Special Relativity - Conclusion

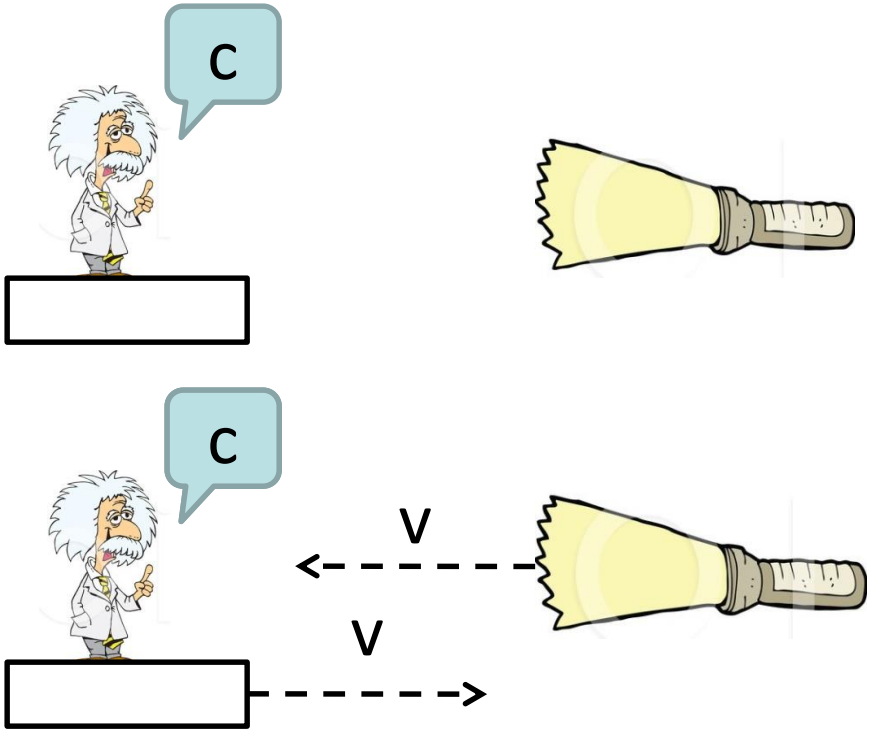
Paul McGrath
pmcgrath@uwaterloo.ca

CEMC
Centre for Education in Mathematics and Computing
Faculty of Mathematics, University of Waterloo

cemc.uwaterloo.ca

Postulates of Special Relativity

1) The laws of physics are identical in all (inertial) frames.
2) Speed of light is independent of motion of the source.



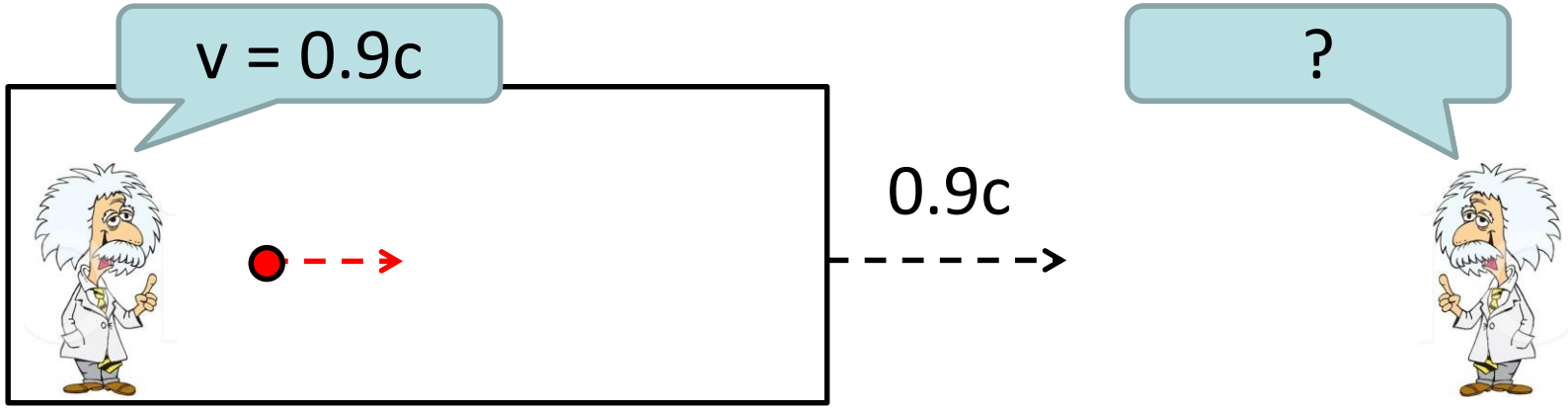
Einstein sees flashlight at rest, light with speed c.

Einstein sees flashlight moving toward him but, by (2), speed is still c.

⇨ Speed of light is the same in all (inertial) frames!

“Adding” Velocities

Question:
A ball is thrown at 0.9c to the right on a train moving at 0.9c to the right. For an observer on the ground, how fast is the ball moving?

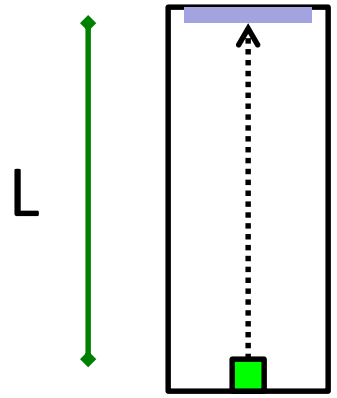


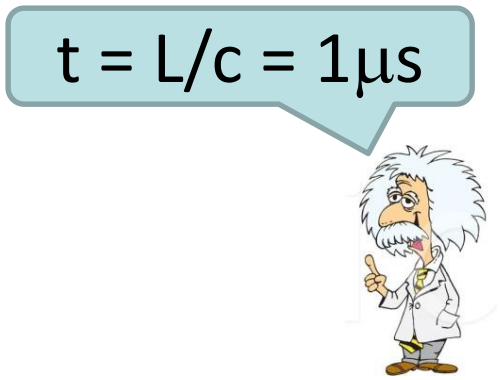
$$v_{CA} = \frac{v_{CB} + v_{BA}}{1 + \frac{v_{CB} v_{BA}}{c^2}}$$

Velocity composition law

Time Dilation

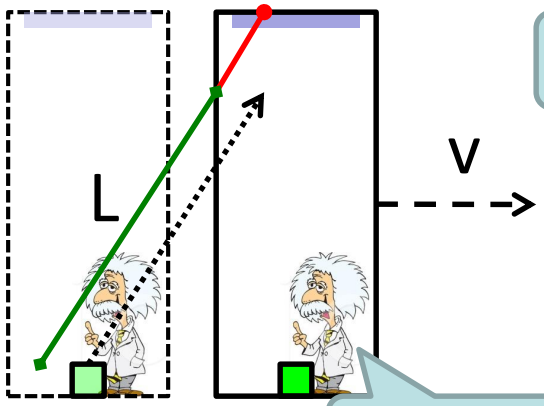
1.





