UNIVERSITY OF WATERLOO

Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

Grade 7/8 Math Circles

March 3/4/5 2020

Applications

Energy

What is energy?

Energy is the ability to do ____work____. It is a property that allows things to ____move____. It takes energy for us to...

- Lift our arms.
- Jump up and down.
- Walk to school.

...and so much more.

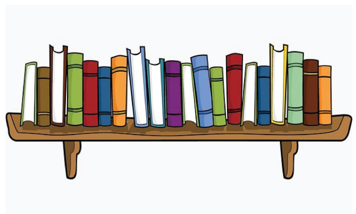
Retrieved from https://www.disneyclips.com/images/goofy5.html

Some types of energy:

- **Kinetic energy:** The energy an object possesses from being in ____motion____, because something had to transfer energy to it in order for it to move.

Retrieved from http://clipart-library.com/running-cartoons.html

- **Gravitational potential energy:** Energy that is ____stored____ due to an object being ______above the ground______. When objects aren't moving, they can have the potential to move due to the energy stored in them. For example, a book on a shelf is not moving, but it has gravitational potential energy because if someone were to nudge it, it would fall to the ground due to gravity (therefore it would move).



Retrieved from https://www.pinterest.ca/pin/588353138794971521/

- Light energy.
- Heat energy.

Exercises:

1. Write down 2 examples of things that have kinetic energy.

2. Write down 2 examples of things that have gravitational potential energy.

A Closer Look at Kinetic Energy

Harry Potter, 60 kg, is running away from a group snatchers in the woods at a speed of 10 m/s. When he is caught, the snatchers tell him that if he can calculate how much kinetic energy he had while running, they won't capture and bring him to Voldemort. How can Harry do this?

Calculating kinetic energy:

$$KE = \frac{1}{2} \times m \times v^2$$

Where...
KE stands for kinetic energy, meaured in joules (J).
m is the mass in kg.
v is the speed in m/s.

So, Harry must use...

$m = 60$ kg
$v = 10$ m/s

To calculate...

$KE = \frac{1}{2} \times 60 \times 10^2$
$KE = 3000$ J

Exercise:

You pitch a baseball to a batter. Using a radar gun, you find that the speed at which you pitched the baseball was 27 m/s. If the baseball has a mass of 149 g, what is the kinetic energy of the baseball after you let go of the ball?

We have,
$v = 27$ m/s
$m = 0.149$ kg

We use,
$KE = \frac{1}{2} \times 0.149 \times 27^2$

To get,
$KE = 54$ J

A Closer Look at Gravitational Potential Energy

Harry Potter is in the middle of trying to retrieve an egg from a dragon for his first task in the triwizard tournament. Unfortunately, while flying on his broom he falls and finds himself hanging off the ledge of a building, 45 m above the ground. He eventually recovers and is able to retrieve the egg, but loses points for poor technique because he fell off of his broom during the task. The judges tell him that he may earn back the points he lost if he can calculate what his gravitational potential energy was while he was hanging from the building. How can Harry do this?

Calculating gravitational potential energy:

$$PE = m \times g \times h$$

Where...
PE is the gravitational potential energy, measured in joules (J).
m is the mass in kg.
g is the gravitational constant (9.8 m/s^2).
h is the height of the object above the ground in meters (m).

So, Harry must use...

$m = 60$ kg (from the previous question)
$g = 9.8$ m/s^2
$h = 45$ m

To calculate...

$PE = 60 \times 9.8 \times 45$
$PE = 26{,}460$ J

Exercise:

You race to get on the Leviathan first thing in the morning at Canada's Wonderland. The total mass of everyone on the ride when you go on is 3360kg. When you get to the top and stop, the car is 93m above the ground. How much potential energy does the car have?

