Function Building

Last time we tried to solve some problems in simple climate modeling using functions that had jumps in them. In this problem set out to play with functions with jumps. This is relatively simple mathematics, but it turns out to be very useful in engineering (recall my demo of turning the lights on and off). The basic building block is the Heaviside step function, or $H(x)$:

$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The function has two possible outputs, 0 and 1. For negative inputs (x values) the output is zero, and when x reaches 0 the output suddenly jumps. Actually different sources define what happens at $x = 0$ in different ways, but we won't worry about it for now.

Question 1: Sketch each of the following functions for the domain given
i) $xH(x)$ $-2 \leq x \leq 2$.
ii) $x^2H(x)$ $-2 \leq x \leq 2$.

iii) $(H(x+1) - H(x-1))\ -2 \leq x \leq 2$.
iv) $x^2(H(x+1) - H(x-1))\ -2 \leq x \leq 2$.

v) $\sin(H(x))$ $-4\pi \leq x \leq 4\pi$
vi) $H(\sin(x))$ $-4\pi \leq x \leq 4\pi$.
vii) $H(\sin(x))\sin(x)$ $-4\pi \leq x \leq 4\pi$.

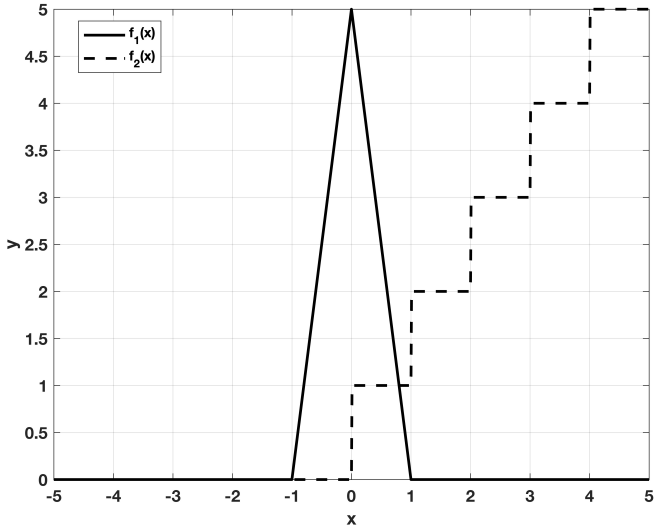

Figure 1

Question 2: Figure out how to write functions for each of the functions in the above picture.

Question 3: Describe the behaviour of each of the following functions: $H(f(x))$, $H(f(x))f(x)$.

Rough Work