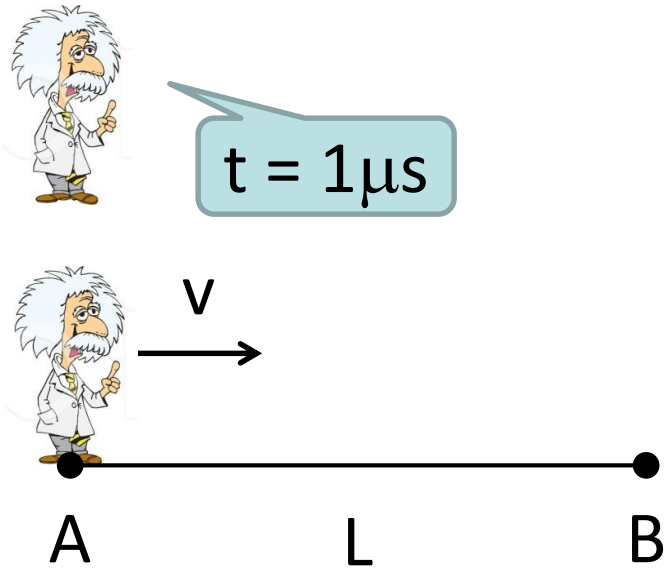
Light takes **one** microsecond to travel floor to ceiling in rest frame.

2.



Time must move **slower** in the moving frame!

Length Contraction

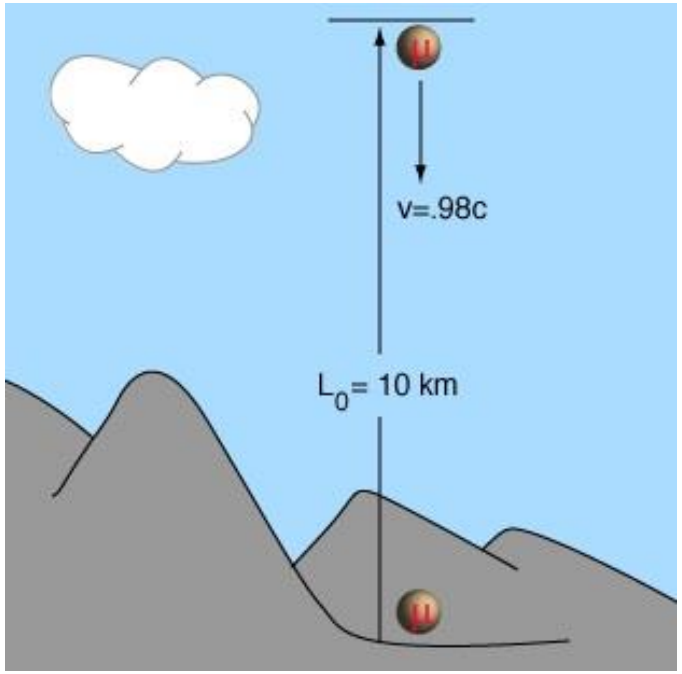


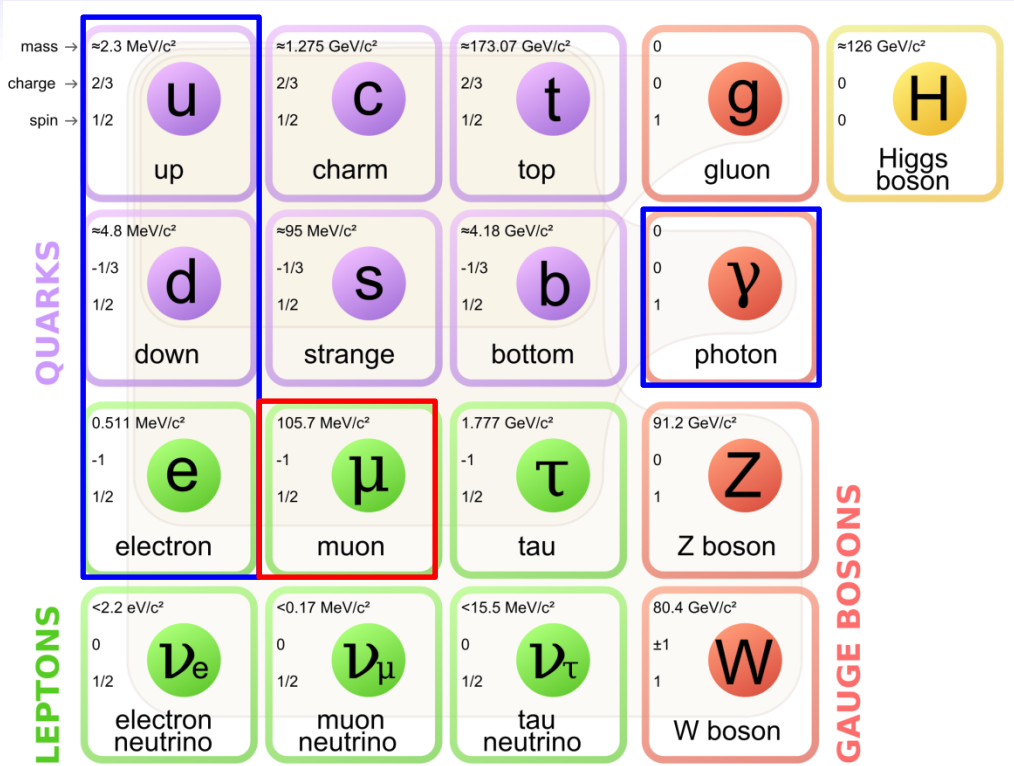
Moving observer measures **less time** for trip along rod of length L, but sees end of rod (point B) approaching at same speed v.

Only possible if the rod (which to them is moving) appears contracted.

Muon Decay

Muons live for about 2 μs on average.

