

# Special Relativity - Conclusion

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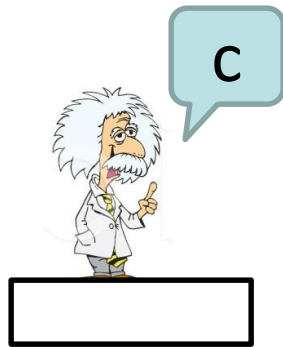
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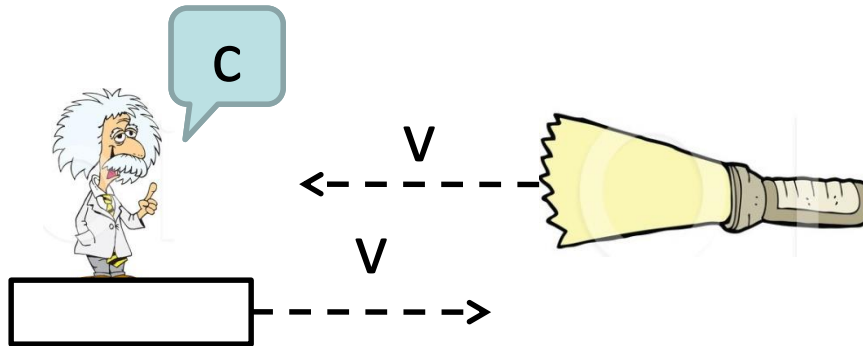
# Postulates of Special Relativity

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- 1) The laws of physics are identical in all (inertial) frames.
- 2) Speed of light is independent of motion of the source.



Einstein sees flashlight at rest, light with speed  $c$ .



Einstein sees flashlight moving toward him but, by (2), speed is still  $c$ .

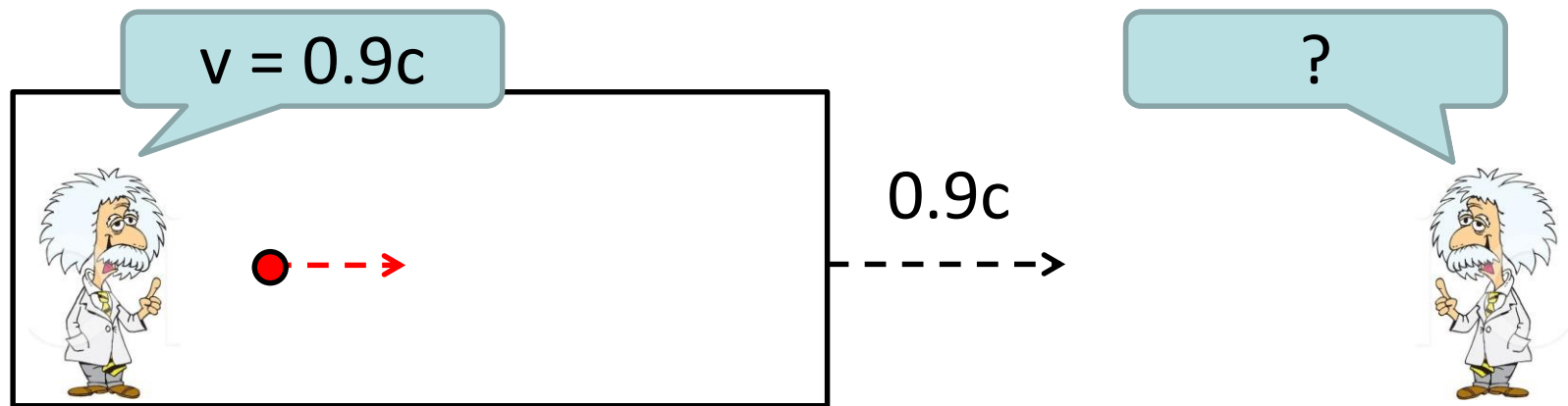
➔ Speed of light is the same in all (inertial) frames!



# “Adding” Velocities

Question:

A ball is thrown at  $0.9c$  to the right on a train moving at  $0.9c$  to the right. For an observer on the ground, how fast is the ball moving?



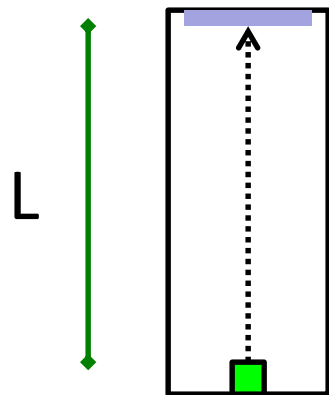
$$V_{CA} = \frac{V_{CB} + V_{BA}}{1 + \frac{V_{CB} V_{BA}}{c^2}}$$

Velocity  
composition law



# Time Dilation

1.

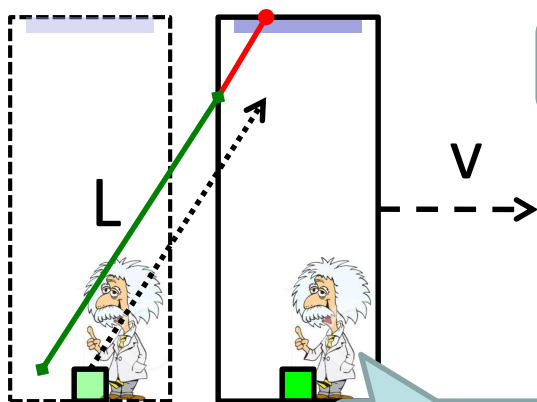


$$t = L/c = 1\mu\text{s}$$



Light takes **one** microsecond to travel floor to ceiling in rest frame.

2.



$$t > 1\mu\text{s}$$



Time must move **slower** in the moving frame!

$$t = 1\mu\text{s}$$

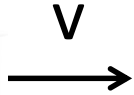


# Length Contraction

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$$t = 1\mu\text{s}$$



A

L

B

$$t < 1\mu\text{s}$$

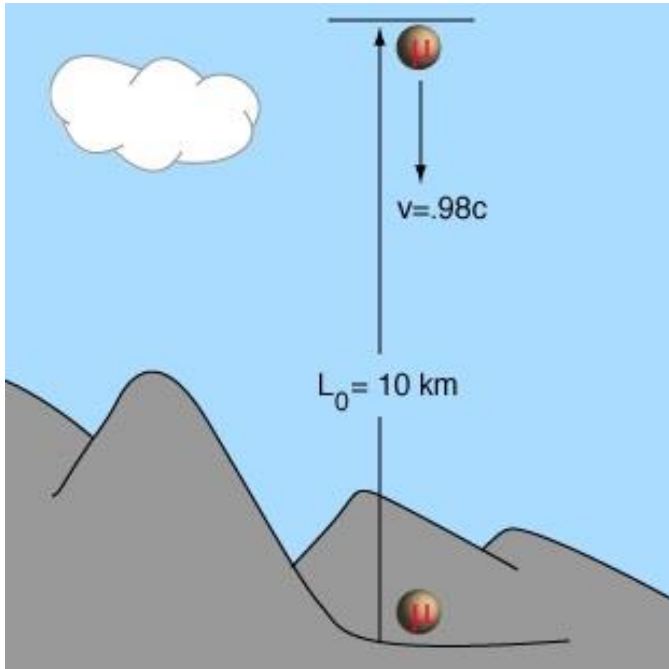
Moving observer measures **less time** for trip along rod of length  $L$ , but sees end of rod (point B) approaching at same speed  $v$ .