Retrieved from www.themeparktourist.com

We use,

m = 3360 kg

h= 93 m

g= 9.8 m/s^2

To get,

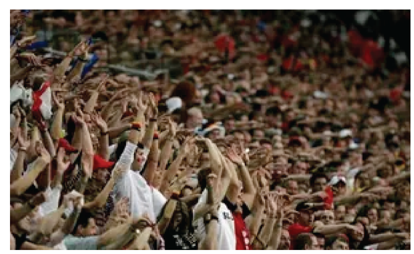
$PE = 3360 \times 9.8 \times 93$

PE = 3,062,304 J

The Wave

Retrieved from www.sciencefocus.com

Retrieved from www.disneyclips.com



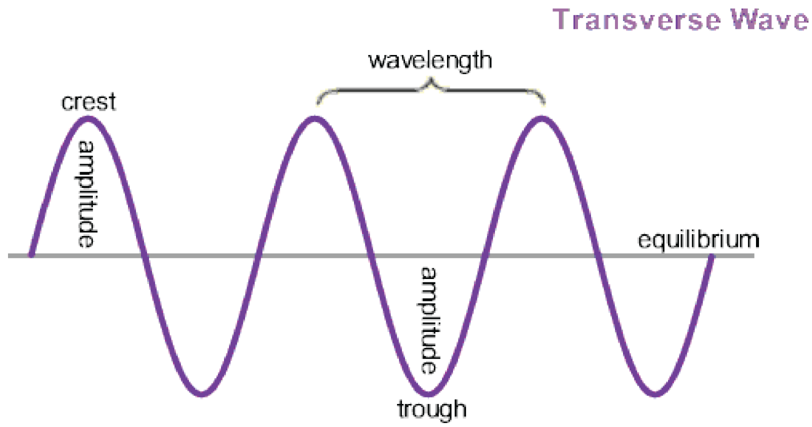
Retrieved from www.theguardian.com

What is a wave?

Waves are a method of ___energy transfer___. A wave is something that travels through space and transfers energy from one place to another.

For example, ___light___ is a wave. Light travels through the air as a wave and tranfers light energy to objects. Think of how plants get energy from the sun during photosynthesis. This is due to the waves of light transferring energy from the sun to the plants, so that they are able to create sugar.

What does a wave look like?



Crest: All the ____highest points____ (or maxima) of the wave.
Trough: All the ____lowest points____ (or minima) of the wave.
Wavelength (λ): The ____horizontal distance____ between two consecutive ____crests____ or two consecutive ____troughs____.
Equilibrium: The ____midpoint____ between the crests and the troughs.
Amplitude: The ____vertical height____ from the equilibrium line to a ____crest or trough____.

Frequency and Wavespeed

Frequency (f): The number of times one full wavelength passes a fixed point in ____one second____ (meausured in cycles/s, otherwise known as Hertz (Hz)). Frequency

$$f = \frac{\text{\# of cycles}}{\text{Time in seconds}}$$

Wavespeed (v): How ____fast____ the wave is travelling in m/s (the same way you would measure the speed of a car).

The Wave Equation

$$v = f \times \lambda$$

Where we measure v in m/s, f in Hz and λ in m.

This equation tells us that if we know the frequency and wavelength of the wave, we can find its wavespeed.

Example:

Your teammate is up to bat and hits a homerun! The sound wave from the bat hitting the ball has a frequency of 2000 Hz and a wavelength of 17 cm. How fast is this wave travelling?

We have $v = f \times \lambda$, we want to find v.
The question tells us that f = 2000 Hz and λ = 0.17 m.
v = 2000 × 0.17
v= 340 m/s
This is close to the speed of sound in air, 343 m/s.

Exercise:

1. The wavelength of a beam of red light is 0.00000068 m. The frequency of this beam of light is 441,000,000,000,000 Hz. What is the speed of this light wave?

v = 0.00000068 × 441,000,000,000,000
v = 299,880,000 m/s

Genetics

What is genetics?

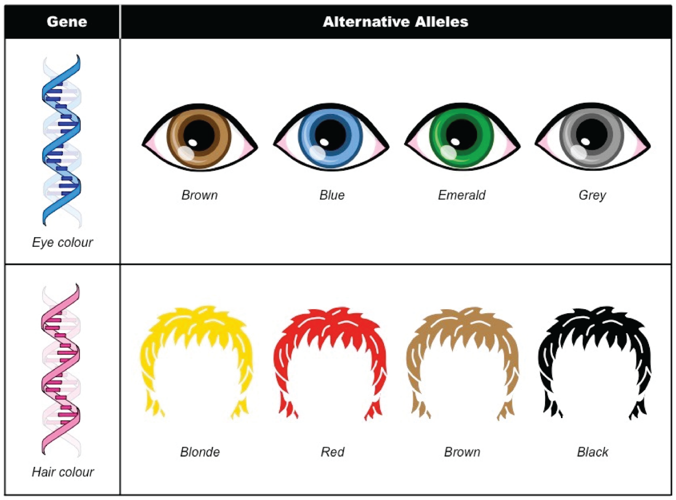
Genetics is the study of ____traits____ that living things ____inherit from their parents____. You have probably noticed that people tend to look similar to their parents. This is because they get, or "inherit" their parents' genes.