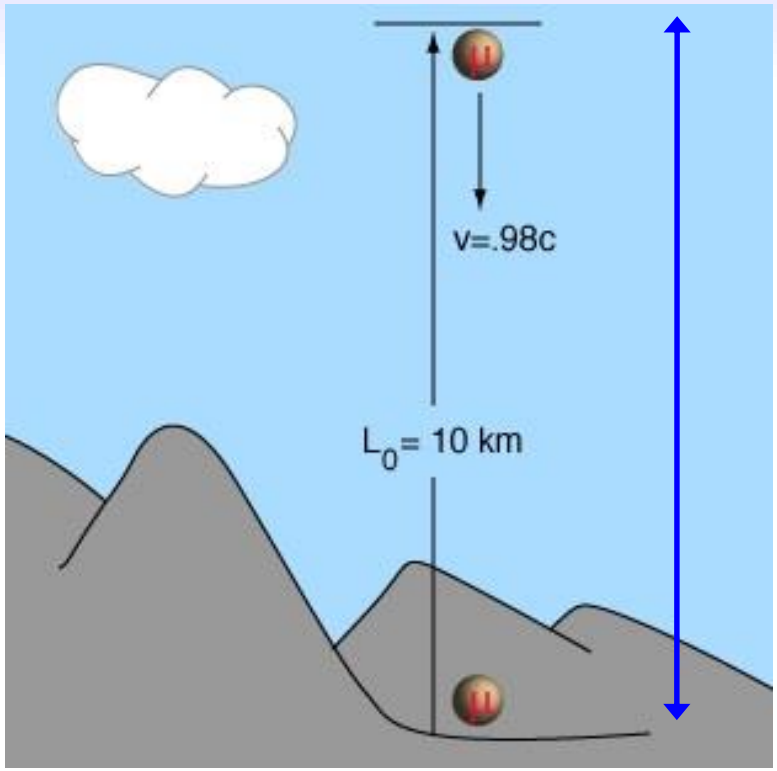




Trip from upper atmosphere to ground takes 30 μs.

But their "clocks" run slow from our frame (**time dilation**) so many more make it to the ground than expected (Rossi-Hall experiment).

Muon Decay

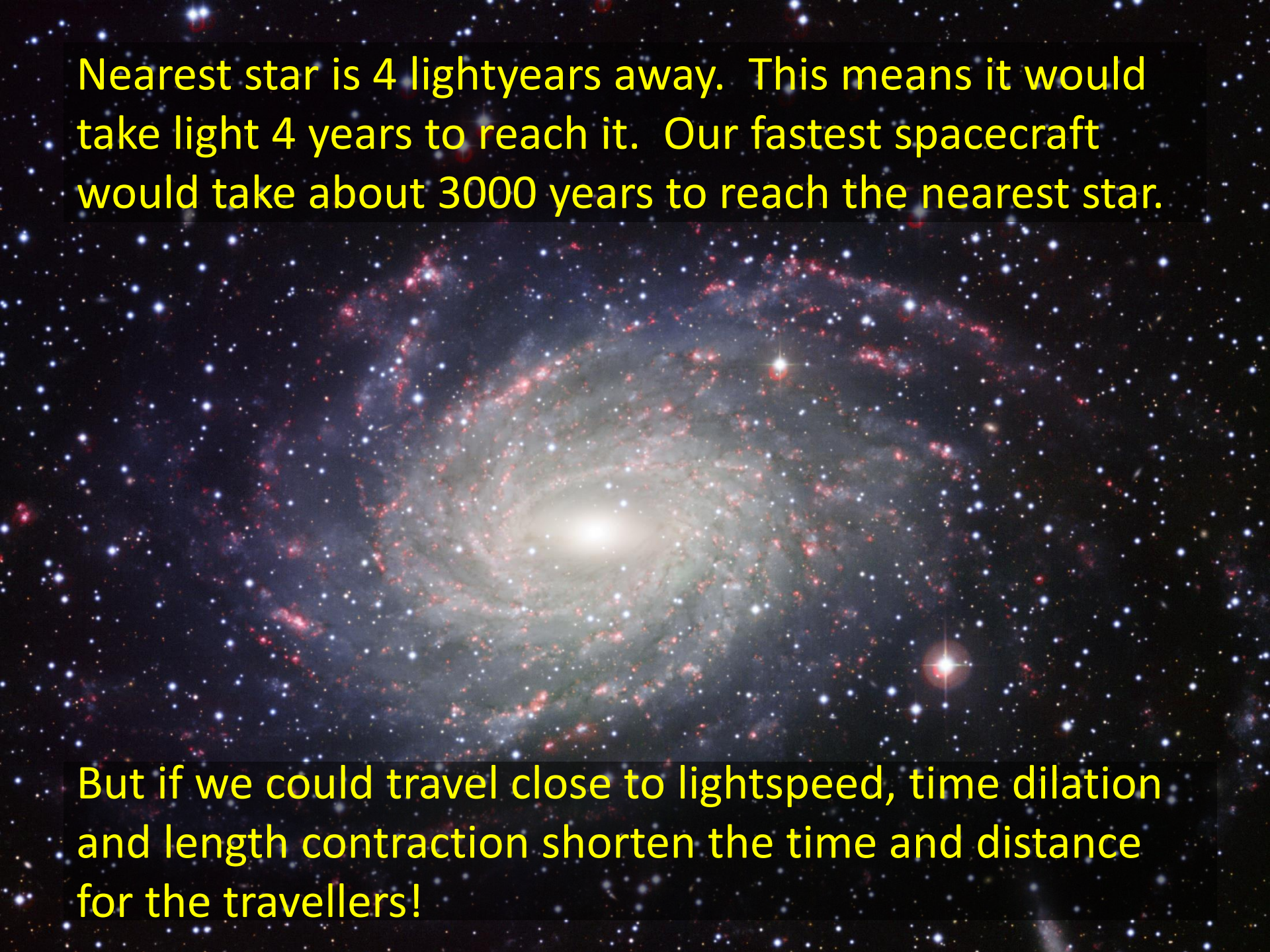


Do the muons notice that their time is slowed down?

No, they see the distance from the atmosphere to the ground **length contracted**.

At 99% light speed, our 10 km looks to them like 1 km which they can cover in 4 μs their time.

Interestingly, they also see *our* clocks running slowly. There is no preferred frame, ***everything is relative***!