Only possible if the rod (which to them is moving) appears contracted.



# Muon Decay

Muons live for about  $2 \mu\text{s}$  on average.



	mass → $\approx 2.3 \text{ MeV}/c^2$ charge → $2/3$ spin → $1/2$ <b>u</b> up	mass → $\approx 1.275 \text{ GeV}/c^2$ charge → $2/3$ spin → $1/2$ <b>c</b> charm	mass → $\approx 173.07 \text{ GeV}/c^2$ charge → $2/3$ spin → $1/2$ <b>t</b> top	mass → $0$ charge → $0$ spin → $1$ <b>g</b> gluon	mass → $\approx 126 \text{ GeV}/c^2$ charge → $0$ spin → $0$ <b>H</b> Higgs boson	
QUARKS	mass → $\approx 4.8 \text{ MeV}/c^2$ charge → $-1/3$ spin → $1/2$ <b>d</b> down	mass → $\approx 95 \text{ MeV}/c^2$ charge → $-1/3$ spin → $1/2$ <b>s</b> strange	mass → $\approx 4.18 \text{ GeV}/c^2$ charge → $-1/3$ spin → $1/2$ <b>b</b> bottom	mass → $0$ charge → $0$ spin → $1$ <b><math>\gamma</math></b> photon		
	mass → $0.511 \text{ MeV}/c^2$ charge → $-1$ spin → $1/2$ <b>e</b> electron	mass → $105.7 \text{ MeV}/c^2$ charge → $-1$ spin → $1/2$ <b><math>\mu</math></b> muon	mass → $1.777 \text{ GeV}/c^2$ charge → $-1$ spin → $1/2$ <b><math>\tau</math></b> tau	mass → $91.2 \text{ GeV}/c^2$ charge → $0$ spin → $1$ <b>Z</b> Z boson	GAUGE BOSONS	
	LEPTONS	mass → $< 2.2 \text{ eV}/c^2$ charge → $0$ spin → $1/2$ <b><math>\nu_e</math></b> electron neutrino	mass → $< 0.17 \text{ MeV}/c^2$ charge → $0$ spin → $1/2$ <b><math>\nu_\mu</math></b> muon neutrino	mass → $< 15.5 \text{ MeV}/c^2$ charge → $0$ spin → $1/2$ <b><math>\nu_\tau</math></b> tau neutrino		mass → $80.4 \text{ GeV}/c^2$ charge → $\pm 1$ spin → $1$ <b>W</b> W boson

Trip from upper atmosphere to ground takes  $30 \mu\text{s}$ .

But their “clocks” run slow from our frame (**time dilation**) so many more make it to the ground than expected (Rossi-Hall experiment).

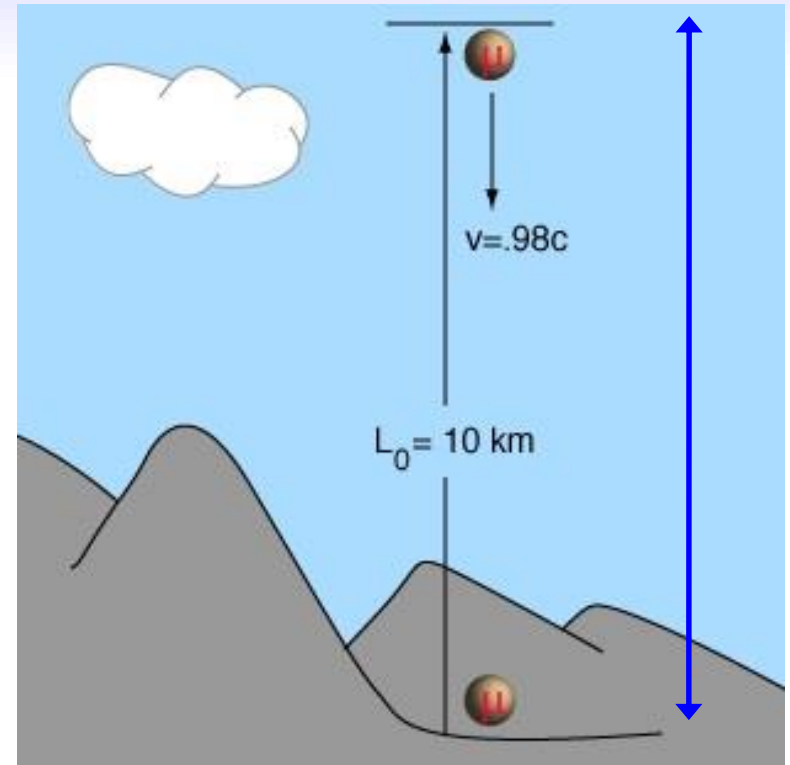


# Muon Decay

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Do the muons notice that their time is slowed down?

No, they see the distance from the atmosphere to the ground **length contracted**.



At 99% light speed, our 10 km looks to them like 1 km which they can cover in  $4 \mu\text{s}$  their time.

