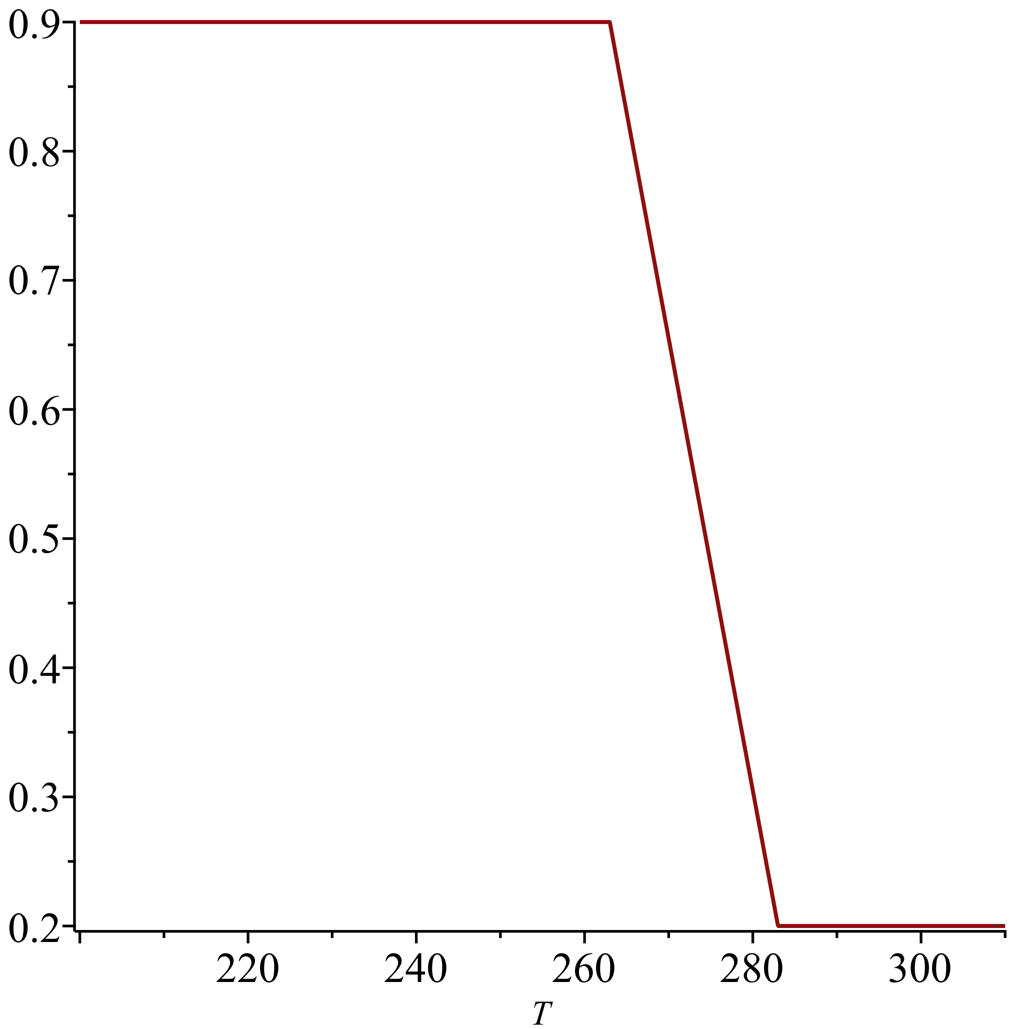
> alpha $:= 0.9 - \frac{(\text{Heaviside}(T-263) - \text{Heaviside}(T-283)) \cdot 0.7}{20} \cdot (T-263) - \text{Heaviside}(T-283) \cdot 0.7$

$$\alpha := 0.9 - \frac{(0.7\,\text{Heaviside}(T-263) - 0.7\,\text{Heaviside}(T-283))\,(T-263)}{20} - 0.7\,\text{Heaviside}(T-283) \qquad (1)$$

This is the way to build the albedo profile so it has two constant regions and one linear region using the Heaviside step function

> *plot*(alpha, $T = 200..310$)