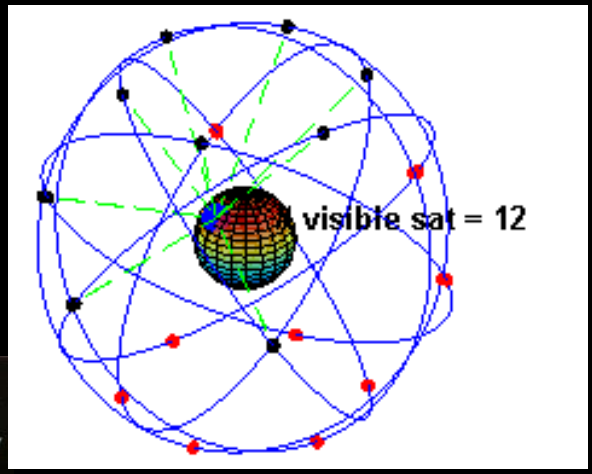


Nearest star is 4 lightyears away. This means it would take light 4 years to reach it. Our fastest spacecraft would take about 3000 years to reach the nearest star.

But if we could travel close to lightspeed, time dilation and length contraction shorten the time and distance for the travellers!

Global Positioning System (GPS)

Signals from 4 satellites needed for accurate location (1 for time, 3 for position).



0.06 s

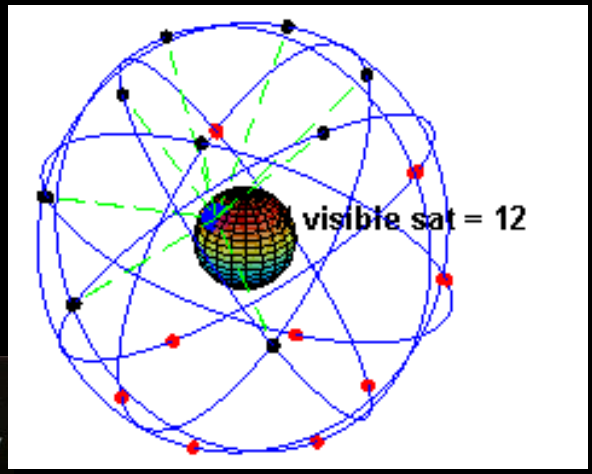
0.08 s

0.12 s

0.10 s

Global Positioning System (GPS)

Signals from 4 satellites needed for accurate location (1 for time, 3 for position).

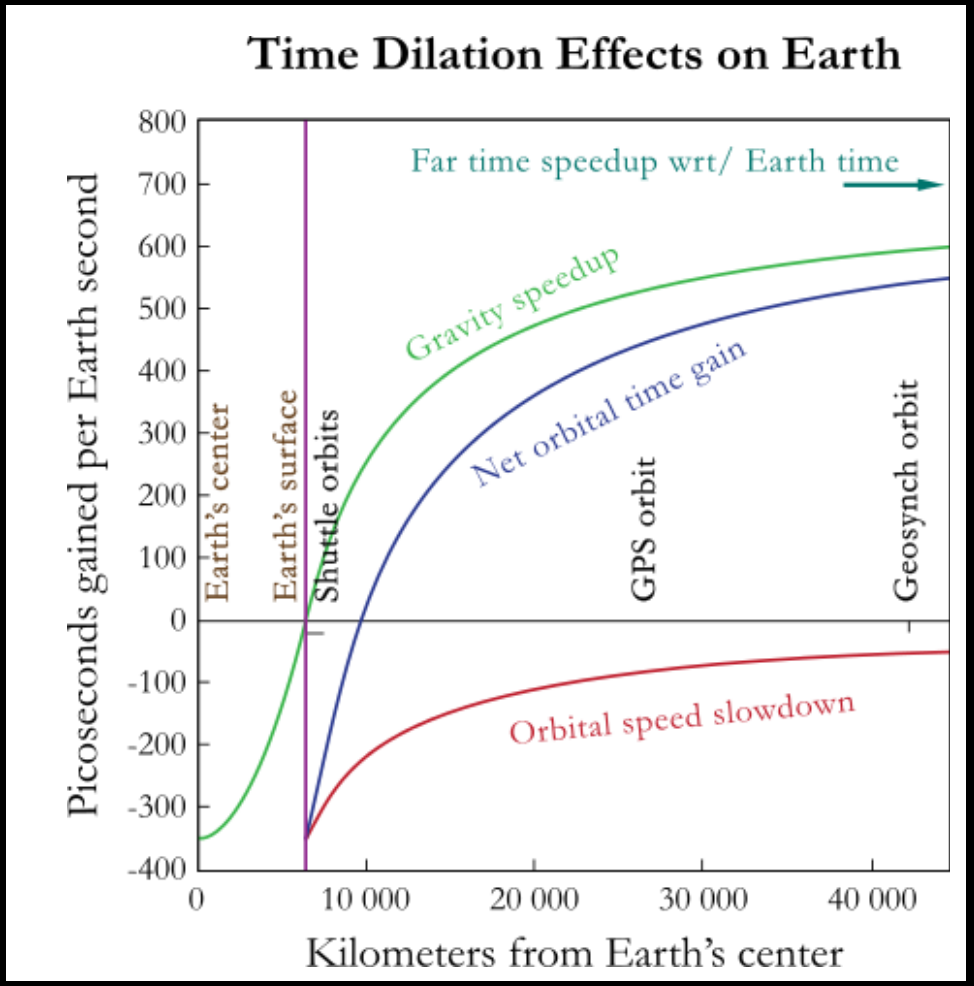


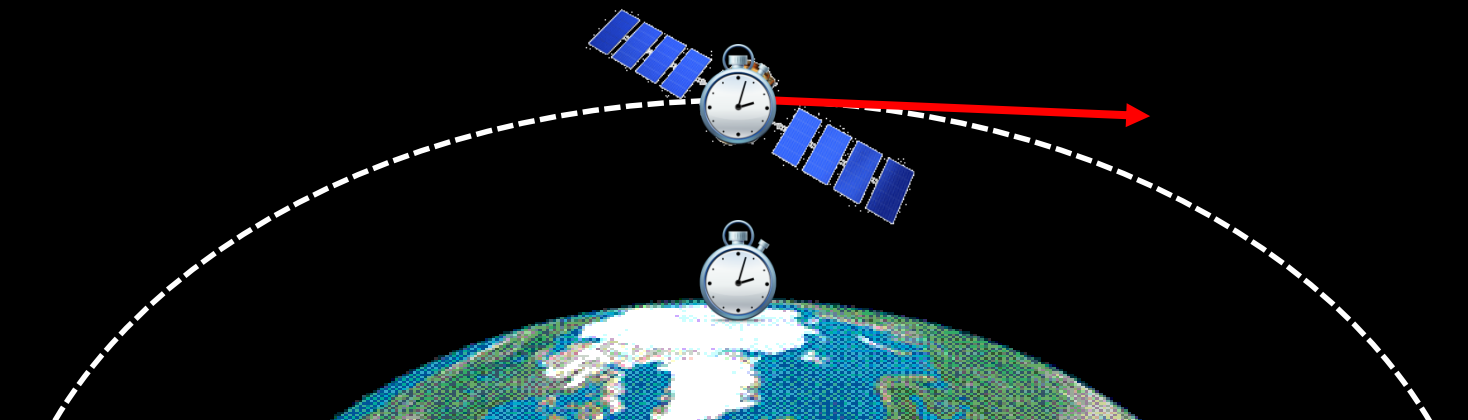
The satellites' clocks must be updated regularly.