Interestingly, they also see *our* clocks running slowly. There is no preferred frame, ***everything is relative!***





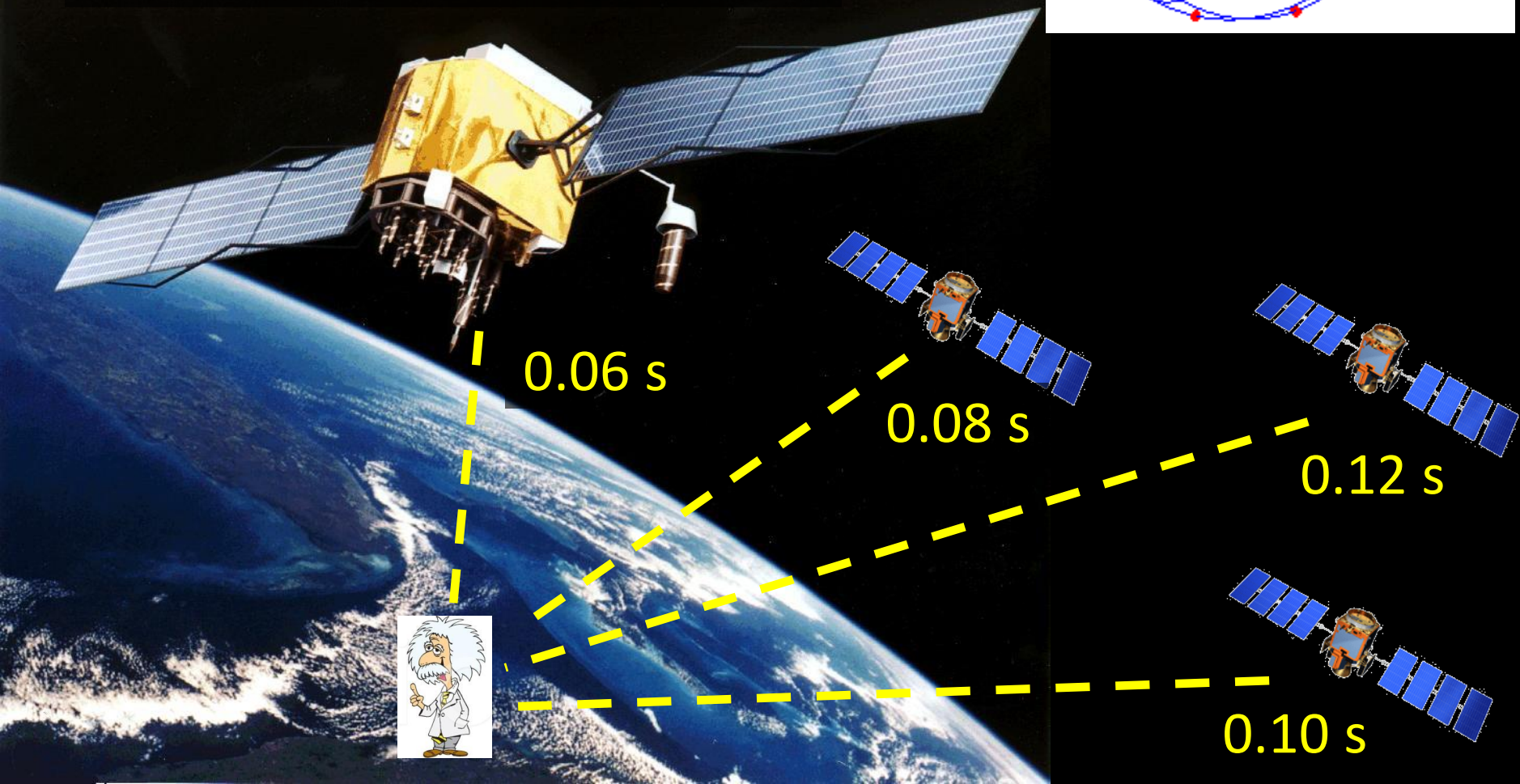
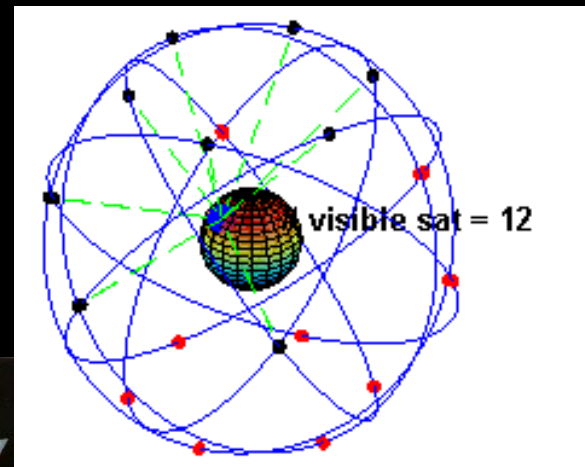
Nearest star is 4 lightyears away. This means it would take light 4 years to reach it. Our fastest spacecraft would take about 3000 years to reach the nearest star.

But if we could travel close to lightspeed, time dilation and length contraction shorten the time and distance for the travellers!



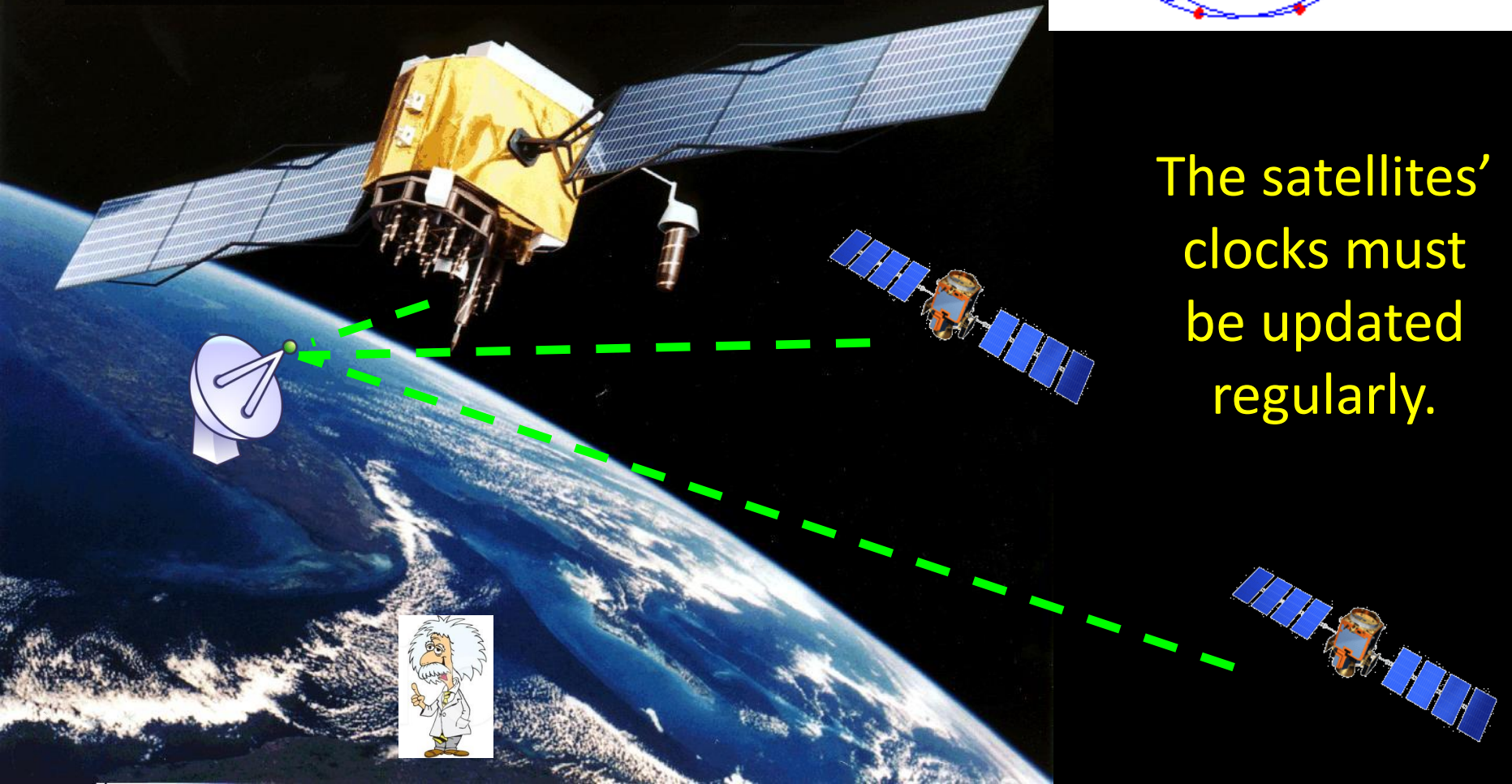
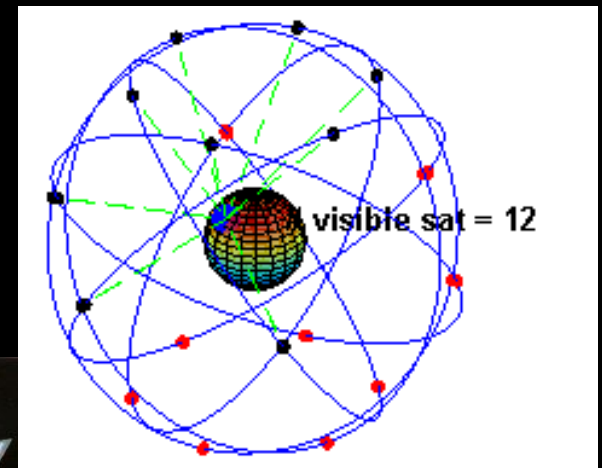
# Global Positioning System (GPS)

