

## Grade 7/8 Math Circles

October 14, 2020

### *BCC Prep - Solutions*

## Introduction

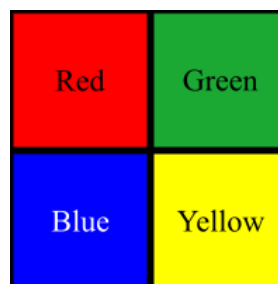
The BCC (Beaver Computing Contest) is a contest that focuses on computational and logical thinking, with no necessary coding experience. It's a way to start thinking like a Computer Scientist without having the technical background. Taking place in early November of each year, the grade 7/8 level of the contest consists of 15 questions, divided into 3 parts: A, B and C, with each getting increasingly difficult. Today, we are taking a look at some approaches to solving these problems and doing some practice. More information about the contest can be found here: <https://cemc.uwaterloo.ca/contests/bcc.html>

## Warm-up

Let's start with some practice questions from previous contests. Try to solve both problems on your own and when you think you have the answer, click on the link to watch a video that briefly goes through the solutions.

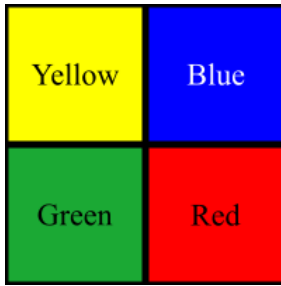
### Warm-up 1. *Rotation Game (BCC 2018)*

Beavers play a simple game. The game always begins with the starting position show on the right. From this starting position, rotation instructions are followed. All the rotations are clockwise and one quarter of a complete turn.



The possible instructions are:

- 1R – meaning rotate the squares one time,
- 2R – meaning rotate the squares two times,
- 3R – meaning rotate the squares three times.

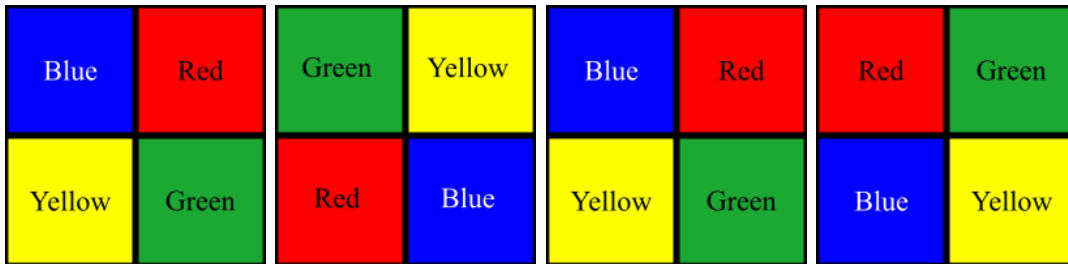


For example, if the first instruction is 2R, the top-left square will be Yellow as shown on the left. From the starting position, what colours will the top-left square be after each of the instructions 1R, 2R, 2R, and 3R are followed in order?

*Solution:*

Red Blue Green Blue Red

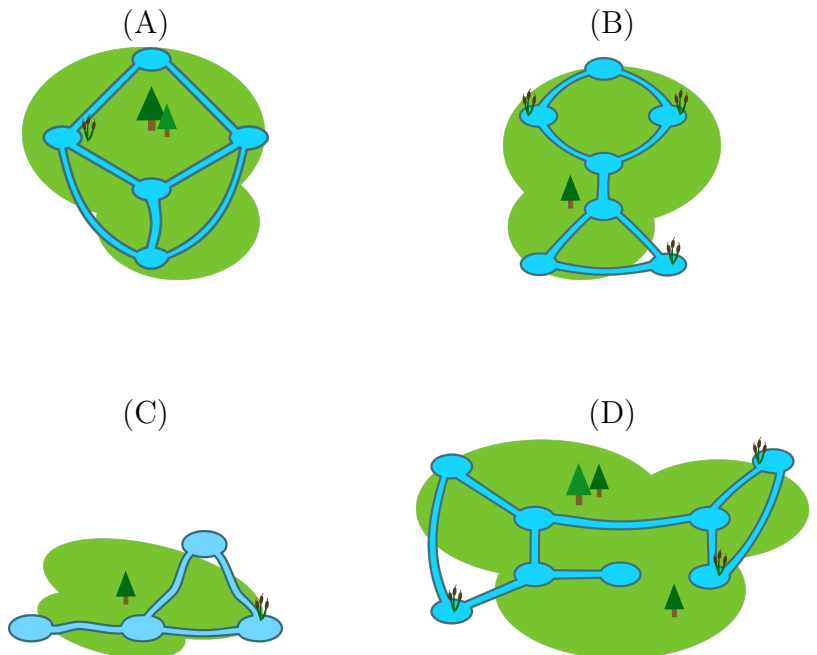
The top-left square is Red for the starting position and the results after each rotation are shown below.



