UNIVERSITY OF WATERLOO

Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

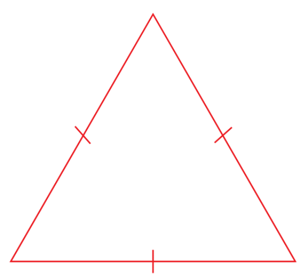
Grade 7/8 Math Circles

Wednesday, March 31, 2021

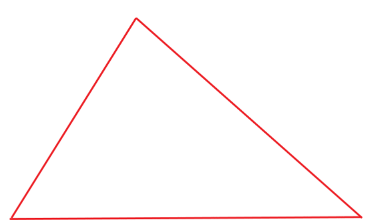
Triangles - Solutions

1. Draw an example of the triangle described.

   (a) equilateral

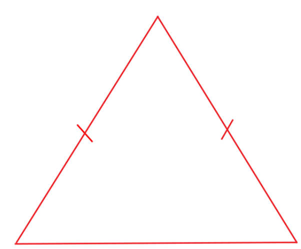
   (b) scalene

   (c) obtuse right

   There is no triangle that fits this definition. Since the sum of the interior angles of a triangle is 180°, if there is an obtuse angle, than there cannot be a right angle. Similarly, if there is a right angle, there cannot be an obtuse angle as well.

   (d) isosceles acute

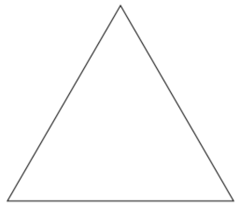
   

2. Why are all equilateral triangles classified as acute triangles? Explain using a diagram and properties of triangles.

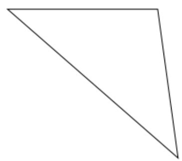
   The sum of the three interior angles of a triangle equals 180°. In an equilateral triangle, all three angles are equal so, we can divide 180° by 3 to get the measure of each angle. Therefore, each angle is 60° in an equilateral triangle. Since 60° is less than 90° and thus, an acute angle, then all the angles in an equilateral triangle are acute. So, equilateral triangles satisfy the definition for acute triangles.

3. Identify the hypotenuse of the following triangles by labeling the side with an h, if there is one.

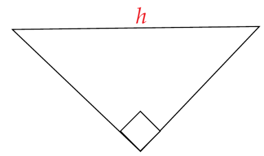
(a)



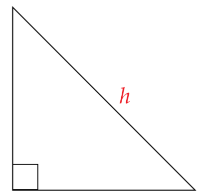
(b)



(c)



(d)

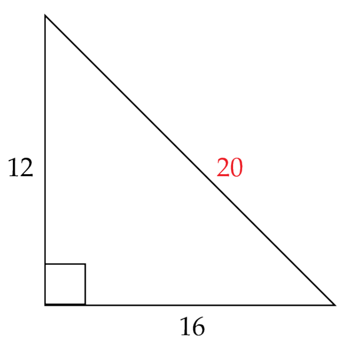


4. Rearrange the Pythagorean Theorem to come up with an equation for each variable. The first one is done for you.

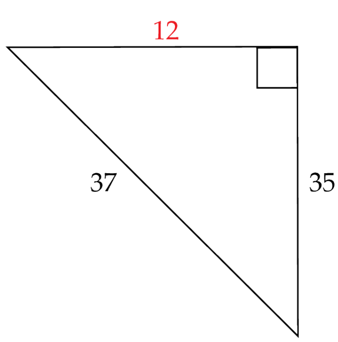
   (a) $c = \sqrt{a^2 + b^2}$ (b) $a = \sqrt{c^2 - b^2}$ (c) $b = \sqrt{c^2 - a^2}$

5. Use the Pythagorean Theorem to solve for the missing side.

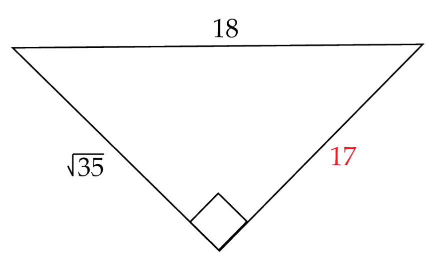
(a)