Signals from 4 satellites  
needed for accurate location  
(1 for time, 3 for position).



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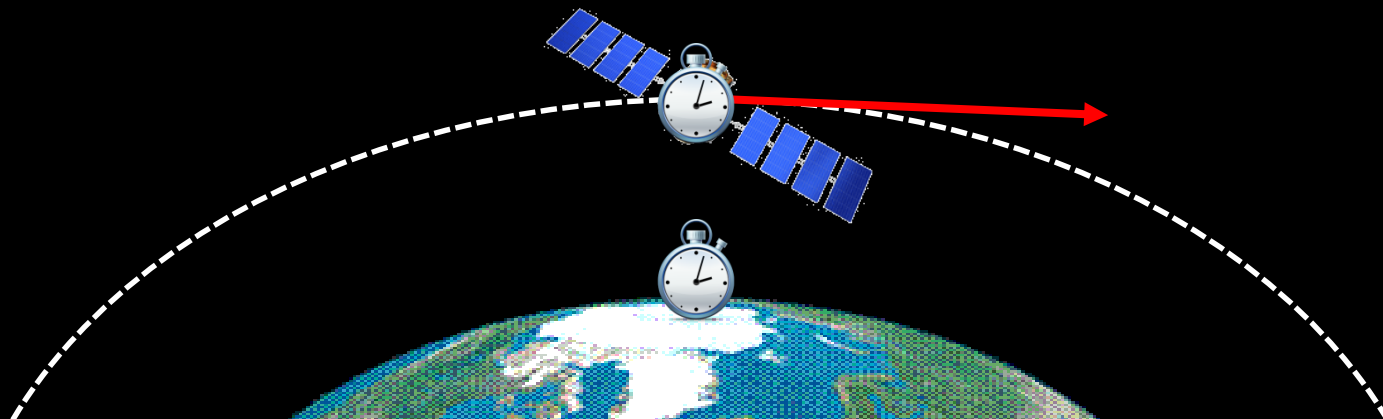
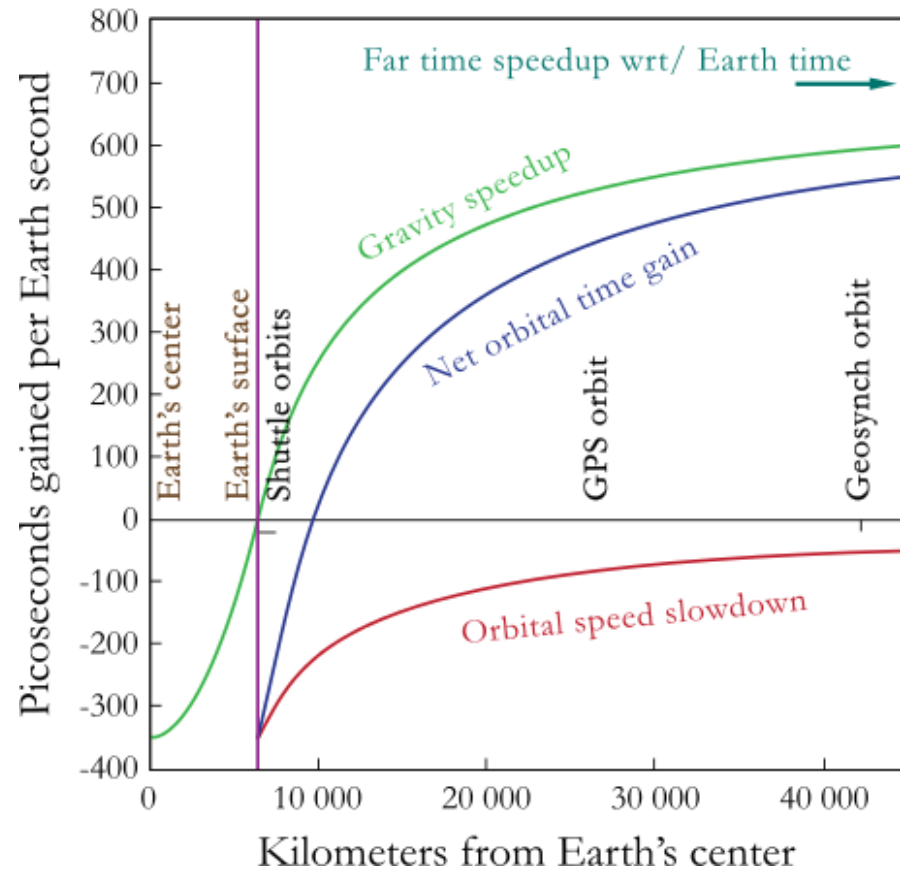
The satellites' clocks must be updated regularly.