**Warm-up 2. Pond Planning (BCC 2016)**

Beaver neighbourhoods consist of rivers flowing between ponds. Patricia is grumpy and wants to build one dam in each neighbourhood that will cause trouble. That is, she wants to block a single river so that beavers will not be able to travel between all pairs of ponds in the neighbourhood.

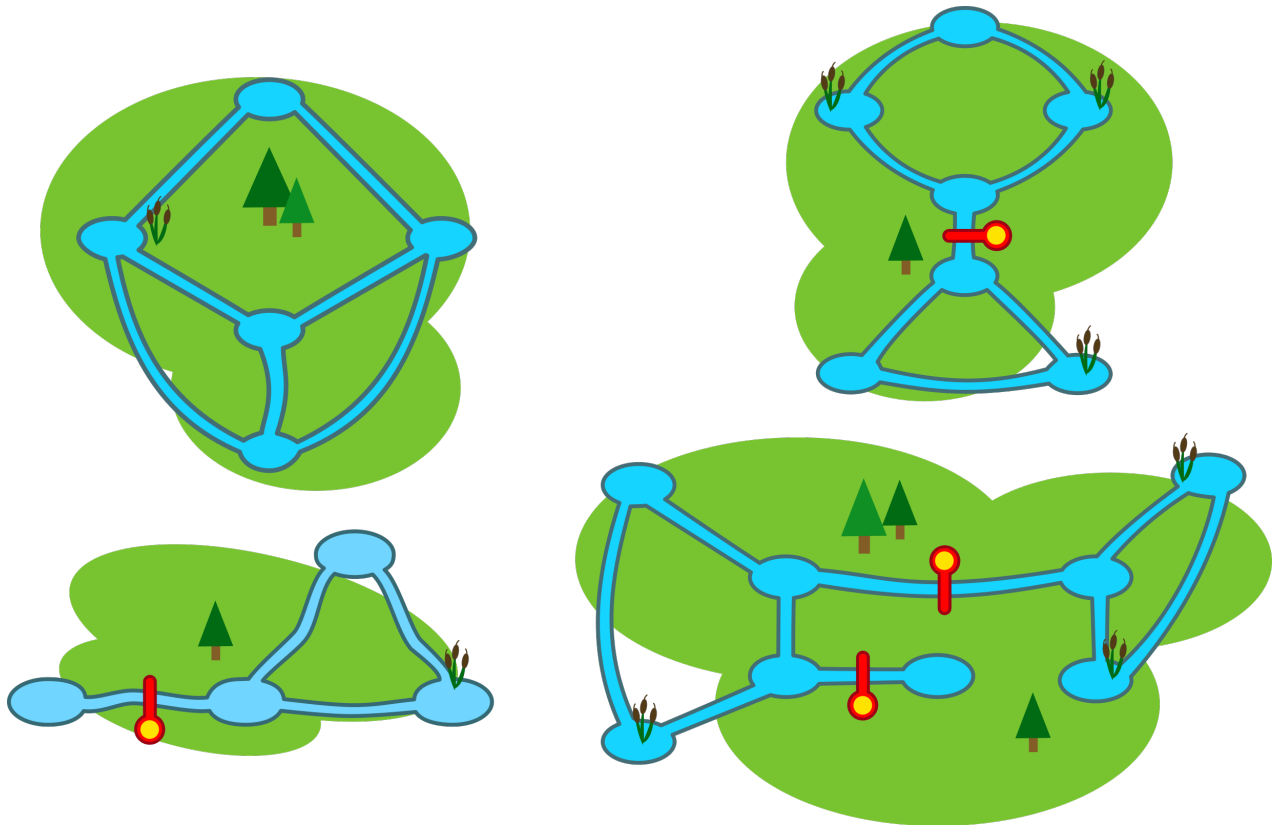
In which of the following neighbourhoods is Patricia unable to build her dam?



