Part II: For the Teacher

Curriculum Areas

Problem 1 – Number Sense
Problem 2 – Number Sense and Measurement
Problem 3 – Number Sense
Problem 4 – Measurement, Spatial Sense, and Pattern and Algebra
Problem 5 – Measurement
Problem 6 – Number Sense

Hints and Suggestions

Problem 1
Hint 1 – Which questions could not have an answer of 0.19344?
Hint 2 – What do you think would be the last two digits of 3.01×0.605 ?.

Problem 2
Hint 1 – How many years did it take Nanting to grow 0.3 m?

Extension: Hint 1 – If she did continue to grow at the same rate, how tall would she be in 6 years? In 12 years?

Problem 3
Hint 1 – How do you know that the smallest tent must be either red or blue?
Hint 2 – How many campers are in the red, blue, and orange tents altogether? How many are in the other three tents?

Suggestion: Have students draw a simple diagram of the tents, with colours, and any known information written beside each tent.

Problem 4
Hint 1 – Look at the figure with 2 rows. Can you rearrange the 1 cm squares to form a bigger square? What about the figure with 3 rows?

Problem 5
Hint 1 – How many shorter sides equal the longer side of each rectangle? How do you know?
Hint 2 – If the shorter side were 1 unit in length, what would be the length of the perimeter?

Extension: Hint 1 – How many rectangles make up the area of 84 cm^2?
Hint 2 – What is the area of each rectangle?

Problem 6
Suggestion: Play the game as a class to introduce the game. It may also be beneficial for the students to try playing as a group with a single set of squares, as this will provoke discussion of the best strategies.

Solutions and Notes

Problem 1

Mandeep knows that 0.19344 must be 0.31 x 0.624 because the last digit in the other two products, 3.01 x 0.605 and 6.15 x 0.313, must be a 5. Further, he knows 3.01 x 0.605 will end with the digits 05 because of the 0's in 3.01 and 0.605. Thus 6.15 x 0.313 = 1.92495.

Problem 2

If Nanting grows 0.3 metres in the three years from grade 6 to grade 9, then she must be growing at 0.1 m per year. Thus in five years from grade 5 to grade 10, she grows 0.5 metres, giving her a height of 1.3 m + 0.5 m = 1.8 m now.

Extension: If Nanting were to grow this way for another 6 years, she would be 1.8 + 0.6 = 2.4 metres tall. Although this is quite tall, it is possible. If she did so for another 12 years, she would be 3 metres tall, which is improbable. Growing the same amount each year at any time in her life is also improbable. Discuss this with students. Perhaps they can suggest periods in their own lives when they grew the fastest.

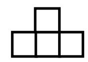
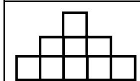
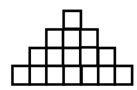
Problem 3

a) We know the orange tent has 10 campers, and there are 13 campers in the red and blue tents, which must have 6 in one and 7 in the other, since one of them has 6, the smallest group. This leaves 48 – (10 + 13) = 25 campers for the yellow, green, and purple tents. Since the purple tent has 2 more than the blue tent, it must be either 6 + 2 or 7 + 2, leaving either 25 – 8 = 17 or 25 – 9 = 16 campers for the yellow and green tents. But they each have an equal number, so it must be 16. Thus the blue tent has 7 campers, the red tent 6, the yellow and green tents 8 each, the purple tent 9, and the orange tent 10.

b) If the orange tent had 11 campers, there would be 48 – (11 + 13) = 24 campers for the yellow, green and purple tents. Thus we would need 8 campers in the purple tent, in order to have an even number left over for the yellow and green. But this would leave 16 campers for the yellow and green tents, meaning they each also have 8 campers, leaving three tents with 8 campers. This contradicts the given condition that only two tents, yellow and green, have the same number of campers. Thus it is not possible to have 11 campers in the orange tent.

Problem 4

a) The completed table:

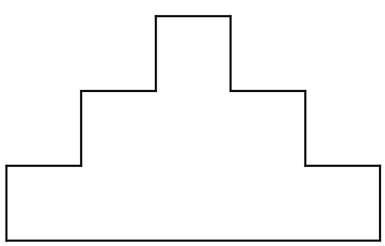
Figure	Number of Rows	Area (cm^2)	Length of Bottom Row cm
	1	1	1
	2	4	3
	3	9	5
	4	16	7
	5	25	9
	7	49	13

Each successive area is obtained from the previous area by adding the length (i.e. the number of squares) in the bottom row of the new figure.

b) It is clear from the table in a) that the area is the square of the number of rows.

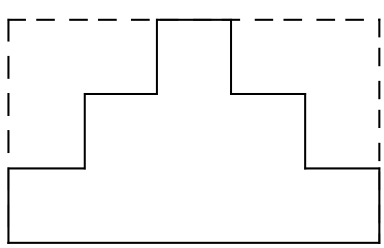
c) The figure with bottom row length of 15 would have 8 rows, since the length of the bottom row grows by 2 each time the number of rows increases by 1. (So the figure with 7 rows has length 11 + 2 = 13, and with 8 rows has length 13 + 2 = 15). Thus the area of the figure is 8 x 8 = 64 cm^2.
In general, the length of the bottom row is 1 less than twice the number of rows. So the number of rows is half of {1 + length of bottom row}. Thus if the length of the bottom row is 25, the number of rows is $\frac{1}{2} \times (1 + 25) = 13$. Thus the area of the figure with bottom row of length 25 cm is 13 x 13 = 169 cm^2.