# GPS

Satellites experience *relativistic* and *gravitational* time dilation.

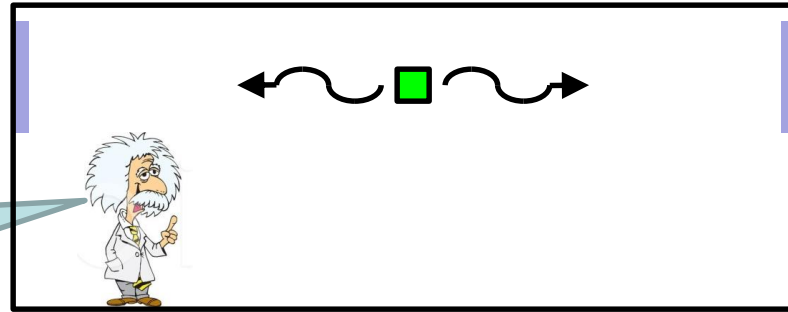
Clocks gain about  $40 \mu\text{s}$  per day which translates to about a 10 km position error.

### Time Dilation Effects on Earth

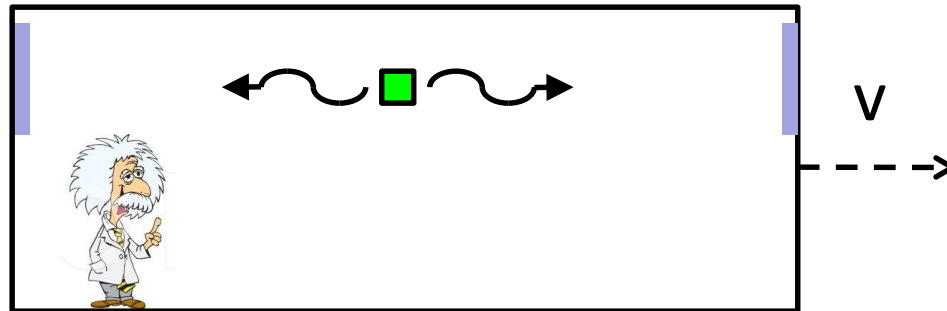


# Relativity of Simultaneity

Same time.



Left first!



The order of events is different for different observers!

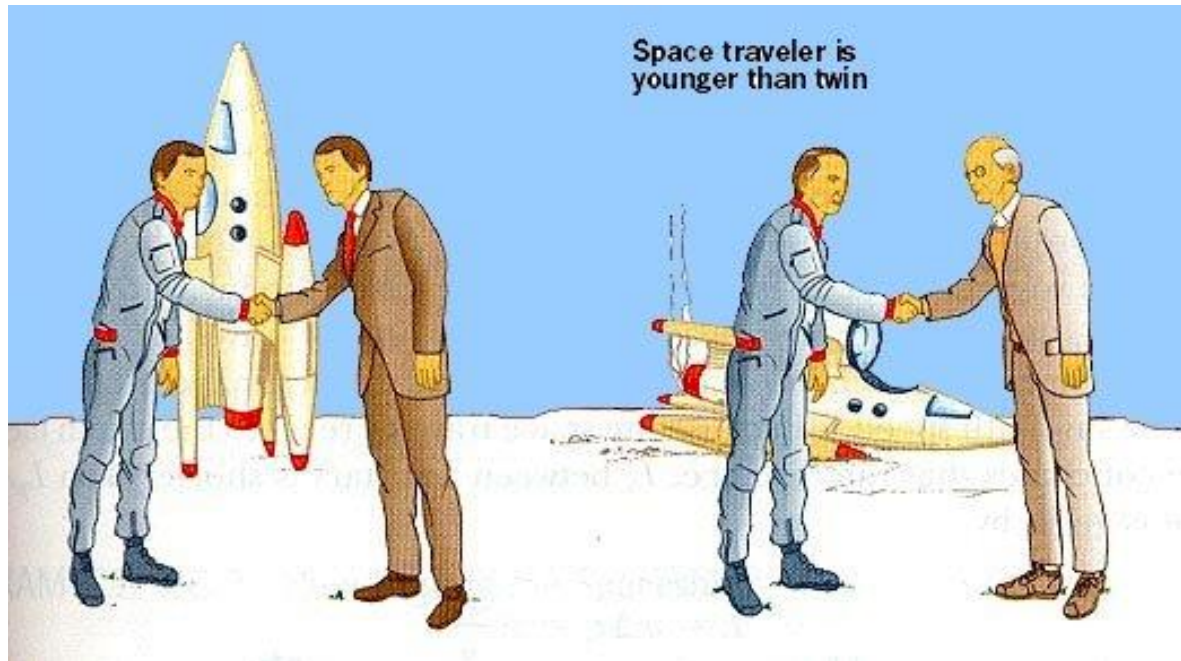




# Twin Paradox

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No absolute space or absolute time –  
everyone must carry around their own rulers and clocks!



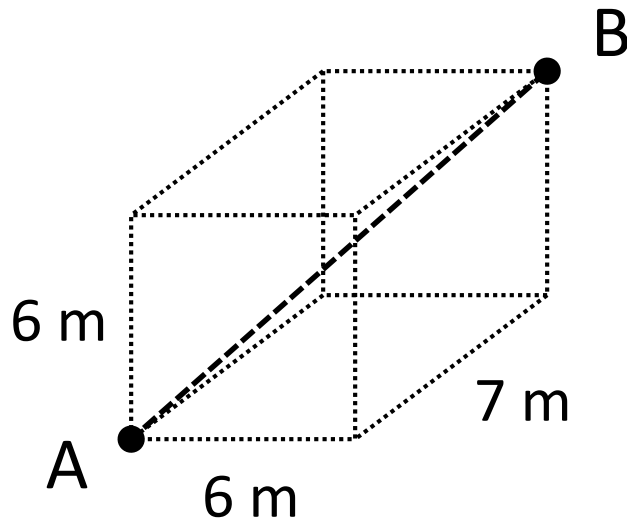
An astronaut will come back to Earth having aged less than us (by about a millisecond).