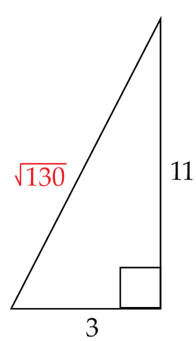
(b)



(c)



(d)



6. Consider the following values for a, b, and c, such that, a, b are the lengths of the legs

of a right triangle (in cm) and c is the length of the hypotenuse (in cm). Solve for the missing length. Where applicable, leave your answer in radical form and simplify the radical.

(a) $a = 8$, $b = 15$, $c = 17$

(b) $a = 20$, $b = 21$, $c = 29$

(c) $a = 2\sqrt{10}$, $b = 6$, $c = 2\sqrt{19}$

(d) $a = 9$, $b = \sqrt{31}$, $c = 4\sqrt{7}$

(e) $a = 2$, $b = 4$, $c = 2\sqrt{5}$

(f) $a = \frac{1}{2}$, $b = \frac{\sqrt{3}}{2}$, $c = 1$

7. Ximena lives 5 km due north of the University of Waterloo. Ty lives 13 km due west of the University. How far apart do they live from each other?
If we start at the University, the distance between the Ty and Ximena would be the hypotenuse of a right-angled triangle with the distance between the University and Ty's house and the University and Ximena's house as the two legs of the triangle. So, we have a triangle with side lengths 5 km and 13 km and we need to find the length of the hypotenuse. Using Pythagorean Theorem, we get

$$(5)^2 + (13)^2 = c^2$$
$$25 + 169 = c^2$$
$$c^2 = 194$$
$$c = \sqrt{194}$$

Therefore, they live $\sqrt{194}$ km apart from each other.

8. Maya is in a helicopter 165 m above the ground. The launching pad is 30 m away from where the helicopter is hovering. What is the distance between Maya's helicopter and the launching pad?
The distance between Maya's helicopter and the launching pad is the hypotenuse of a right-angled triangle. In this situation, the distance between the helicopter and the ground it is hovering above, and the distance between where the helicopter is hovering above and the launching pad are the two legs of the triangle. Again, the side lengths of the two legs are known and thus, we can use Pythagorean Theorem to solve for the

hypotenuse. So,

$$\begin{aligned}(165)^2 + (30)^2 &= c^2\\ 27,225 + 900 &= c^2\\ c^2 &= 28,125\\ c &= \sqrt{28,125}\\ c &= 75\sqrt{5}\end{aligned}$$

Therefore, Maya's helicopter is $75\sqrt{5}$ km from the launching pad.

9. A slide is 7 m long from the top of the slide to the ground and goes across 5 m. If Sandy is standing at the top of the slide, how many meters above the ground is she?
From the top of the of slide to the ground is one leg of a right-angled triangle and the length across is the other. Then, the length of the slide to the ground is the hypotenuse of the right triangle formed. Since we are given the length of the slide from top to bottom and the length across, we can rearrange the Pythagorean Theorem to find the distance from the top of the slide to the ground. So,

$$\begin{aligned}(5)^2 + (b)^2 &= (7)^2\\ b^2 &= (7)^2 - (5)^2\\ b^2 &= 49 - 25\\ b^2 &= 24\\ b &= 2\sqrt{6}\end{aligned}$$

Hence, Sandy is $2\sqrt{6}$ m above the ground.