*Solution:* (A)

If putting a dam in a river causes the trouble Patricia wants, we call it a *weak link*. The correct answer is (A). No matter which single river is blocked, there is still a route between

any possible pair of ponds that avoids the dam. The other areas have the following weak links (see the dams in the picture): the middle river in the second picture, the left river in the third picture, the two middle rivers in the fourth picture. There is no weak link in the first picture.



Warm-up Problem Solutions Video: <https://youtu.be/m1NLq99NLyI>

## Strategies

Although the BCC questions don't require knowledge of coding, they require logical and computational thinking. Below, a few strategies are listed to approach these types of problems. Read through the strategies, and we will put some of them into use this week.

- Each question on the BCC has a story and a question. The story will give you the background information that you need to solve the question. Start by reading the story and question, then go back and reread the story. Reread the question and story until you understand what it is asking you to find and what you are given to find it.
- After understanding the story and question, start writing down what you know. Follow that with writing down what you can deduce from what you know.
- Break the problem into pieces. Focus on one step or case at a time.
- Remove answers that are impossible and ones that you can show won't be the solution. This may be easier than solving for the solution directly.
- Make a chart or diagram to help organize what is given in the story and what you learn from it.
- Try asking the question in your own words or in a different way. You may be able to find a simpler question that, if answered, will give you the answer to the original question.
- Have fun! The BCC and other contests are all meant to be fun.

## Examples

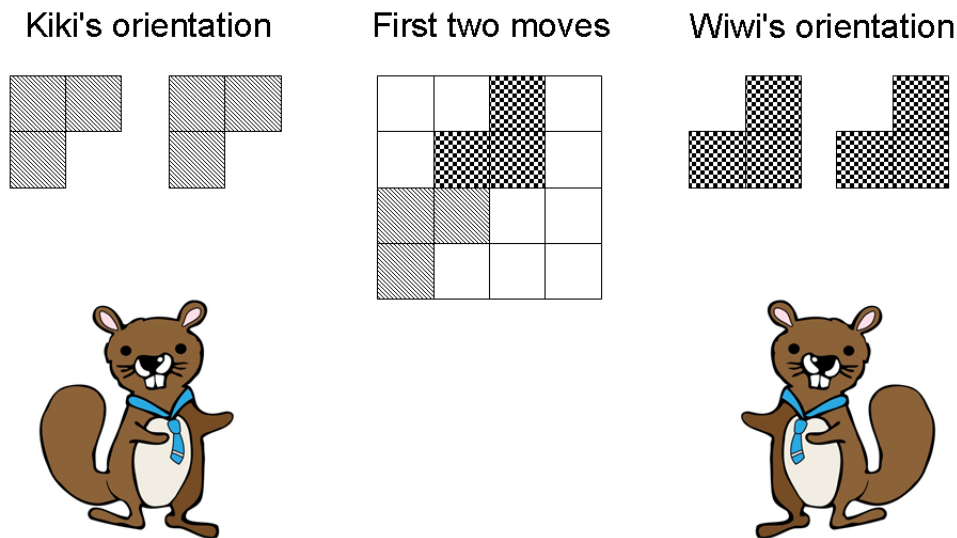
Try each of the following questions first on your own. If you get stuck, try using one of the strategies listed above. Associated with each of the following problems is a video. The video will be an explanation of one or more strategies for solving the given question. Once you have tried the problem on your own, watch the video.

