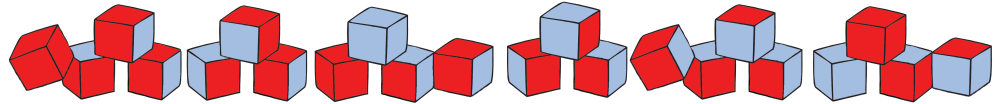


Problem



Reena is making cubes for a game in which the cubes are selected from a bag. She has red paint and blue paint. She paints each face of a cube either red or blue. How many different cubes can Reena make? (Two cubes are not considered different if one can be rotated to match the other.)

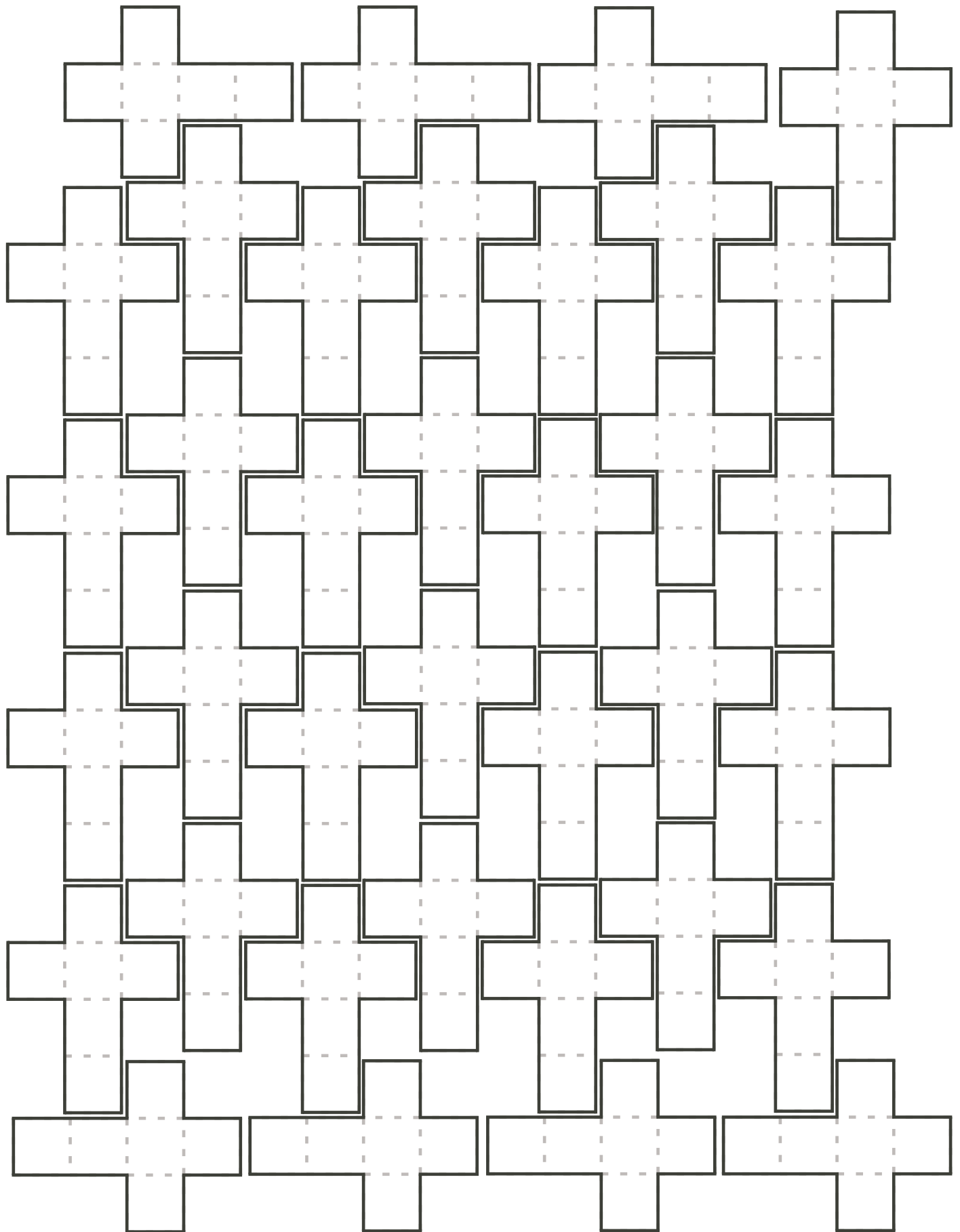


Extension:

Suppose Reena makes one face yellow. Could she make at least 10 different cubes with the remaining faces each painted either red or blue?

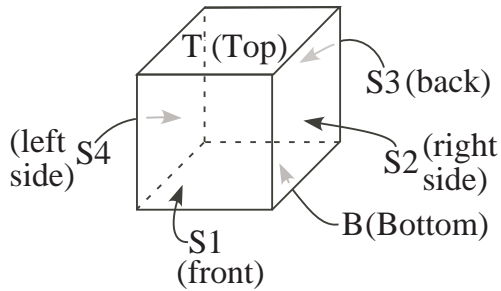
Hints

Suggestion: Give the students a page of cube nets (see below) to colour in different ways and form into cubes. Have them work in groups if desired. Encourage them to compare one another's cubes, rotating them to see whether their colourings are truly different.



Solution

There are 10 ‘different’ cubes Reena can make as follows:

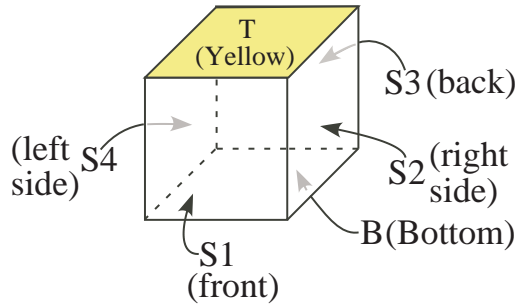


- 2 cubes of solid colours, 1 red and 1 blue;
- 2 cubes with top in one colour and the other 5 faces in the other colour;
- 1 cube with T, S1, S2 in red and B, S3, S4 in blue (i.e., 3 faces which meet in a corner with the same colour, the other 3 the second colour);
- 2 ‘banded’ cubes, with S1, S2, S3, S4 all the same colour, and T and B the second colour;
- 2 cubes with 2 adjacent sides, say S1 and S2, the same colour, and the other 4 sides the second colour;
- 1 cube with a ‘partial band’, say S1, S2, S3 of one colour and the remaining ‘partial band’ B, S4, T in the second colour.

Coloured cube nets can also be used to show that there are only 10 such cubes.

Extension:

1. Allowing for one face to be yellow permits many more cubes. Here are some possibilities:



- 2 with all five remaining sides in one colour (red or blue);
- 2 with 1 adjacent side (say S1) in red (blue), and the other four in blue (red);
- 2 with 2 adjacent sides (S1, S2) in red (blue) and the other three faces in blue (red);
- 2 with 1 adjacent side (S1) and the bottom B in red (blue) and the other three sides in blue (red);
- 2 with opposite sides (S1 and S3) in red (blue), and the remaining faces (S2, B, S4) in blue (red).

Encourage students to come up with more than 10 different cubes. Colouring cube nets is, again, a very good way for students to discover the many possibilities. Here is one possible set of nets for the above ten possibilities plus two more with all four sides S_1, S_2, S_3, S_4 of red (blue) and the reverse.