10. Compute the following trig ratios. Round to two decimal points.

(a) $\sin(39°) = 0.63$

(b) $\tan(25°) = 0.47$

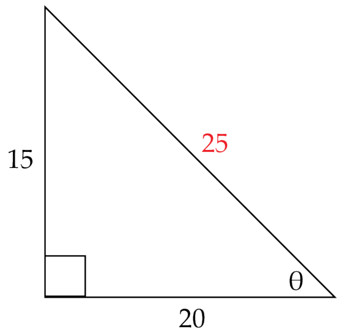
(c) $\cos(40°) = 0.77$

(d) $\tan(16°) = 0.29$

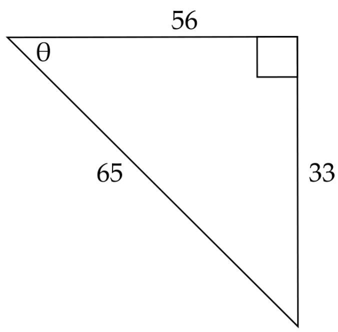
(e) $\sin(57°) = 0.84$

(f) $\cos(8°) = 0.99$

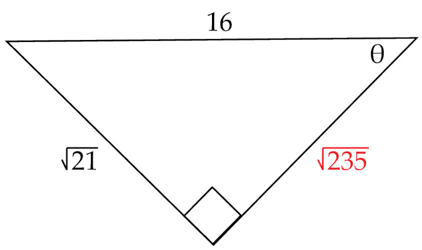
11. Find all three primary trig ratios for the given triangles.

(a) 

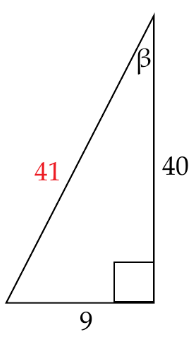
$$\sin\theta = \frac{15}{25} = \frac{3}{5} \quad \cos\theta = \frac{20}{25} = \frac{4}{5} \quad \tan\theta = \frac{15}{20} = \frac{3}{4}$$

(b) 

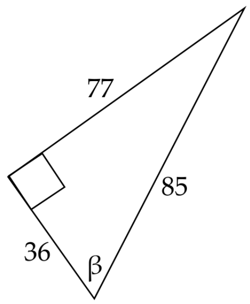
$$\sin\theta = \frac{33}{65} \quad \cos\theta = \frac{56}{65} \quad \tan\theta = \frac{33}{56}$$

(c) 

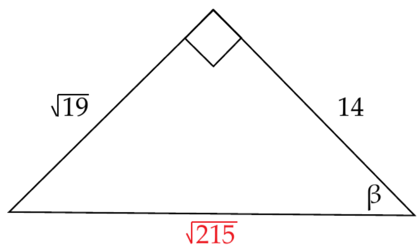
$$\sin\theta = \frac{\sqrt{21}}{16} \quad \cos\theta = \frac{\sqrt{235}}{16} \quad \tan\theta = \frac{\sqrt{21}}{\sqrt{235}} = \frac{\sqrt{4935}}{235}$$

(d) 

$$\sin\theta = \frac{9}{41} \quad \cos\theta = \frac{40}{41} \quad \tan\theta = \frac{9}{40}$$

(e) 

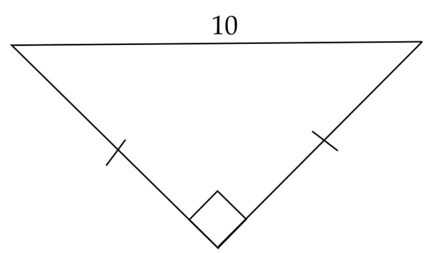
$$\sin\theta = \frac{77}{85} \quad \cos\theta = \frac{36}{85} \quad \tan\theta = \frac{77}{36}$$

(f) 

$$\sin\theta = \frac{\sqrt{19}}{\sqrt{215}} = \frac{\sqrt{4085}}{215} \quad \tan\theta = \frac{\sqrt{19}}{14}$$

$$\cos\theta = \frac{14}{\sqrt{215}} = \frac{14\sqrt{215}}{215}$$

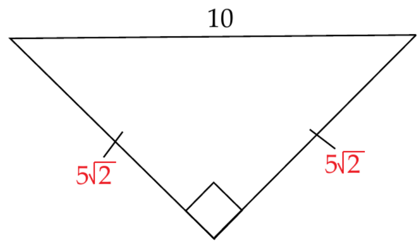
12. Jackie drew the triangle below. Find all the sides of the triangle and the primary trigonometric ratios.