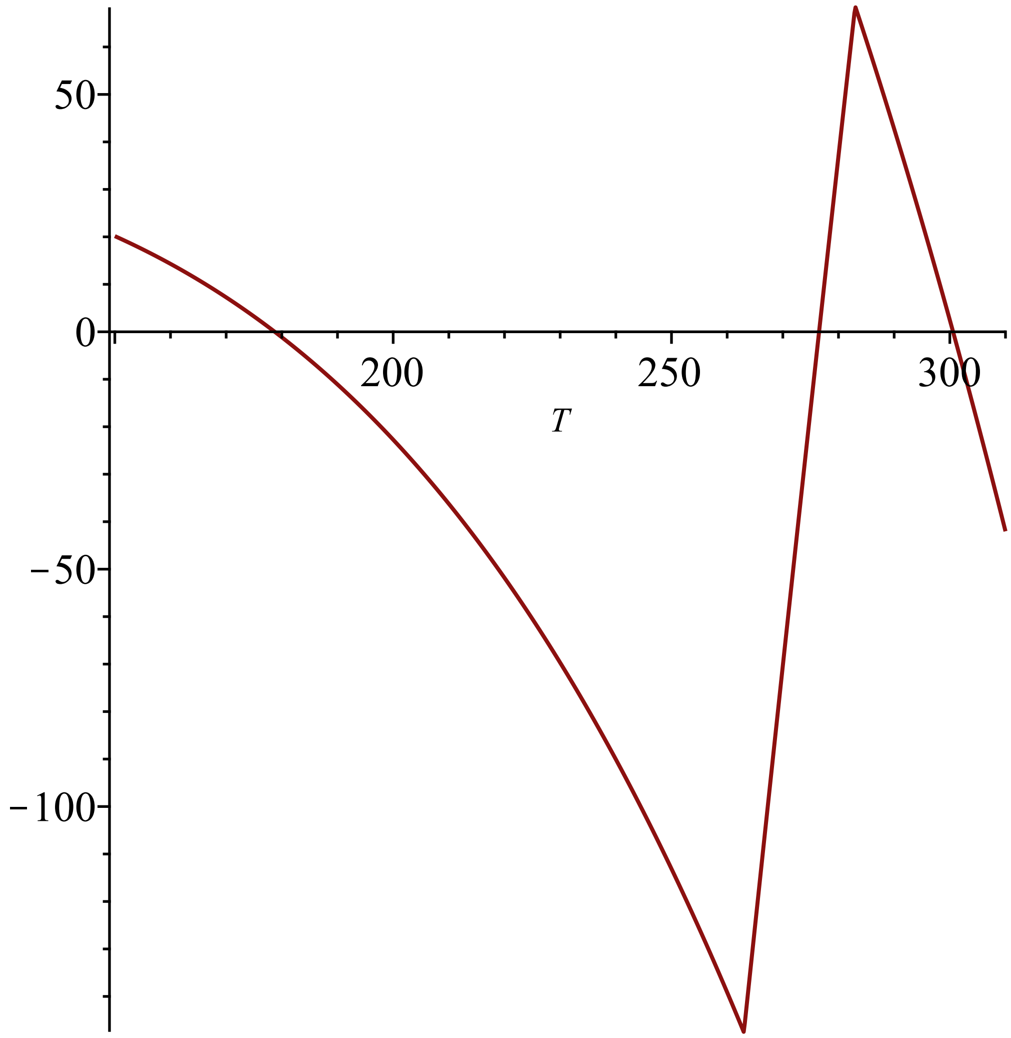
Here is a plot of the albedo

> *myrhs* $:= S \cdot (1 - \text{alpha}) - \text{sigma} \cdot T^4$

$$myrhs := S\left(0.1 + \frac{(0.7\,\text{Heaviside}(T-263) - 0.7\,\text{Heaviside}(T-283))\,(T-263)}{20} + 0.7\,\text{Heaviside}(T-283)\right) - \sigma\,T^4 \qquad (2)$$

This is the RHS of the climate equation

> *plot*(*subs*($S = 400$, sigma $= 0.7 \cdot 5.6\text{e}-8$, *myrhs*), $T = 150..310$)



This is a plot of the right hand side. You can see that we cross the T-axis three times, meaning there are three equilibrium points.