**Example 1.** *L-Game (BCC 2016)*

Kiki and Wiwi are playing L-Game on a 4x4 board. The player who can no longer play a piece loses. They take turns placing L-shaped pieces one at a time with Kiki play first so that

- every piece placed by Kiki is oriented as shown below,
- every piece placed by Wiwi is oriented as shown below,
- every piece is placed entirely on the board, and
- no two pieces overlap.

The diagram below illustrates a possible board after each player has placed a piece once.

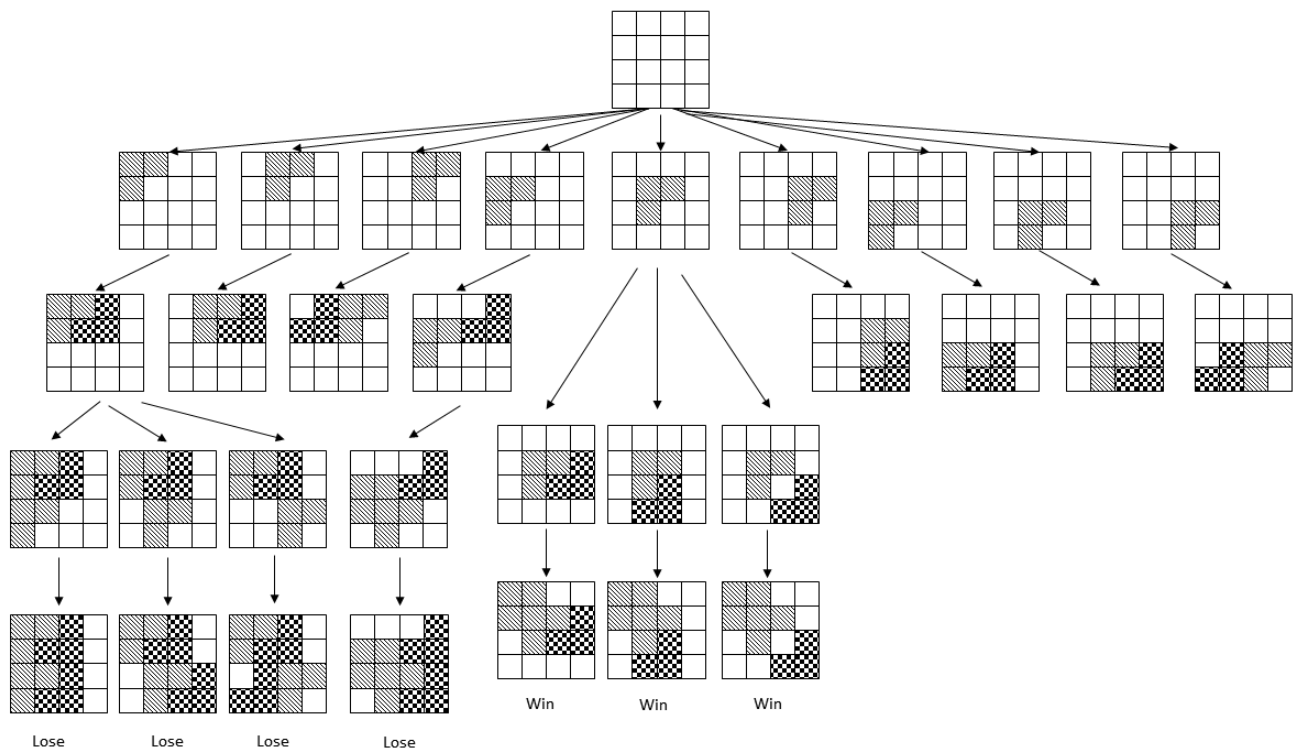


Starting from an entirely empty board, how many of Kiki's nine possible first moves guarantee that Kiki will win no matter what moves anybody makes?