Extension: 1) Since the area of 144 cm^2 is the square of the number of rows, we see that the number of rows is 12. Thus the length of the bottom row is 2 x 12 – 1 = 23.
2) As an example of one way to find the perimeter, consider the figure with 3 rows; each line segment has length 1 cm.



Number of vertical segments = 2 x number of rows.

Number of horizontal segments = 2 x length of bottom row.



Alternatively, note that the perimeter of the figure with 3 rows is the same as that of the rectangle outlined by dotted lines.

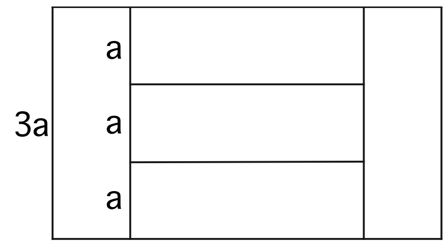
Length of vertical sides = 2 x number of rows.
Length of horizontal sides = 2 x length of bottom row.

Either way, the perimeter is 2 x number of rows + 2 x length of bottom row. For the figure with 3 rows, this gives (2 x 3) + (2 x 5) = 16. In general, since the length of the bottom row now is (2 x number of rows) – 1, the total perimeter for n rows is 2 x n + 2 x (2 x n – 1) = 6 x n – 2.

The completed table reflects this by revealing that each time the number of rows increases by 1, the perimeter increases by 6.

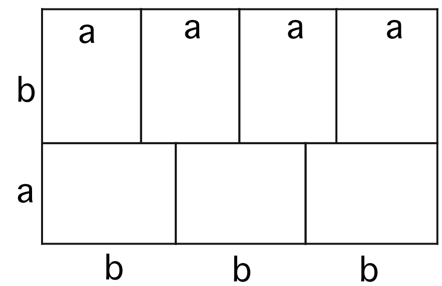
Number of Rows	Perimeter
1	4
2	10
3	16
4	22
5	28
6	34

Problem 5
Let a represent the length of the shorter side of the rectangle. The diagram then reveals that the longer side must have length 3 a.
Thus the perimeter is $2 \times (3a + a + 3a + a) = 16a = 32$, so $a = 2$
Thus the area is $3a \times 5a = 60\,\text{cm}^2$.



Extension: There are 7 rectangles making up the total area of 84 cm^2. Thus the area of one rectangle is 12 cm^2. The dimensions are whole numbers, and thus are either 6 cm by 2 cm, or 4cm by 3 cm, or 1 cm by 12 cm. The diagram shows that 3 times the length of the rectangle equals 4 times the width. Thus the rectangles are 3 cm by 4 cm, and the perimeter is

$$2 \times (4a + a + a + b) = 2 \times (5a + b) = 38 \text{ cm}.$$



Problem 6
Discussion questions:

- The lowest possible digit is “0” (5+5, 6+4, or 4+6); the highest is 9 (3+6, 4+5, 5+4, 6+3).
- There are two ways to get the digit 2 (1+1 or 6+6); there are six ways to get a 7 (1+6, 2+5, 3+4, 4+3, 5+2, 6+1) Knowing this enables you to guess that a 7 is more likely to be rolled than a 2.
- Generally, a “0” should be placed as the units digit, if possible, and a 9 as the thousands, if the goal is to achieve the largest possible number; a 5 could go in the tens digit. But these decisions will obviously depend on what digits have already been placed.

Note: There are two ways to get the digit 1 (5+6 or 6+5), two ways to get 2 (see above), two ways to get 3 (1+2 or 2+1). There are 3 ways to get a 0 (see above), or a 4 (1+3, 2+2, 3+1). There are 4 ways to get a 5 (1+4, 2+3, 3+2, 4+1), or a 9 (3+6, 4+5, 5+4, 6+3). There are five ways to get a 6 (1+5, 2+4, 3+3, 4+2, 5+1), or an 8 (2+6, 3+5, 4+4, 5+3, 6+1). There are six ways to get a 7 (see above). So a 9 is less likely than a 6, 7, or 8. If you roll a 9, the decision is easy but if you roll a 6, 7, or 8 you have to decide whether you want to take a chance on getting a 9 later on, or place the 6, 7, or 8 in the thousands slot, knowing it’s more likely you’ll get a 0, 1, 2, 3, 4, than a 9.

Extension: Probably fewer blanks is more challenging. To extend the problem further, change from addition to multiplication. Roll the dice, multiply the two numbers showing, and record the units digit on one square. Explore how the strategies change. (Now there is one way to roll a 1, 3, or 9 two ways to get 8, and three ways to get 0, 2, 4, 5, or 6.)