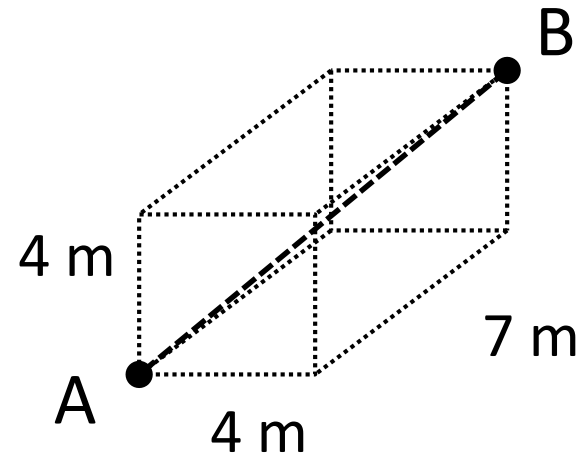
# Minkowski Spacetime

Distance between events is observer dependent.



11 m

$$\sqrt{6^2 + 6^2 + 7^2} = 11$$



9 m

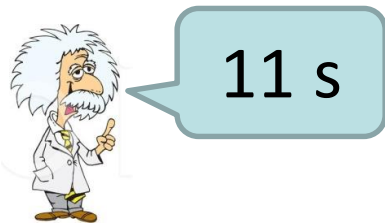
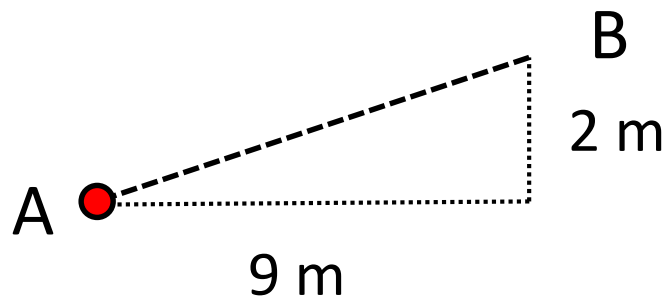
$$\sqrt{4^2 + 4^2 + 7^2} = 9$$



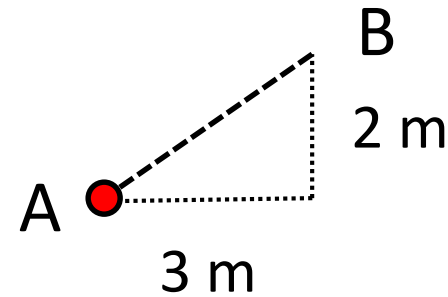
# Minkowski Spacetime

(Change speed of light to 1 m/s.)

Time between events is observer dependent.



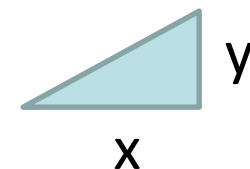
$$-11^2 + 9^2 + 2^2 = -6^2$$



$$-7^2 + 3^2 + 2^2 = -6^2$$

But Minkowski noticed something any two events share.

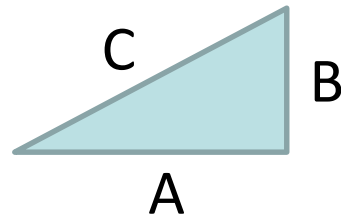
$$-t^2 + x^2 + y^2 = -I^2$$



# Minkowski Spacetime

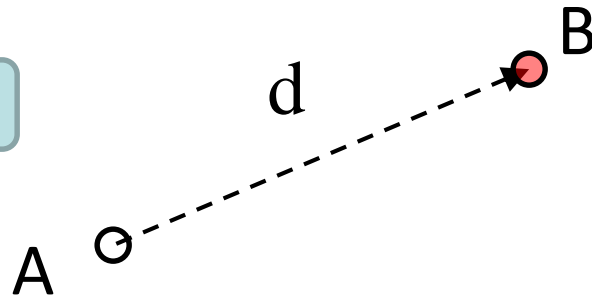
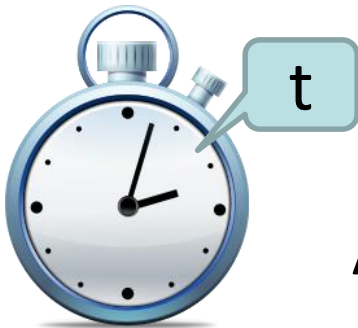
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Space and time (without gravity) obey the rules of a geometry where the Pythagorean equation,



$$A^2 + B^2 = C^2$$

is replaced by

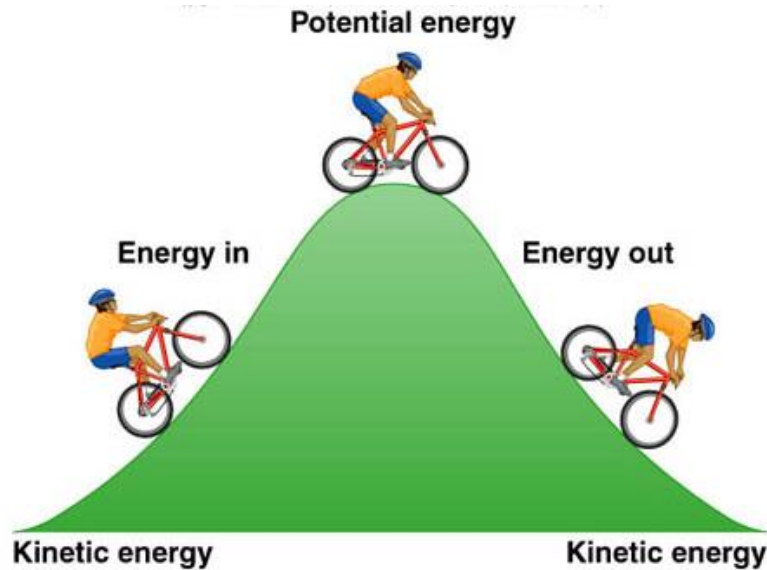


$$-c^2t^2 + d^2 = -I^2$$

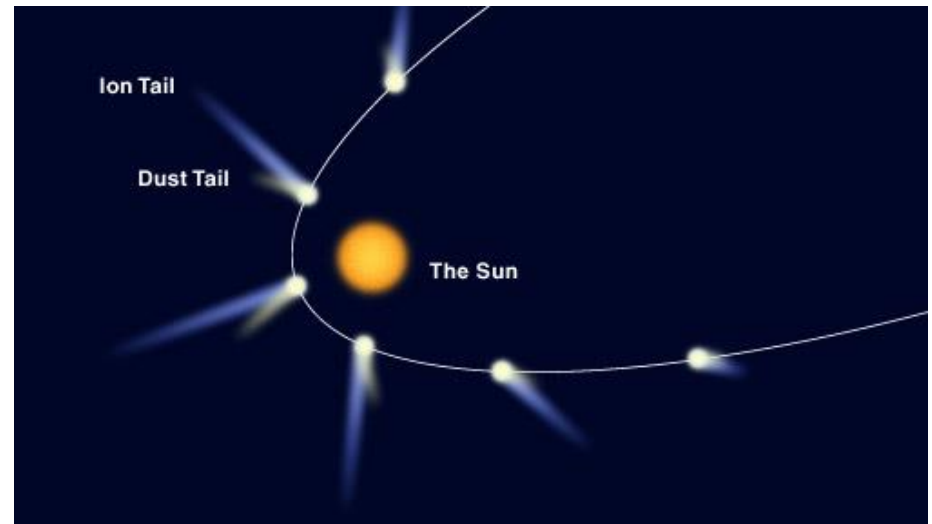
All observers will agree on the *interval*,  $I$ ,  
between any two events.