L-Game Solution Video: <https://youtu.be/R-DwEYzAgIw>

*Solution: 1*

There is only one first move that will guarantee that Kiki wins. By placing a piece in the middle position, Kiki is guaranteed to win the game. No matter how Wiwi places a piece on their first turn, Kiki can only place a piece in the top left corner on their second turn. Then, Wiwi cannot place a piece according to the rules. If Kiki places a piece in any other position on their first turn, then there is always at least one way that they can lose the game. The following diagram details some of the possibilities and symmetry that can be used to rule out many of the positions.



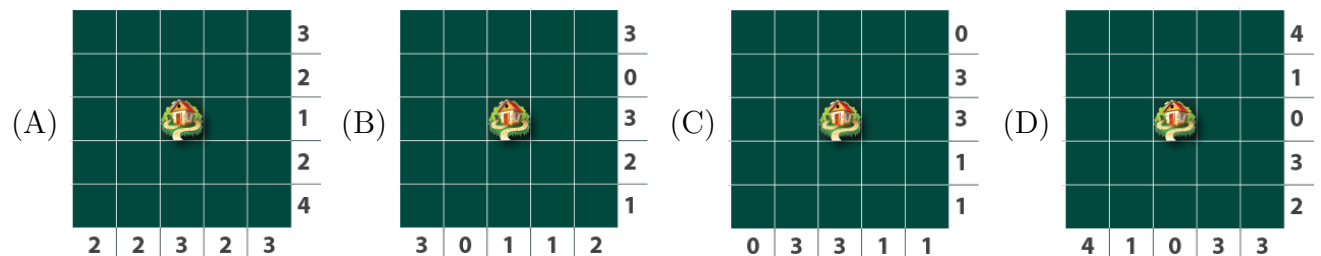
**Example 2.** *Farmer's Report (BCC 2019)*

Farms are divided into square fields. There is always a farmhouse in the centre square. Every year, farmers must decide whether a field will grow wheat or grass. They must report the total number of wheat fields in each row and column. An example of a report is shown on the right.



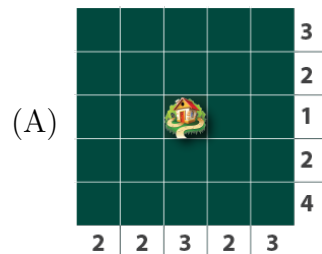
The totals given are accurate because there is one wheat field in each row, one wheat field in the left column, two wheat fields in the middle column, and no wheat fields in the right column.

In each of the following reports, each dark green square, except the centre square containing the farmhouse, represents either a wheat field or a grass field. Which report could contain accurate totals?

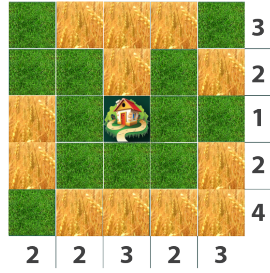


Farmer's Report Solution Video: [https://youtu.be/bMdvK7\\_1-20](https://youtu.be/bMdvK7_1-20)

*Solution:*

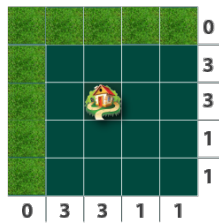


The following configuration shows that Option A could be accurate.

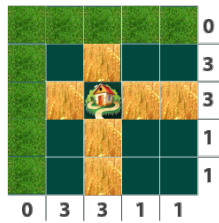


In Option B, the sum of the numbers at the bottom of the diagram is  $3 + 0 + 1 + 1 + 2 = 7$  and the sum of the numbers to the right of the diagram is  $3 + 0 + 3 + 2 + 1 = 9$ . These must both equal the total number of wheat fields so Option B is inaccurate. Option D is inaccurate for a similar reason.

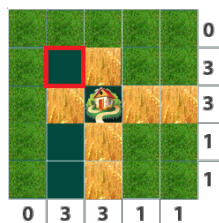
In Option C, the sums of rows and columns match, so we have to argue in a different way. The first row and the first column must have 0 wheat fields.