GPS

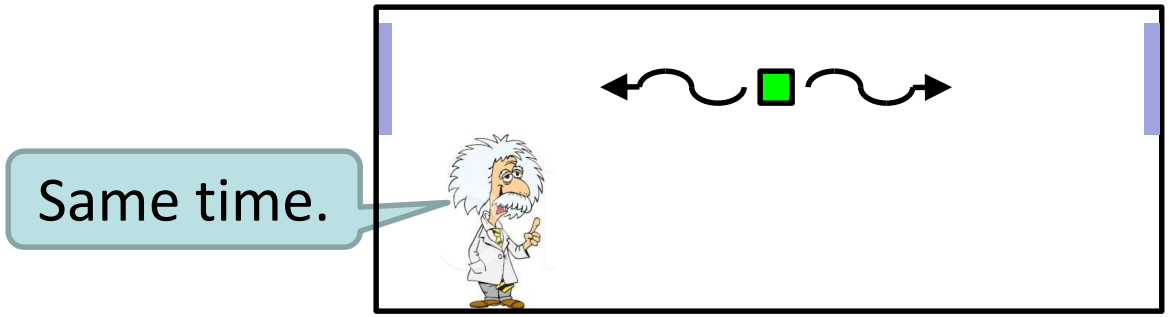
Satellites experience *relativistic* and *gravitational* time dilation.

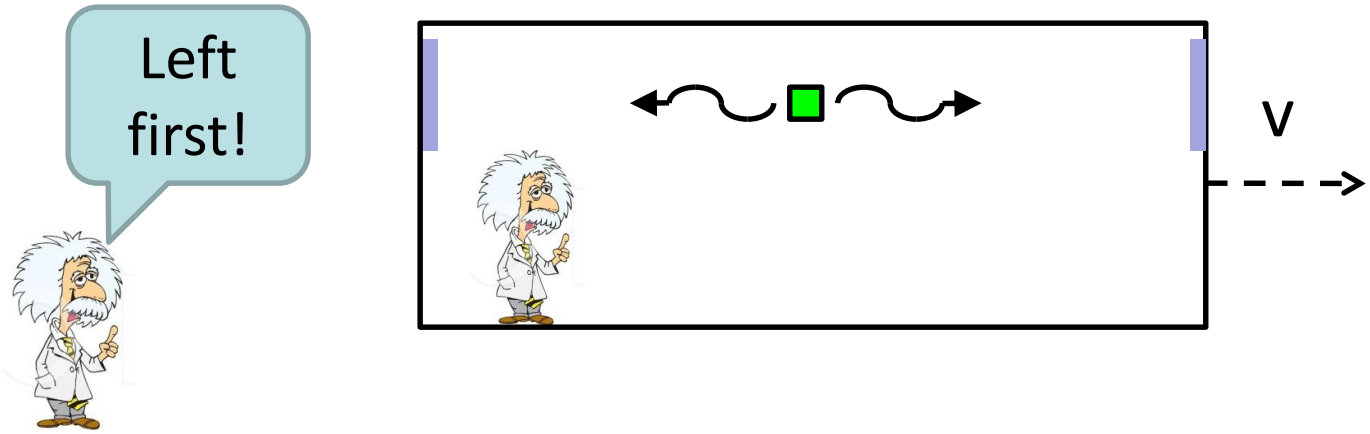
Clocks gain about 40 μs per day which translates to about a 10 km position error.





Relativity of Simultaneity

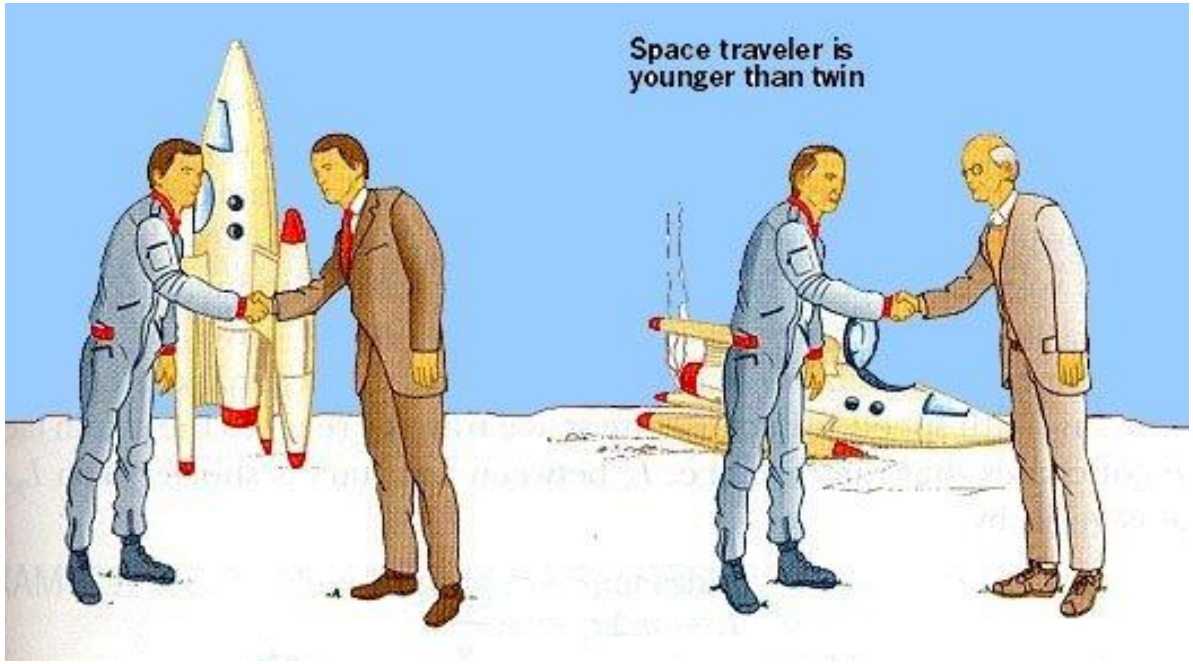




The order of events is different for different observers!

