

Problem



Hockey Games (Suggested for groups of 2 to 4 students)

Here are the current standings of the top four teams in the North-Eastern Hockey League:

Team	Wins	Losses	Points
Majors	15	2	30
All-Stars	13	4	26
Champs	12	5	24
Generals	10	7	20

No tie games are allowed in this league.

- Each team has three games left, one game against each of the other teams in the top four. How many games must be played? Make a list of the games that must be played.
- If the Majors lose all three games, what is the highest place they could achieve in the final standings? Explain.
- Which teams could finish with a greater number of wins than the Majors? Explain.
- What teams could end up in a tie for the highest standing? Explain.
- What is the highest place the Generals could obtain in the final standings? Explain.



Hints

Part a)

Hint 1 - In listing the games to be played, be careful not to list the same game twice.

Part b)

Hint 1 - Remember that when one team wins, another team loses.

Part c) & e)

Hint 1 - Remember that there has to be a total of six wins and 6 losses.

Solution

Current Standings are:

Team	Wins	Losses	Points
Majors	15	2	30
All-Stars	13	4	26
Champs	12	5	24
Generals	10	7	20

- a) There are six games left to play, as follows: 1. Majors vs All-Stars, 2. Majors vs Champs, 3. Majors vs Generals, 4. All-Stars vs Champs, 5. All-Stars vs Generals, 6. Champs vs Generals.
- b) Since the standing is determined solely by the number of wins, we can ignore the losses and points. Further, since we're asked for the highest the Majors can place, we first see whether they can stay on top.

This could happen for the following final standings in the Wins column:

Majors	15	15	15
All-Stars	14 (1 win)	15 (2 wins)	15 (2 wins)
Champs	14 (2 wins)	14 or 13 (2 or 1 win)	15 (3 wins)
Generals	13 (3 wins)	12 or 13 (2 or 3 wins)	11 (1 win)

Note that the last two columns are ties for first place.

- c) Since there are only three more games for each team, the only team that could surpass the Majors is the All-Stars, if they win all three games, and the Majors lose all three of their games. In this case the final standings in the Wins column could be either of the following:

Majors	15
All-Stars	16 (3 wins)
Champs	13 or 12 (2 or 1 win)
Generals	12 or 13 (1 or 2 wins)

- d) The Majors and All-Stars could tie for highest standing, or the Majors, All-Stars and Champs could have a three-way tie, as shown in the following final standings in the Wins column:

Majors	15	16 (1 win)	15
All-Stars	15 (2 wins)	16 (3 wins)	15 (2 wins)
Champs	14 or 13 (2 or 1 win)	14 or 13 or 12 (2, 1 or 0 wins)	15 (3 wins)
Generals	12 or 13 (2 or 3 wins)	10 or 11 or 12 (0, 1 or 2 wins)	11 (1 win)

- e) The Generals could end up in a three-way tie for second place by winning all three of their games, as shown in the following final standings for the Wins column:

Majors	18 (3 wins)	17 (2 wins)
All-Stars	13	13
Champs	12	13 (1 win)
Generals	13 (3 wins)	13 (3 wins)

Comment: In each case, the final win column must total 56. Thus one way to explain possible outcomes is to add 0, 1, 2, or 3 to each team's original number of wins.