This is an isosceles triangle so, the two unknown side lengths are equal. Since both side lengths are equal, we can replace a and b with x in our equation. Using the Pythagorean Theorem, we get

$$\begin{aligned} x^2 + x^2 &= 10^2 \\ 2x^2 &= 100 \textit{ Note:} \text{ There are two } x^2\text{s so, we can rewrite it as } 2x^2 \\ x^2 &= 50 \\ x &= \sqrt{50} \\ x &= 5\sqrt{2} \end{aligned}$$

Hence, we can label all the side lengths as shown in the picture below.



Recall that an isosceles triangle has two equal angles so, the two remaining angles other than the right angle, are equivalent. Therefore, regardless of which angle we use, the trigonometric ratios will be the same. Therefore, we get $sin\theta = \frac{5\sqrt{2}}{10} = \frac{\sqrt{2}}{2}$ $cos\theta = \frac{5\sqrt{2}}{10} = \frac{\sqrt{2}}{2}$ tan

13. Elmdale Public School is building an accessibility ramp at the front of the school. To meet safety guidelines, the ramp must have an incline no more than 35°.

    (a) In the current design, the ramp is 24 feet long and 10 feet wide. Does the ramp satisfy the safety conditions? Explain.
    If the incline satisfies the incline requirement, then it should satisfy the corresponding angle ratio. The sine of an angle is the ratio between the opposite side and the hypotenuse which in this case would be the height of the ramp and the length of the ramp. It is a good choice for a trig ratio to use since the sine ratio increases with the size of the angle. Note that the cosine ratio decreases as the angle becomes larger.
    Using Pythagorean Theorem, we get that the height of the ramp is $2\sqrt{119}$ feet. The, the ratio between the height and the length of the ramp is $\frac{2\sqrt{119}}{24} = \frac{\sqrt{119}}{12} \approx 0.91$. Since $\sin(35°) \approx 0.57$, the incline of the ramp is larger than 35°. Thus, it does not meet the safety guidelines.

    (b) Pablo is submitting a design for the ramp. In his design, the ramp has an incline of 30°. If the length of the ramp is 15 feet, how tall is the ramp? (*Hint:* $\sin(30°) = \frac{1}{2}$)
    The sine of an angle is the ratio between the opposite side and the hypotenuse. If we know the angle of the incline of the ramp, then we know the ratio between the height of the ramp and the length of the ramp.
    Since $\sin(30°) = \frac{1}{2}$, that means that the ratio between the height of the ramp and the length of the ramp is 1 : 2. Using ratios, if the length of the ramp is 15 feet, then the height must be half of 15. Therefore, the height of the ramp is 7.5 m.

14. A square has a diagonal of $4\sqrt{2}$ cm.

(a) What is the side length of the square?

Observe that the diagonal of a square is the hypotenuse for a right triangle that is formed using two sides of the square. So, we can use the Pythagorean Theorem to solve for the side length. Since all the sides of square are equal, a and b must be the same and can be replaced by the same letter x. Then,

$$\begin{aligned} a^2 + b^2 &= (4\sqrt{2})^2 \\ x^2 + x^2 &= 16 \times 2 \\ 2x^2 &= 32 \text{ \textit{Note:} There are two } x^2\text{s so, we can rewrite it as } 2x^2 \\ x^2 &= 16 \\ x &= \sqrt{16} \\ x &= 4 \end{aligned}$$

Hence, the side length of the square is 4 cm.

(b) What is the perimeter and area?

Using the perimeter and area formulas and the side length of 4 cm that was solved for in part (a), then

$$\begin{aligned} P &= 4s \\ &= 4(4) \\ &= 16 \end{aligned}$$

$$\begin{aligned} A &= s^2 \\ &= (4)^2 \\ &= 16 \end{aligned}$$

So, the perimeter of the square is 16 cm and the area of the square is 16 cm^2.