Twin Paradox

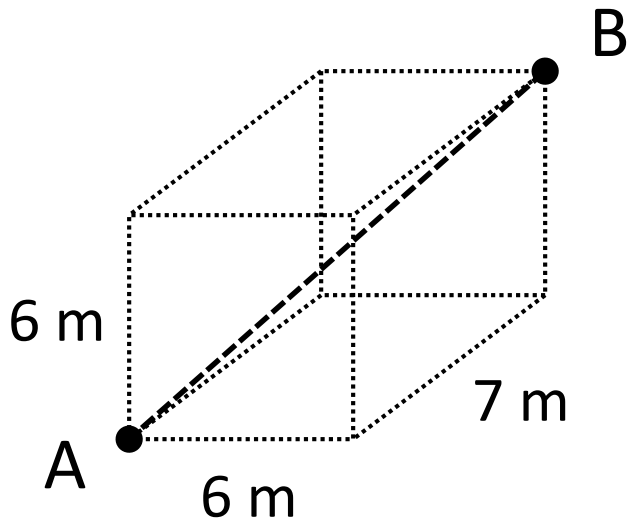
No absolute space or absolute time –
everyone must carry around their own rulers and clocks!

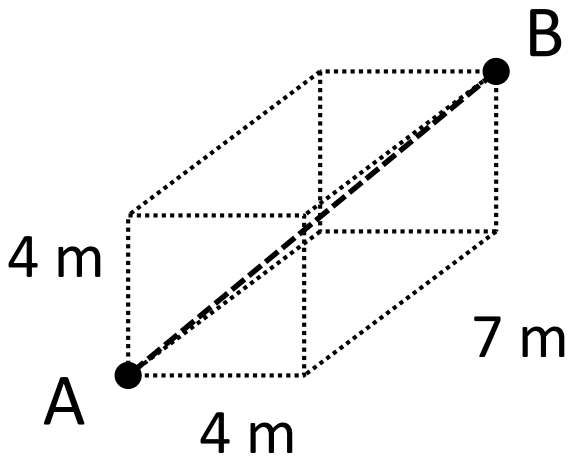


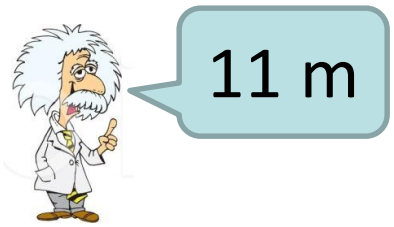
An astronaut will come back to Earth having aged less than us (by about a millisecond).

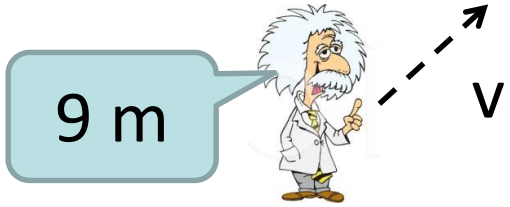
Minkowski Spacetime

Distance between events is observer dependent.









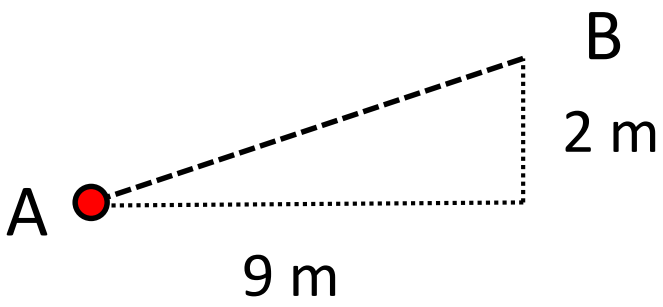
$$\sqrt{6^2+6^2+7^2}=11$$

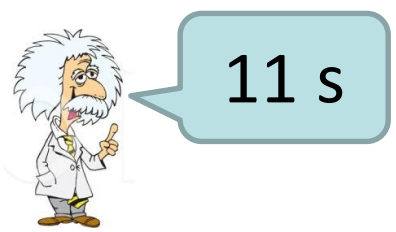
$$\sqrt{4^2+4^2+7^2}=9$$

Minkowski Spacetime

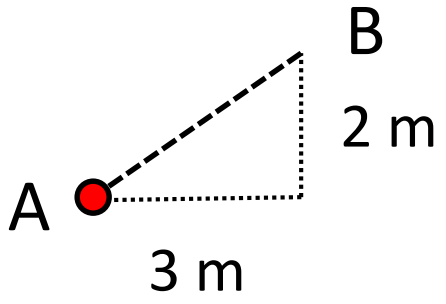
(Change speed of light to 1 m/s.)

Time between events is observer dependent.





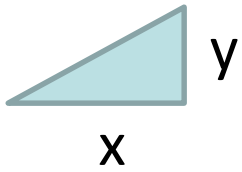
$$-11^2 + 9^2 + 2^2 = -6^2$$



$$-7^2 + 3^2 + 2^2 = -6^2$$

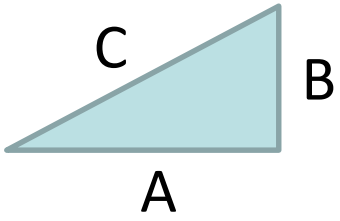
But Minkowski noticed something any two events share.

