Math Circles - Grade 11/12
February 17 - 23, 2021
Some problems that I like

Before trying the problems below, please watch the following video https://youtu.be/MejsohEuo0U

In all of the problems below, you are asked to determine the value of an expression involving some variables. In each of them, try to solve the problem without actually determining the value of the individual variables.

1. Suppose that x is a real number such that $x + \dfrac{2}{x} = 7$. Determine $x^2 + \dfrac{4}{x^2}$.

2. Suppose that x, y are positive real numbers that satisfy

$$xy = \frac{1}{9}$$
$$x(y + 1) = \frac{7}{9}$$
$$y(x + 1) = \frac{5}{18}$$

   Determine $(x + 1)(y + 1)$.

3. Suppose that x and y satisfy the equations

$$3 \sin x + 4 \cos y = 5$$
$$4 \sin y + 3 \cos x = 2$$

   Determine $\sin(x + y)$.

4. A list of numbers $x_1, x_2, \cdots, x_{100}$ has the following property: given any integer $1 \le k \le 100$, x_k is k less than the sum of the other 99 numbers. Determine $x_1 + x_2 + \cdots + x_{100}$.