Emmy Noether - Circle 2 for 2006-2007

Part I: Problems

Problem 1

Ticket prices in Sportsville are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Adults</th>
<th>Children</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOCKEY:</td>
<td>$2.00</td>
<td>$1.00</td>
</tr>
<tr>
<td>BASKETBALL:</td>
<td>$3.00</td>
<td>$1.50</td>
</tr>
<tr>
<td>SOCCER:</td>
<td>$5.00</td>
<td>$2.50</td>
</tr>
</tbody>
</table>

a) If Mr. and Mrs. Sportsfan take their three children to see the soccer game, and pay for their tickets with a $20 bill, how much change will they get?

b) If they paid a total of $10.50 for all five tickets for the same game, which game did they see?

c) Suppose the family has only $8 to spend on tickets. If they don’t all necessarily go to the same game, in how many different ways can they buy tickets so that every family member gets to see a game?

Problem 2

a) Suppose you have to place the house numbers from 1 to 100 on a row of 100 new homes. How many 3’s would you have to buy in order to do all the numbering?

b) For what digit(s) other than 3 is the answer the same as in a)? Explain your reasoning.

c) The remaining digits are required a different number of times. How many would you have to buy of each of these digits?

d) What is the total number of digits you have to buy for all the houses?
Problem 3

a) Part of Jan’s yard is fenced to make a pen for her dog. The pen measures 10 metres wide by 20 metres long. She wants to enlarge the pen as she has adopted a second dog. She plans to move one side of the pen outward to increase the area by 40 square metres. What is the least number of metres of additional fencing Jan will need to enclose the enlarged pen if she re-uses all the existing fencing?

b) Suppose Jan has 10 metres of fencing stored in the garage. If she uses this instead to enlarge the pen (keeping it rectangular), what is the greatest number of square metres of area she can add to the pen, assuming she re-uses all existing fencing?

Problem 4

Picture two identical squares held together by a pin at A, so that the top square can be rotated about A while the bottom square remains fixed. Rotate the top figure partially as shown in the diagram. Trace the outside edges to form a polygon.

a) How many sides has this polygon?

b) What is the least number of sides for a polygon formed by rotating the top square about the pin at A? How many different figures can you form with this (least) number of sides?

c) Could you form a polygon with more than 6 sides this way?

d) Repeat parts b) and c) if the top figure is an equilateral triangle with side length the same as the square on the bottom, with the pin A at one vertex of the triangle (and one corner of the square).

Extension:
Suppose instead that the triangle of part d) is the isosceles triangle that is one-half the square. Are your answers the same as in d)?
Problem 5

The Grade 6 class at Happy Valley School is holding a cookie sale and lottery to raise money for the Red Cross. They have 60 small boxes, each containing 6 cookies and a small card printed with one of the six letters Y, O, U, W, I, N. The letters are distributed as follows: 15 boxes have cards with a Y, 13 with an O, 12 with a U, 10 with a W, 6 with an I, 4 with an N. To win the lottery, you have to buy boxes containing at least one of each of the six letters.

a) Could you win if you bought just six boxes? Are you likely to win?

b) How many winners could there be?

c) Assuming the boxes look the same and are sold in random order, what is the probability that a buyer selecting one box will get one with a Y? an O? an N?

d) How many boxes of cookies would you have to buy in order to guarantee that you have some set of six boxes containing Y, O, U, W, I, N (i.e., to guarantee that you’ve won the lottery)?

e) If the students sell all the cookies for $4 per box, how much will they make? If each lottery winner gets $50, and baking the cookies cost $30, what is the greatest amount they could make for the Red Cross?

Extension:
If you get together with three friends and borrow enough money from someone else to buy all 60 boxes of cookies to share among you, how much of your own money will each of you have to pay back to the person who lent you the money?
Hockey Games (Suggested for groups of 2 to 4 students)

Here are the current standings of the top four teams in the North-Eastern Hockey League:

<table>
<thead>
<tr>
<th>Team</th>
<th>Wins</th>
<th>Losses</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Majors</td>
<td>15</td>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>All-Stars</td>
<td>13</td>
<td>4</td>
<td>26</td>
</tr>
<tr>
<td>Champs</td>
<td>12</td>
<td>5</td>
<td>24</td>
</tr>
<tr>
<td>Generals</td>
<td>10</td>
<td>7</td>
<td>20</td>
</tr>
</tbody>
</table>

No tie games are allowed in this league.

a) Each team has three games left, one game against each of the other teams in the top four. How many games must be played? Make a list of the games that must be played.

b) If the Majors lose all three games, what is the highest place they could achieve in the final standings? Explain.

c) Which teams could finish with a greater number of wins than the Majors? Explain.

d) What teams could end up in a tie for the highest standing? Explain.

e) What is the highest place the Generals could obtain in the final standings? Explain.