Retrieved from https://neurosciencenews.com/lupus-genetics-14023/

What are genes?

Genes are ____segments of your DNA____ that ____code____ for different ____traits____ in your body. For example, you have genes that code for your eye colour, hair colour and whether you are right or left handed.

What about alleles?



As mentioned, we have genes that code for eye colour, but not everyone has the same colour of eyes. This means that there exists different ____types____ of each ____gene____. We refer to the different types of genes as ____alleles____. The genes that code for our eye colour may have a ________blue allele________ or a ________brown allele________, giving different people different colours of eyes.

Retrieved from https://ib.bioninja.com.au/standard-level/topic-3-genetics/31-genes/alleles.html

How can we guess at what eye colour the baby of two parents will have?

For eye colour, everyone inherits ____one____ allele from their ____mom____ and ____one____ from their ____dad____. They can either inherit a ____blue____ allele or a ____brown____ allele from each parent. So, if B = brown and b = blue, the possible combinations of alleles each person can have are...

- ____BB____
- ____Bb____
- ____bb____

So, we can guess that if you have the combination BB, you have brown eyes, whereas if you have the combination bb, you will have blue eyes. What if you have the combination Bb? Brown is known as the dominant allele, meaning that you only need one brown allele to have brown eyes. So, if you have the combination Bb, you will have brown eyes because the dominant allele overpowers the recessive one (blue).

Example:

Ron Weasley and Hermione Granger have a child named Rose. If Ron has blue eyes (bb) and Hermione has brown eyes (Bb) , without looking at Rose's eyes, what is the probability that she will have brown eyes?

Hermione

Ron	B	b
b	Bb	bb
b	Bb	bb

To calculate the probability of having brown eyes, we can first write this as a fraction and then a percentage.

We know that there are four different possible combinations of alleles that Rose can have from our table, so our denominator will be 4.

How many of these combinations will cause Rose to have brown eyes? 2.

So the probability that Rose will have brown eyes, written in fraction form, is $\frac{2}{4}$ or $\frac{1}{2}$.

Written as a percentage, the probability that she will have brown eyes is 50%.

Exercise:

Lupin and Tonks have a son named Teddy. If both Lupin and Tonks have brown eyes with alleles Bb, what can you guess that Teddy's eye colour will be?

Tonks

Lupin	B	b
B	BB	Bb
b	Bb	bb

3 out of 4 of these combinations will cause Teddy's eyes to be brown, so Teddy has a $\frac{3}{4}$, or 75% chance of having brown eyes. We can therefore make a good guess that his eyes will be brown.

Problem Set

* Indicates challenge questions.

1. List the types of energy (kinetic or gravitational potential energy) that the following items possess:

   (a) A car driving along the road.

   Kinetic energy.

   (b) A ball sitting still on the top of a hill.

   Gravitational potential energy.

   (c) A ball soaring through the air.

   Kinetic energy and gravitational potential energy. The ball is moving *and* it is above the ground.

   (d) A phone I hold still in my hand at eye level.

Gravitational potential energy.

(e) A skier skiing down the middle of a hill.

Kinetic energy and gravitational potential energy. The skier is moving *and* is above ground level.

(f) A bowling ball rolling along the alley.

Kinetic energy.

2. (a) A sprinter is running along the track at 4 m/s. If the mass of this sprinter is 70 kg, what is their kinetic energy?

   Answer: 560 J
   $v = 4$ m/s, $m = 70$ kg.
   Following the kinetic energy equation, $KE = \frac{1}{2} \times 70 \times 4^2$.
   $KE = 560$ J

   (b) If another sprinter has a kinetic energy of 398 J and a mass of 65 kg, how fast are they running?

   Answer: 3.5 m/s

   We have to rearrange the KE equation for v to find the speed.