Thus there is only one way to cover the fields in the third row and in the third column by wheat.



Then, all others fields in columns 4 and 5 must be covered by grass.

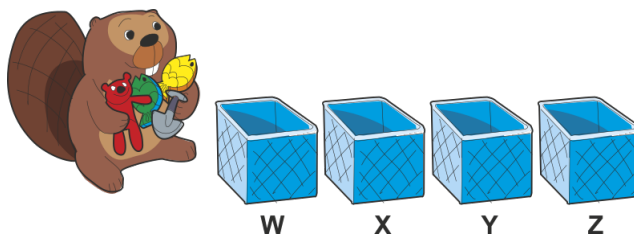


Now, it is impossible to plant wheat on two more fields in the second row because only one more field (marked in red) is available. Therefore Option C is inaccurate.



**Example 3.** *Triple Trouble (BCC 2019)*

A beaver puts each of four toys into boxes labeled W, X, Y, and Z. Each box can hold any number of toys.



At least one of the three conditions in each row of the table shown is satisfied.

a toy is in X	no toy is in Y	no toy is in Z
a toy is in W	a toy is in X	no toy is in Z
no toy is in X	no toy is in Y	a toy is in Z
no toy is in W	no toy is in X	no toy is in Y
no toy is in X	a toy is in Y	no toy is in Z

What is the minimum possible number of empty boxes?

Triple Trouble Solution Video: <https://youtu.be/M4rhcneVqFw>

*Solution:* 1

Notice that if there is a toy in each box, then no condition in the fourth row is satisfied. This means there must be at least one empty box. To show that it is possible to have exactly one empty box, consider placing a toy in boxes X, Y, and Z. (We could find this combination by trying the four possible combinations of three boxes.) Since a toy is in box X, the first two rows have a condition satisfied. Since a toy is in box Z, the third row has a condition satisfied. Since there is not a toy in box W, the fourth row has a condition satisfied. Finally, since there is a toy in box Y, the fifth row has a condition satisfied. We have shown that the minimum possible number of empty boxes is one.

## Problem Set

### 1. *Dogs versus Beavers (BCC 2015)*

Beavers and dogs compete. The nine participants scored the following points: 1, 2, 2, 3, 4, 5, 5, 6, 7. We know that no dog scored more than any beaver, but one dog had the same score as a beaver and two dogs had the same score. How many dogs took part in the competition?

*Solution:* 6

If no dog scored more than any beaver, we can order the animals in a row so that a separator can be used to separate the dogs and beavers. For example: (dogs in front) 1, 2, 2, 3, | 4, 5, 5, 6, 7 (beavers at end), where | means the separator between dogs and beavers. If two dogs scored 5, then since all dogs scored less than beavers, then two dogs must have also scored 2. This, however, does not allow for the fact that a dog and a beaver tied. Hence, the two 5s must be a dog and a beaver, which means that the separator between the dogs and the beavers must be between the two 5's.

(dogs) 1, 2, 2, 3, 4, 5 | 5, 6, 7 (beavers).

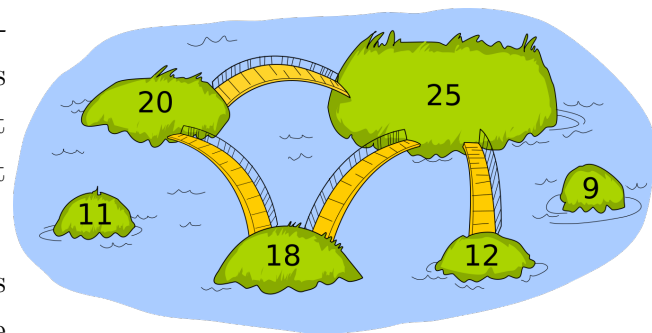
Therefore, 6 dogs participated in the competition.

### 2. *Connect the Islands (BCC 2018)*

People of Kastoria use only one rule to decide where bridges are to be built:

They choose one number called the *bridge number*. If the sum of the populations of two islands is greater than the bridge number, a bridge is built between the islands. Otherwise, a bridge is not built between the two islands.