$$-t^2 + x^2 + y^2 = -I^2$$



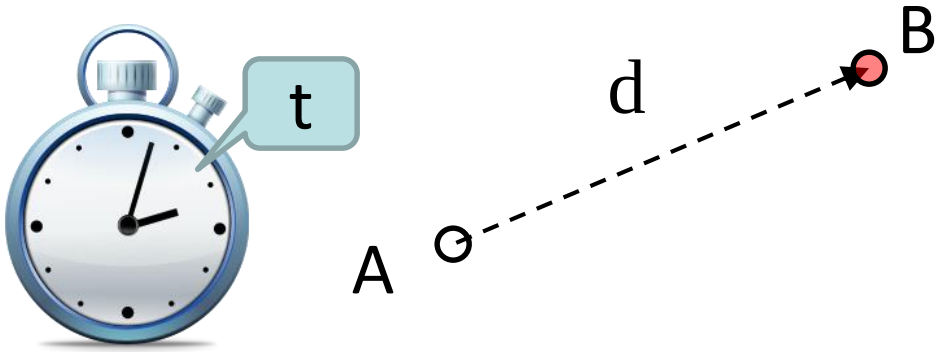
Minkowski Spacetime

Space and time (without gravity) obey the rules of a geometry where the Pythagorean equation,



$$A^2 + B^2 = C^2$$

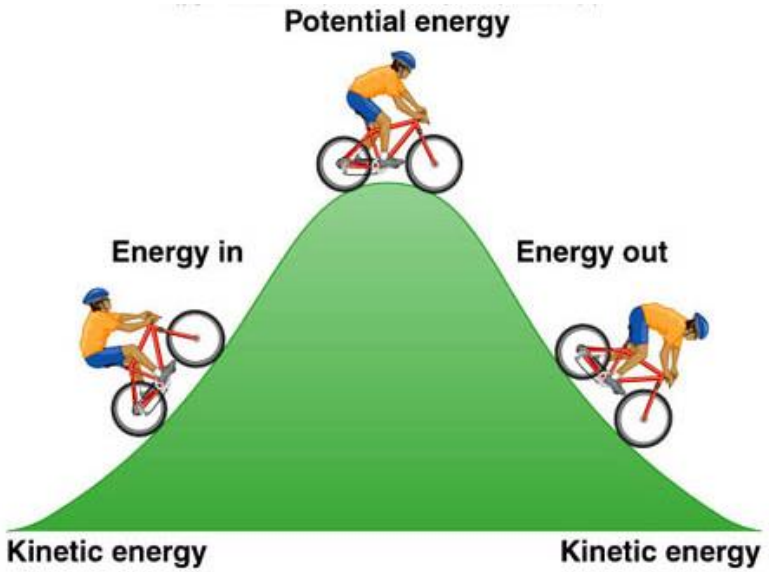
is replaced by



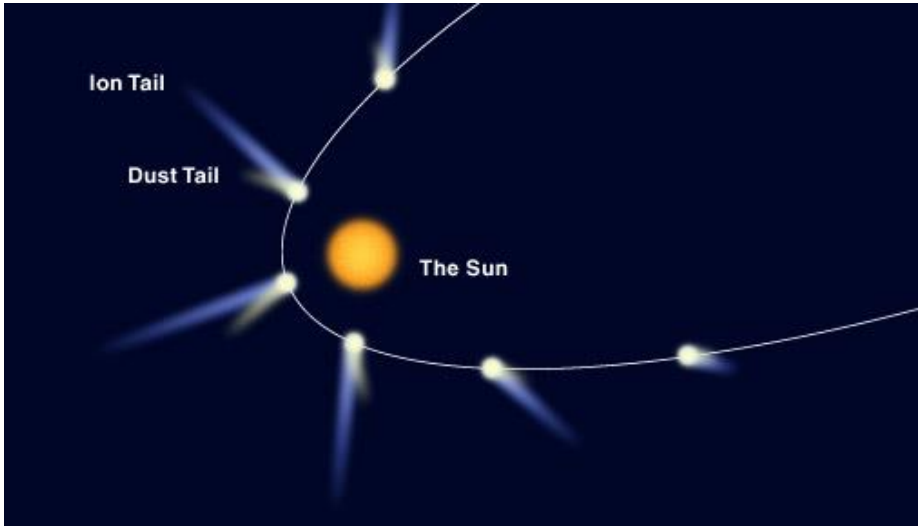
$$-c^2t^2 + d^2 = -I^2$$

All observers will agree on the ***interval***, I, between any two events.

Energy-Mass Equivalence



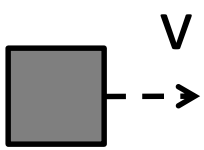
Objects can have kinetic (motion) energy and/or potential (stored) energy. But they also have an inherent energy simply by having mass (matter).



First clue is that light has momentum and energy!

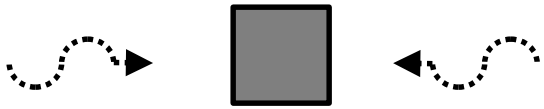
Before:

After:



Light has momentum, so mass will start moving.

Before:



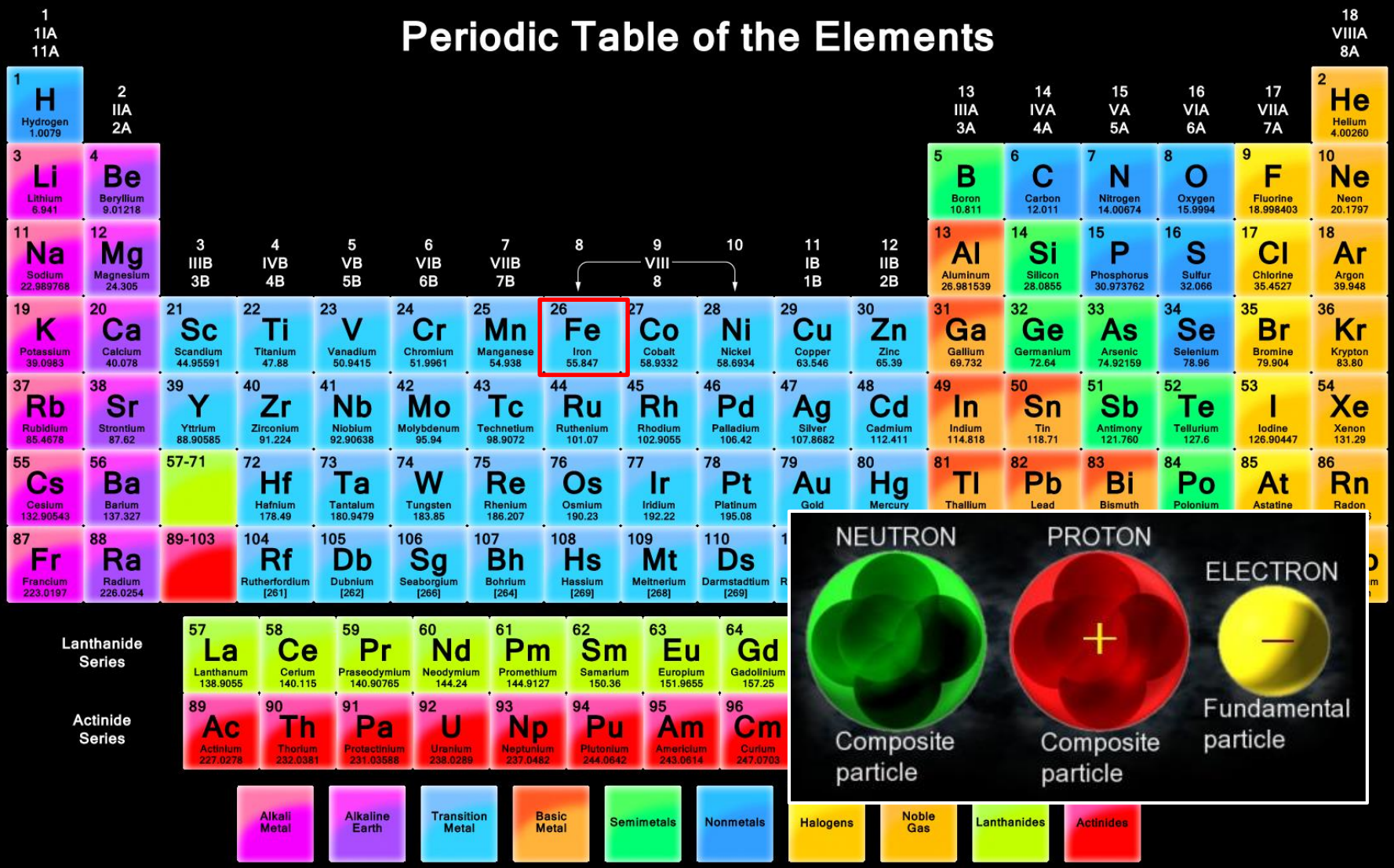
Total momentum is zero, so mass doesn't move.