# Energy-Mass Equivalence



Objects can have kinetic (motion) energy and/or potential (stored) energy. But they also have an inherent energy simply by having mass (matter).



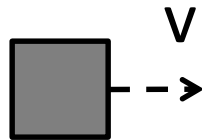
First clue is that light has momentum and energy!



Before:

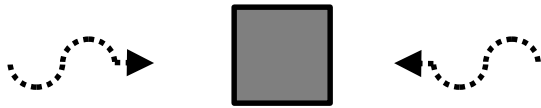


After:



Light has momentum, so mass will start moving.

Before:



After:



Total momentum is zero, so mass doesn't move.

But where does the light energy go?



Block gets heavier!

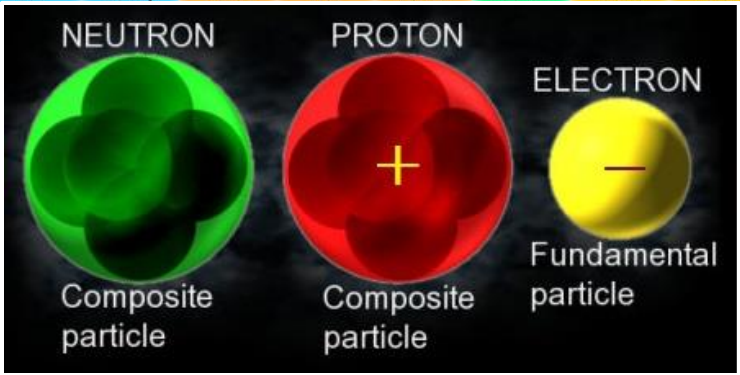
**Energy** can be converted to **mass** and vice versa,  **$E=mc^2$** .