>

> $alpha1 := 0.7 - 0.5\cdot \text{Heaviside}(T - 273)$

$$\alpha 1 := 0.7 - 0.5\ \text{Heaviside}(T - 273) \quad (1)$$

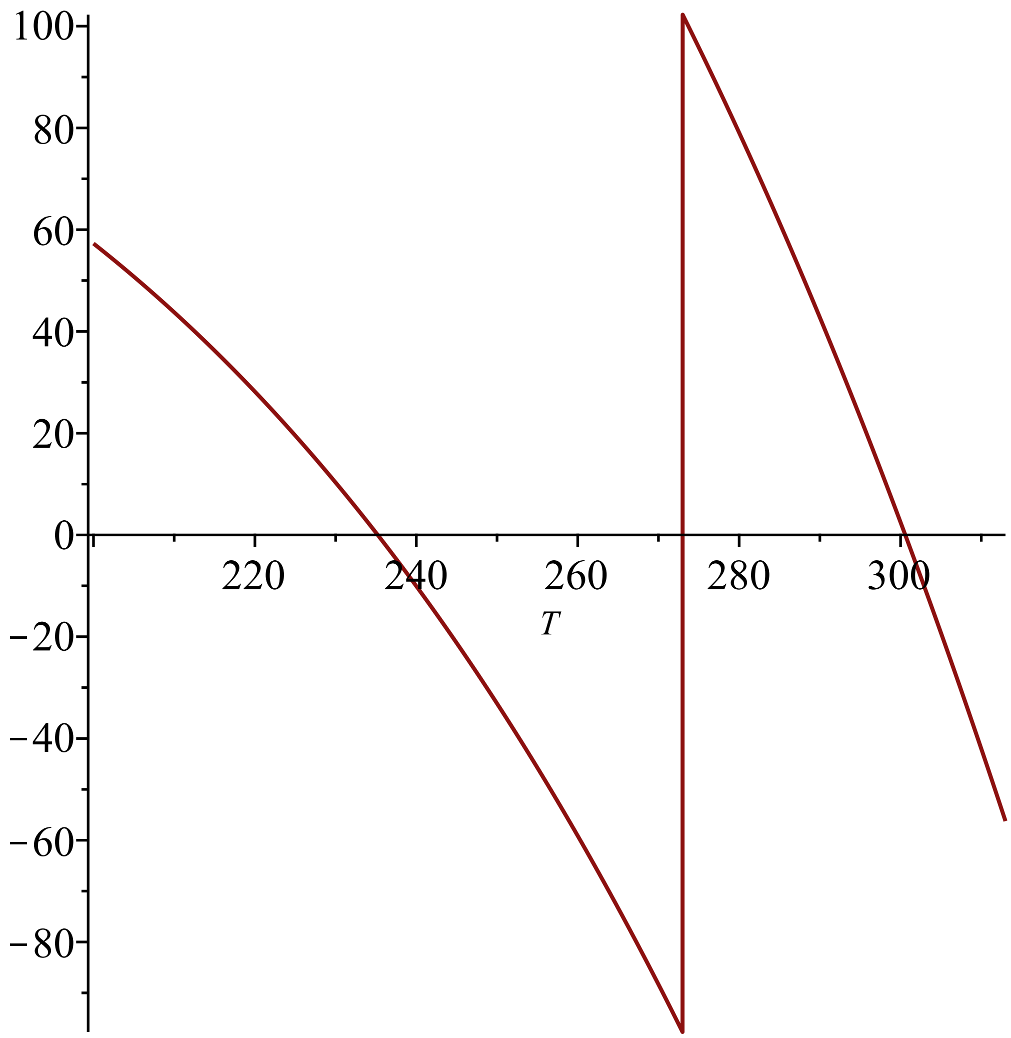
This is the albedo written using the Heaviside step function

> $myrhs1 := S\cdot(1 - alpha1) - \text{sigma}\cdot T^4$

$$myrhs1 := S\ (0.3 + 0.5\ \text{Heaviside}(T - 273)) - \sigma\ T^4 \quad (2)$$

This is the RHS of the climate equation

> $plot(subs(S = 400, \text{sigma} = 0.7\cdot 5.6\text{e}-8, myrhs1), T = 200 ..313)$



This is a plot of the RHS and it shows that there are two equilibrium values, and that the RHS>0 for values of T less than the equilibrium, and the RHS<0 for values of T greater than the equilibrium.

>

> $eq1 := S\cdot 0.3 - \text{sigma}\cdot T^4$

$$eq1 := 0.3\ S - \sigma\ T^4 \quad (3)$$

> $eq2 := S\cdot 0.8 - \text{sigma}\cdot T^4$

$$eq2 := 0.8\ S - \sigma\ T^4 \quad (4)$$

> $solve(subs(S = 400, \text{sigma} = 0.7\cdot 5.6\text{e}-8, eq1), T)$

$$235.2197558, 235.2197558\ I, -235.2197558, -235.2197558\ I \quad (5)$$

> $solve(subs(S = 400, \text{sigma} = 0.7 \cdot 5.6e-8, eq2), T)$

$$300.5840819, 300.5840819\,I, -300.5840819, -300.5840819\,I \quad \textbf{(6)}$$