   $KE = \frac{1}{2} \times m \times v^2$. Move everything on the right side of the equation to the left side, except v.

   $2 \times KE = 2 \times \frac{1}{2} \times m \times v^2$. This cancels out the $\frac{1}{2}$ on the right side ($2 \times \frac{1}{2} = 1$).

   Now we have $2 \times KE = m \times v^2$.

   $\frac{2 \times KE}{m} = \frac{m \times v^2}{m}$. This cancels out the m on the right side ($\frac{m}{m} = 1$).

   Now we have $\frac{2 \times KE}{m} = v^2$.

   Now we have to make v^2 on the right side just v. To do this, we do the opposite operation on the left side of the equation.

   We get $\sqrt{\frac{2 \times KE}{m}} = v$.

   Plugging in our KE and m from the question into our calculator, we get
   $v =$ 3.5 m/s.

3. (a) Mickey is standing at the top of a ladder that is 7.5 m from the ground. If Mickey has a mass of 50 kg, what is Mickey's gravitational potential energy at the top of the ladder?

    Answer: 3675 J
    $h = 7.5$ m, $m = 50$ kg, $g = 9.8$ m/s^2.
    Following the gravitational potential energy equation, $PE = 50 \times 9.8 \times 7.5$.
    $PE = 3675$ J

   (b) Mickey throws Minnie a birthday party in which he orders 50 tons of cheesecake. As a result of this, Mickey eats a lot of leftover cheesecake everyday for multiple weeks. He gains 10 kg.

      i. How much does Mickey weigh now?

         Answer: 60 kg
         Mickey weighed 50 kg in part a), so if he gained 10 kg he now weighs 60 kg.

      ii. Mickey is at the top of the same ladder again. What is his new gravitational potential energy?

         Answer: 4410 J
         We plug our numbers into the same equation as pat a), only instead of $m = 50$ kg, we use $m = 60$ kg.

(c) Mickey signs up for a gym membership once all the cheesecake is gone. As a result, he has lost weight and is in great shape! When Mickey is at the top of his ladder again, his gravitational potential energy is 3307 J. How much weight did Mickey lose since going to the gym?

Answer: 15 kg

We need to find Mickey's new weight first. To do this, we rearrange our PE equation for m.

$\frac{PE}{g} = \frac{1}{g} \times m \times g \times h$. This cancels out g on the right side.

Now we have $\frac{PE}{g} = m \times h$.

$\frac{PE}{g \times h} = \frac{1}{h} \times m \times h$. This cancels out the h on the right side.

Now we have $\frac{PE}{g \times h} = m$.

Now we plug in our values for PE, h and g from the question into our calculator and we get $m = 45$ kg.

Since the question does not ask for Mickey's new weight, but rather how much weight he lost, we subtract 45 from his weight before going to the gym (60 kg) to get 15 kg.

4. Which of the following has more gravitational potential energy: A 0.5 kg ball sitting on top of a shelf that is 1 m above the ground, or a 0.5 kg ball sitting on top of a shelf that is 2 m above the ground?

Answer: The ball that is 2 m above the ground.
The greater the height, the more gravitational potential energy an object has. From looking at our equation for PE, we can see that increasing h will increase the number on the right side of the equation, hence increasing the number on the left side of the equation as well.

5. What happens to the gravitational potential energy when the height is doubled? (Hint: Use the equation $PE = m \times g \times h$).

   (A) It is doubled.
   (B) It is tripled.
   (C) It is halved.
   (D) Changing the height has no effect on the gravitational potential energy.

   Answer: A
   Whatever we do to one side of the equation, we must do to the other. If we multiply the height in our PE equation by 2 (double the height, we are multiplying the entire right side of the equation by 2. This means that we must also multiply the left side of the equation by 2. If we multiply the left side of the equation by 2, we are doubling the PE.

6. *What happens to the kinetic energy when the speed is halved?

   (A) It is halved.
   (B) It is doubled.
   (C) It decreases by a factor of $(\frac{1}{2})^2$.
   (D) It decreases by a factor of $\sqrt{\frac{1}{2}}$.

   Answer: C
   Looking at our KE equation, whatever we do to one side we must do to the other. When we have the speed (divide the speed by 2), we are dividing the whole right side of the equation by 2^2 because the speed is squared (which is the same as multiplying by $(\frac{1}{2})^2$). This means that we must also multiply the left side of the equation by $(\frac{1}{2})^2$. Hence, when the speed is halved, the KE is multiplied by $(\frac{1}{2})^2$ which causes it to decrease.