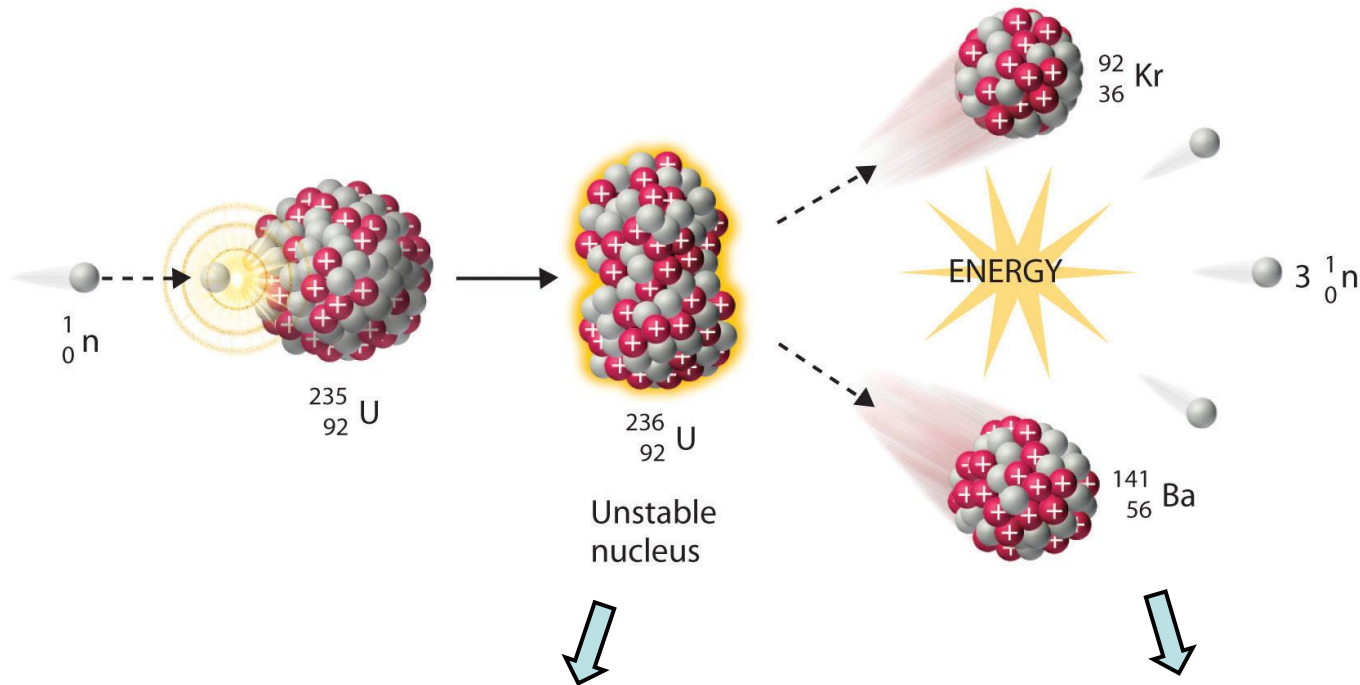
# Periodic Table of the Elements

1 1IA 11A <b>H</b> Hydrogen 1.0079	2 IIA 2A <b>He</b> Helium 4.00260	13 IIIA 3A <b>B</b> Boron 10.811	14 IVA 4A <b>C</b> Carbon 12.011	15 VA 5A <b>N</b> Nitrogen 14.00674	16 VIA 6A <b>O</b> Oxygen 15.9994	17 VIIA 7A <b>F</b> Fluorine 18.998403	18 VIIIA 8A <b>Ne</b> Neon 20.1797																												
3 <b>Li</b> Lithium 6.941	4 <b>Be</b> Beryllium 9.01218	5 <b>B</b> Boron 10.811	6 <b>C</b> Carbon 12.011	7 <b>N</b> Nitrogen 14.00674	8 <b>O</b> Oxygen 15.9994	9 <b>F</b> Fluorine 18.998403	10 <b>Ne</b> Neon 20.1797																												
11 <b>Na</b> Sodium 22.989768	12 <b>Mg</b> Magnesium 24.305	3 IIIB 3B <b>Al</b> Aluminum 26.981539	4 IVB 4B <b>Si</b> Silicon 28.0855	5 VB 5B <b>P</b> Phosphorus 30.973762	6 VIB 6B <b>S</b> Sulfur 32.066	7 VIIB 7B <b>Cl</b> Chlorine 35.4527	8 VIIIB 8 <b>Ar</b> Argon 39.948	9 VIIIB 8 <b>Fe</b> Iron 55.847	10 VIIIB 8 <b>Co</b> Cobalt 58.9332	11 IB 1B <b>Ni</b> Nickel 58.6934	12 IIB 2B <b>Cu</b> Copper 63.546	13 IIB 2B <b>Zn</b> Zinc 65.39	14 IIIB 3B <b>Ga</b> Gallium 69.723	15 IIIB 3B <b>Ge</b> Germanium 72.64	16 IIIB 3B <b>As</b> Arsenic 74.92159	17 IIIB 3B <b>Se</b> Selenium 78.96	18 IIIB 3B <b>Br</b> Bromine 79.904	19 <b>K</b> Potassium 39.0983	20 <b>Ca</b> Calcium 40.078	21 <b>Sc</b> Scandium 44.95591	22 <b>Ti</b> Titanium 47.88	23 <b>V</b> Vanadium 50.9415	24 <b>Cr</b> Chromium 51.9961	25 <b>Mn</b> Manganese 54.938	26 <b>Fe</b> Iron 55.847	27 <b>Co</b> Cobalt 58.9332	28 <b>Ni</b> Nickel 58.6934	29 <b>Cu</b> Copper 63.546	30 <b>Zn</b> Zinc 65.39	31 <b>Ga</b> Gallium 69.723	32 <b>Ge</b> Germanium 72.64	33 <b>As</b> Arsenic 74.92159	34 <b>Se</b> Selenium 78.96	35 <b>Br</b> Bromine 79.904	36 <b>Kr</b> Krypton 83.80
37 <b>Rb</b> Rubidium 85.4678	38 <b>Sr</b> Strontium 87.62	39 <b>Y</b> Yttrium 88.90585	40 <b>Zr</b> Zirconium 91.224	41 <b>Nb</b> Niobium 92.90638	42 <b>Mo</b> Molybdenum 95.94	43 <b>Tc</b> Technetium 98.9072	44 <b>Ru</b> Ruthenium 101.07	45 <b>Rh</b> Rhodium 102.9055	46 <b>Pd</b> Palladium 106.42	47 <b>Ag</b> Silver 107.8682	48 <b>Cd</b> Cadmium 112.411	49 <b>In</b> Indium 114.818	50 <b>Sn</b> Tin 118.71	51 <b>Sb</b> Antimony 121.760	52 <b>Te</b> Tellurium 127.6	53 <b>I</b> Iodine 126.90447	54 <b>Xe</b> Xenon 131.29																		
55 <b>Cs</b> Cesium 132.90543	56 <b>Ba</b> Barium 137.327	57-71 Lanthanide Series	72 <b>Hf</b> Hafnium 178.49	73 <b>Ta</b> Tantalum 180.9479	74 <b>W</b> Tungsten 183.85	75 <b>Re</b> Rhenium 186.207	76 <b>Os</b> Osmium 190.23	77 <b>Ir</b> Iridium 192.22	78 <b>Pt</b> Platinum 195.08	79 <b>Au</b> Gold	80 <b>Hg</b> Mercury	81 <b>Tl</b> Thallium	82 <b>Pb</b> Lead	83 <b>Bi</b> Bismuth	84 <b>Po</b> Polonium	85 <b>At</b> Astatine	86 <b>Rn</b> Radon																		
87 <b>Fr</b> Francium 223.0197	88 <b>Ra</b> Radium 226.0254	89-103 Actinide Series	104 <b>Rf</b> Rutherfordium [261]	105 <b>Db</b> Dubnium [262]	106 <b>Sg</b> Seaborgium [266]	107 <b>Bh</b> Bohrium [264]	108 <b>Hs</b> Hassium [269]	109 <b>Mt</b> Meitnerium [268]	110 <b>Ds</b> Darmstadtium [269]																										
Lanthanide Series			57 <b>La</b> Lanthanum 138.9055	58 <b>Ce</b> Cerium 140.115	59 <b>Pr</b> Praseodymium 140.90765	60 <b>Nd</b> Neodymium 144.24	61 <b>Pm</b> Promethium 144.9127	62 <b>Sm</b> Samarium 150.36	63 <b>Eu</b> Europium 151.9655	64 <b>Gd</b> Gadolinium 157.25																									
Actinide Series			89 <b>Ac</b> Actinium 227.0278	90 <b>Th</b> Thorium 232.0381	91 <b>Pa</b> Protactinium 231.03588	92 <b>U</b> Uranium 238.0289	93 <b>Np</b> Neptunium 237.0482	94 <b>Pu</b> Plutonium 244.0642	95 <b>Am</b> Americium 243.0614	96 <b>Cm</b> Curium 247.0703																									



Elements lighter than iron can fuse and release energy.  
 Elements heavier will radioactively decay and release energy.

# Nuclear Fission



Mass is converted into energy!



# Universal Speed Limit

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As we accelerate an object, we increase its energy...

$$E = mc^2$$



... but then we also increase its mass!

So we have to add *extra* energy to accelerate it further.

As we approach the speed of light, the mass effectively goes to infinity. It becomes impossible to accelerate it any more.





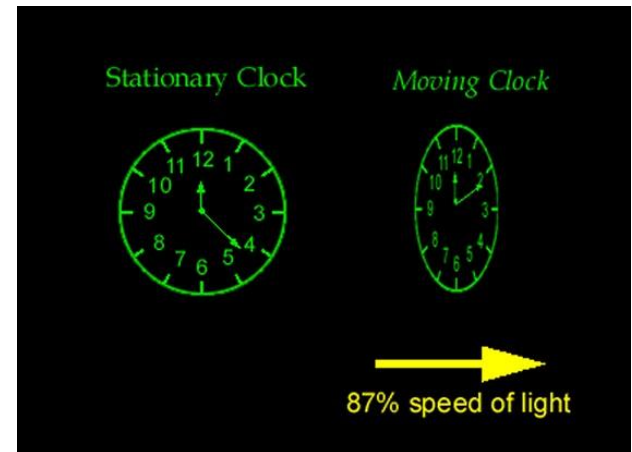
# Consequences of Fixed Light Speed

➡ Moving clocks run slow (time dilation).

➡ Moving objects appear shortened (length contraction).

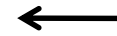
➡ Energy-mass equivalence.

➡ Simultaneity of events is relative.



$$E = mc^2$$

Right one first!



Left one first!

Space and time are intertwined as *spacetime*!

