: 10.3m
Pop: 13.55m
Pop: 5.75m
Pop: 4.38m
Pop: 4.83m
Pop: 3.95m
Pop: 1.82m
Pop: 7.3m
Guangzhou
Zhaoqing
Foshan
Shunde
Dongguan
Huizhou
Jiangmen
Zhongshan
Shenzhen
Zhuhai
Macau
Hong Kong
Pearl River Delta at 2020