7. *The law of conservation of energy tells us that energy cannot be created or destroyed. This means that the total amount of energy of a system has to be the same all the time. While energy may transform between the different types of energy, the total amount of energy will not change.

   (a) I hold a ball in my hand, 2 m off the ground. At this moment, the ball has only gravitational potential energy. When I drop the ball, just before it hits the ground, it has only kinetic energy.

i. If the gravitational potential energy when I'm holding the ball is 54 J, what will the kinetic energy of the ball be just before it hits the ground?

Answer: 54 J

This is because of the law of conservation of energy. When I held the ball, it had only PE, so it's total energy was 54 J. Just before the ball hits the ground, it has only KE, so it's total energy has to be the same as it was before and the 54 J of energy has just been converted from PE to KE.

ii. What is the speed of the ball just before it hits the ground? (Hint: Set the equations for KE and PE equal to each other).

Answer: 6.3 m/s

The way we mathematically write the law of conservation of energy is to set all of our energy at one point, equal to all of the energy at another point. In this case, because we go from having PE to KE, we can write $PE = KE$. When we rewrite this equation with the equations for PE and KE, we get...

$m \times g \times h = \frac{1}{2} \times m \times v^2$. We now rearrange this equation for v (notice that m appears on both sides of the equation, so it cancels out and we do not actually need to know the mass of the ball).

We get $v = \sqrt{2 \times g \times h}$. Now we plug in our numbers from the question into this equation to get 6.3 m/s.

8. If the amplitude of a wave is 2 cm, what is the vertical height between a trough and a crest of this wave?

Answer: 4 cm

Because the amplitude is the vertical height between the equilibrium point and any crest in the wave *and* it is the vertical height between the equilibrium point and any trough in the wave, the vertical height between a crest and a trough is *two* amplitudes.

9. (a) If one full wavelength of a wave passes a fixed point 5 times in one second, what is the frequency of this wave?

Answer: 5 Hz

Because the frequency is the number of cycles per second, if the wave goes through 5 cycles in one second, then the frequency is 5 Hz.

(b) If it takes one full wavelength of another wave 5 seconds to pass a fixed point 20 times, what is the frequency of this wave?

Answer: 4 Hz

Because frequency is the number of cycles per second, we can calculate the frequency using $f = \frac{\text{\# of cycles}}{\text{Seconds}}$. Plugging in 20 for cycles and 5 for seconds, we get $f = 4$ Hz.

10. (a) A wave has a frequency of 10 Hz and a wavelength of 0.8 m, what is the wavespeed of this wave?

Answer: 8 m/s

Plug $f = 10$ Hz and $\lambda = 0.8$ m into our equation $v = f \times \lambda$.

(b) Another wave has a frequency of 103 Hz and a wavespeed of 60 m/s, what is the wavelength of this wave?

Answer: 0.58 m

We have to rearrange our wave equation for λ.

$\frac{v}{f} = \frac{1}{f} \times f \times \lambda$. This cancels out f on the right side.

Now we have $\frac{v}{f} = \lambda$. Plug in the numbers for v and f into this equation to get λ.

11. (a) Neville Longbottom has brown eyes with allele combination BB. Luna Lovegood has blue eyes with allele combination bb. What are the chances that their child has blue eyes? (Express as a percentage).

Answer: 0%

Neville

		B	B
Luna	b	Bb	Bb
	b	Bb	Bb

As seen in the table, there is no possible combination where their child could have the combination bb, so there is no way for their child to have blue eyes.

(b) *A child has blue eyes. His sister has brown eyes, with alleles BB. If both children have the same parents, find the allele combination for the eye colour of each parent.

Answer: Both parents have allele combination Bb.

If one child has blue eyes, this means his allele combination for his eye colour is bb. Hence, he had to have gotten one "b" allele from his mom and one "b" allele from his dad, so both parents have at least one "b" allele. Since his sister has allele combination BB, she had to have gotten one "B" allele from her mom and one "B" allele from her dad, so both parents have at least one "B" allele. Therefore, each parent has the allele combination Bb for eye colour.