But where does the light energy go?

After:

Block gets heavier!

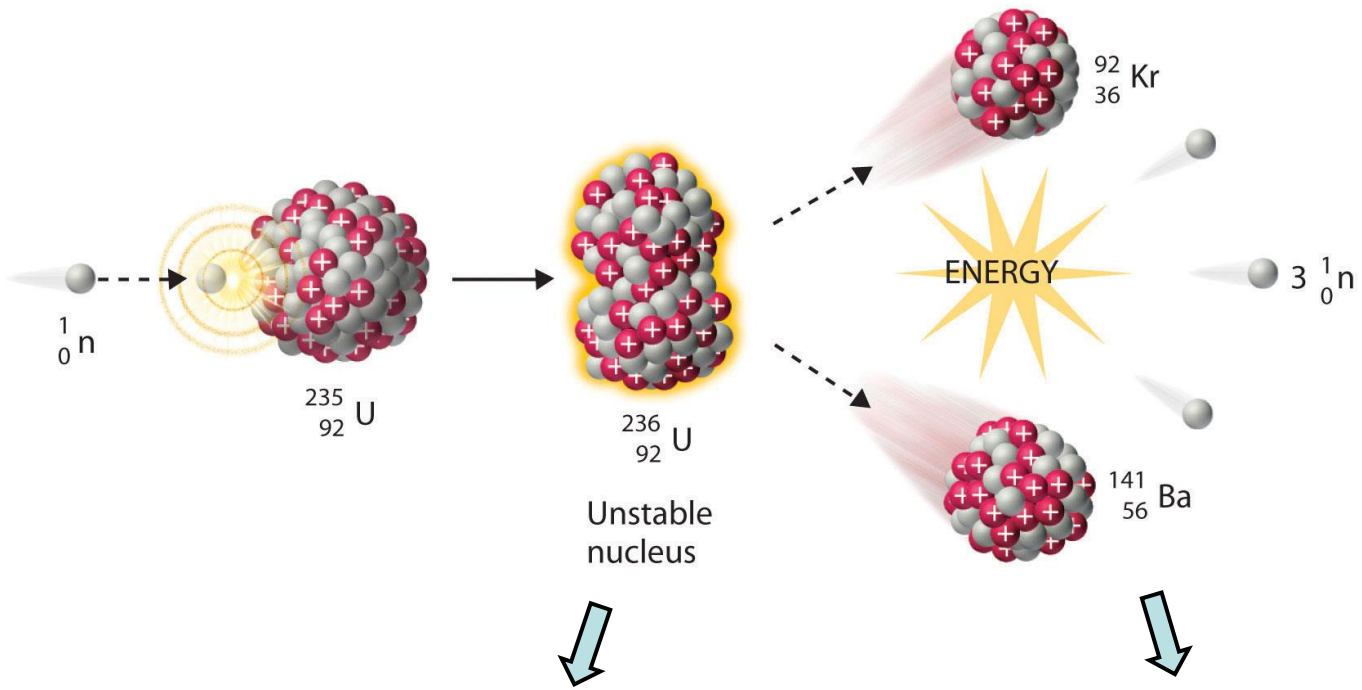
Energy can be converted to **mass** and vice versa, **$E=mc^2$**.



Elements lighter than iron can fuse and release energy.

Elements heavier will radioactively decay and release energy.

Nuclear Fission



Mass is converted into energy!

Universal Speed Limit

As we accelerate an object, we increase its energy...

$$E = mc^2$$

... but then we also increase its mass!

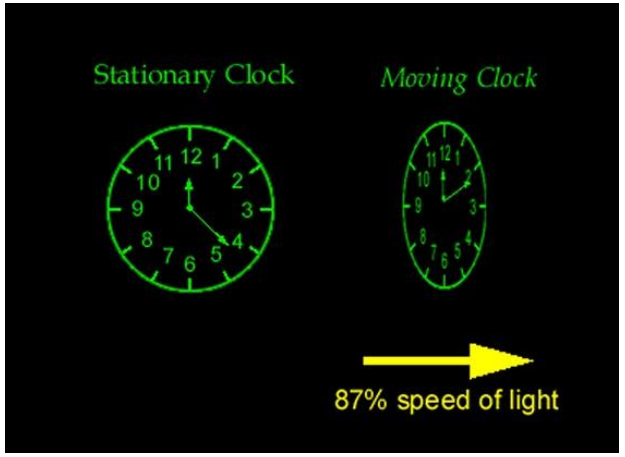
So we have to add *extra* energy to accelerate it further.

As we approach the speed of light, the mass effectively goes to infinity. It becomes impossible to accelerate it any more.

Consequences of Fixed Light Speed

⇨ Moving clocks run slow (time dilation).

⇨ Moving objects appear shortened (length contraction).



⇨ Energy-mass equivalence.

$$E = mc^2$$

⇨ Simultaneity of events is relative.

Space and time are intertwined as ***spacetime***!