The six islands of Kastoria and their populations are shown below. The bridges built using the above rule are also shown. What bridge number was chosen?



- a) 34   b) 35   c) 36   d) 37

*Solution:* c) 36

The chosen number cannot be 34 or 35 because  $11 + 25 = 36$  and there is not a bridge between the islands with populations of 11 and 25.

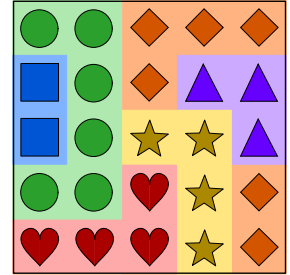
The chosen number cannot be 37 because  $12 + 25 = 37$  and there is a bridge between the islands with populations of 25 and 12.

For each pair of islands connected by a bridge, the sum of their populations is greater than 36. That is,  $18 + 20 = 38$ ,  $18 + 25 = 43$ ,  $20 + 25 = 45$  and  $12 + 25 = 37$ .

Therefore, the chosen number is 36.

3. *So Many Layers (Based on "Wallpapers" (BCC 2017))*

Tanu covered a wall with six overlapping rectangular sheets of wallpaper as shown. Each sheet of wallpaper is designed using a different image in a repeating pattern.



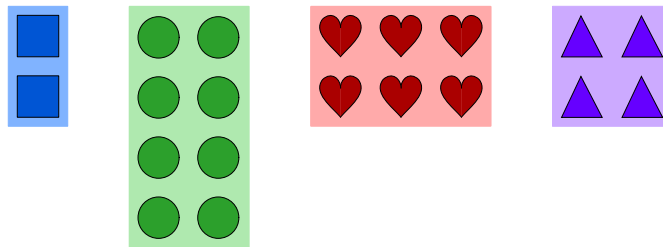
- (a) Give possible dimensions for the six wallpaper sheets Tanu used.
- (b) Determine in what order Tanu placed the wallpaper sheets on the wall.

Try this online activity once you have attempted the problem on your own:

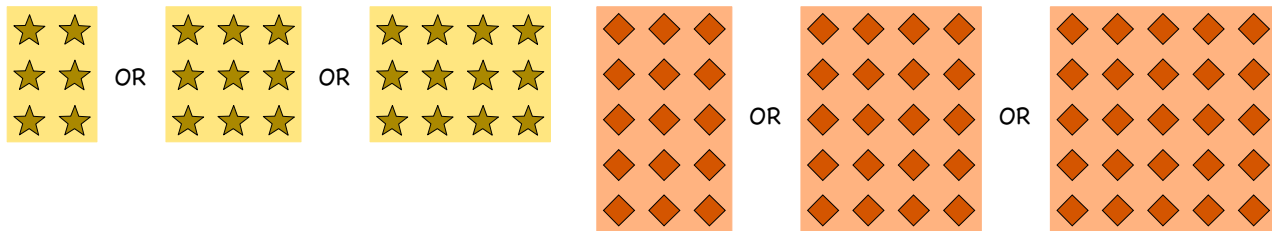
<https://www.geogebra.org/m/c5j6anan>

*Solution:*

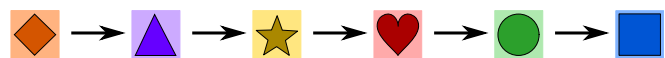
- (a) Since each sheet is rectangular, we can see enough to determine that four of the sheets have the dimensions shown below:



We cannot be sure of the dimensions of the remaining two sheets. Here are the different possibilities for these sheets:



(b) The wallpaper sheets were placed in the following order, from first to last:

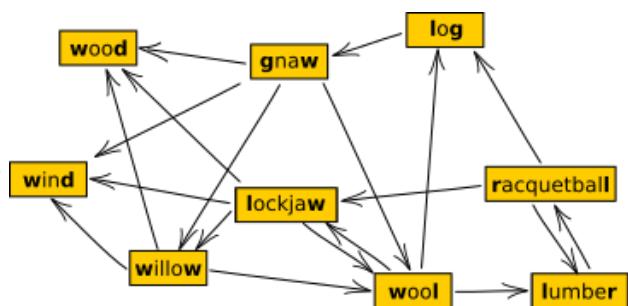


The wallpaper with the blue squares is the only wallpaper that is entirely visible, so it must have been placed last. The wallpaper with the green circles is cut off by the blue squares, so it must have been placed before the blue squares. By similar reasoning, the red hearts were placed before the green circles, the yellow stars were placed before the red hearts, the purple triangles were placed before the yellow stars, and the orange diamonds were placed before the purple triangles. Therefore, the sheets must have been placed in the order indicated above.

*Note that we do not need to know the dimensions of all six wallpaper sheets to determine the order in which they must have been placed. For example, we cannot be sure whether the orange sheet and the green sheet overlap or are placed side-by-side, but this does not stop us from figuring out in which order they were placed.*

#### 4. Longest Word Chain (BCC 2018)

Beavers play a word chain game. One beaver starts by saying a word. The other beaver must say a different word which begins with the last letter of the previous word. Then the first beaver says another word (which was not said yet) using this same rule, and so on. If a beaver is unable to say a new word, that beaver loses the game. These beavers do not know many words. In fact, they can draw their entire vocabulary like this:



Notice that an arrow out of a word points at the next possible word(s) that can be said.

What is the largest possible number of words that can be said in one game?

*Solution:* 8

The beavers can use at most 8 words in one game. One example is:

lockjaw-wool-lumber-racquetball-log-gnaw-willow-wood

(Can you find another game of the same length?)

To be sure that 8 is the largest possible number of words, we have to convince ourselves that it is not possible to use all 9 words. Consider the words *wood* and *wind*. There is no word beginning with *d*, so if either of these words is said, it must be the last word of the game. Since there cannot be two words that are said last, it is not possible to use the entire vocabulary of 9 words.

5. *Timetabling (BCC 2018)*

Bebras Tech offers the following evening classes: Computing (**C**), Geography (**G**), Language (**L**), Math (**M**), and Science (**S**).

Three beavers would like to sign up for these courses:

- Xavier wants to take **C**, **L** and **M**;
- Yvette wants to take **C**, **G**, and **S**;
- Zoey wants to take **L**, **M** and **S**.

Bebras Tech wants to squeeze these courses into as few evenings as possible such that each course is offered on exactly one evening, and beavers can take at most one course per evening.

What is the least number of evenings needed for Bebras Tech to schedule these courses?

- a) 2   b) 3   c) 4   d) 5

*Solution:* c) 4

The following schedule shows that four evenings can be used.

Evening 1: Computing (taken by Xavier and Yvette)

Evening 2: Language (taken by Xavier and Zoey) and Geography (taken by Yvette)

Evening 3: Science (taken by Yvette and Zoey)

Evening 4: Math (taken by Xavier and Zoey)

It is not possible to use only three evenings. To see this, consider Computing, Language, Science, and Math. There are six (unordered) pairs of these subjects and for each pair, at least one beaver wants to take both subjects in the pair:

- Xavier wants to take Computing and Language.
- Yvette wants to take Computing and Science.
- Xavier wants to take Computing and Math.
- Zoey wants to take Language and Science.

- Xavier and Zoey want to take Language and Math.
- Zoey wants to take Science and Math.

This means that these four subjects have to be offered on different evenings, so at least four evenings are required. This is just one possible solution: can you find the other(s)?

#### 6. *More Problems*

Looking for more practice problems? Check for daily practice problems on Piazza or check out the following links:

Practice Contest:

[https://beavercomputingchallenge.ca/index.php?action=user\\_competitions](https://beavercomputingchallenge.ca/index.php?action=user_competitions)

Past Contests and Solutions:

[https://cemc.uwaterloo.ca/contests/past\\_contests.html](https://cemc.uwaterloo.ca/contests/past_contests.html)