>

> $E := \text{sigma} \cdot T^4$

$$E := \sigma T^4 \quad \textbf{(1)}$$

Stefan's Law

> $Epert := subs(T = T0 + DT, E);$

$$Epert := \sigma (T0 + DT)^4 \quad \textbf{(2)}$$

Stefan's law written as a perturbation

> $E0 := \text{sigma} \cdot T0^4$

$$E0 := \sigma T0^4 \quad \textbf{(3)}$$

Stefan's law without perturbations

> $Reldiff := \frac{(Epert - E0)}{E0}$

$$Reldiff := \frac{\sigma (T0 + DT)^4 - \sigma T0^4}{\sigma T0^4} \quad \textbf{(4)}$$

The relative difference

> $expand(Reldiff);$

$$\frac{DT^4}{T0^4} + \frac{4\, DT^3}{T0^3} + \frac{6\, DT^2}{T0^2} + \frac{4\, DT}{T0} \quad \textbf{(5)}$$

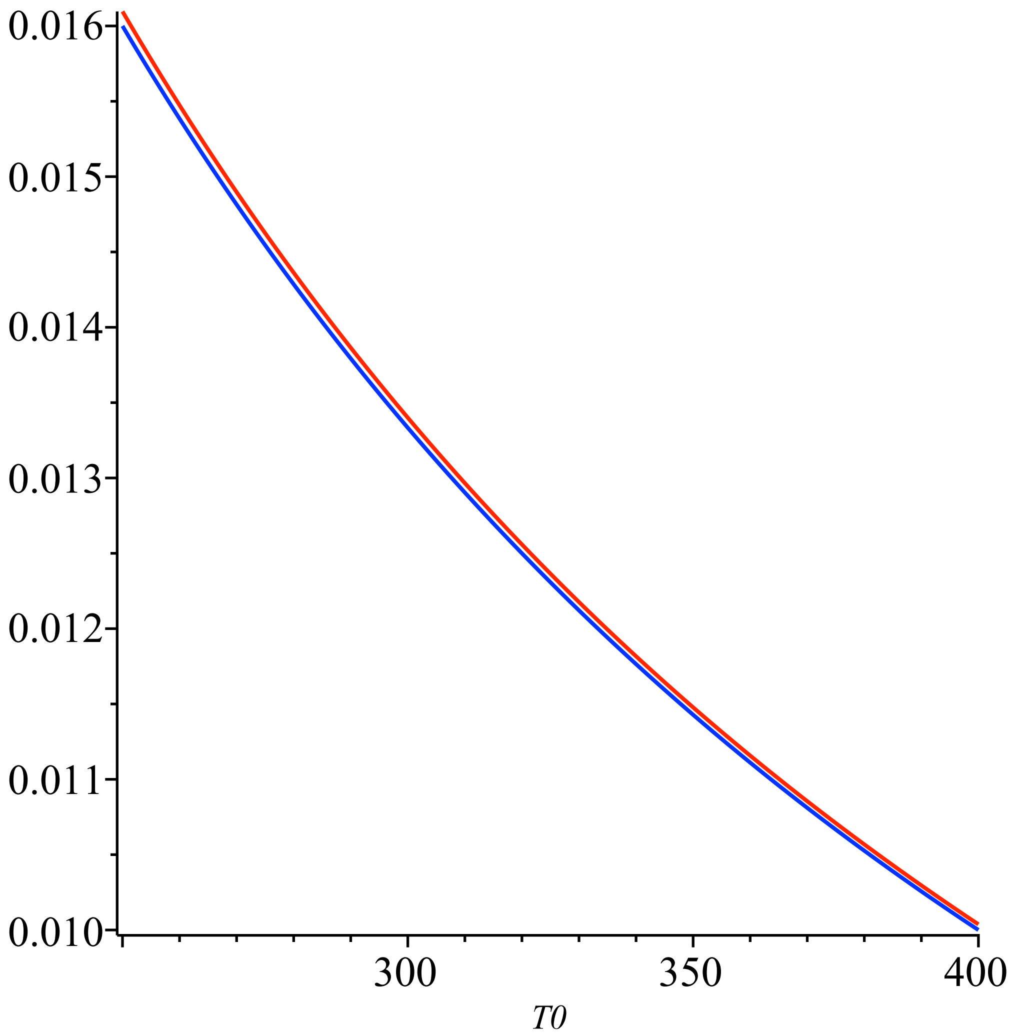
The relative difference expanded out using the binomial theorem

> $Reldifflead := \frac{4 \cdot DT}{T0}$

$$Reldifflead := \frac{4\, DT}{T0} \quad \textbf{(6)}$$

This is the simplest approximation to the relative difference

> $plot([subs(DT = 1, Reldiff), subs(DT = 1, Reldifflead)], T0 = 250..400, color = ['red','blue'])$



And here is a plot showing just how good the approximation is

>