Problem

The area of one side of an Emmy-Os single-serving cereal box is 96 cm$^2$. The area of another side of the same box is 48 cm$^2$. The area of the top of the box is 32 cm$^2$. What is the volume of the box if the length of each edge is a whole number?
Hints

**Hint 1** - Is it possible to draw a diagram of the box?

**Hint 2** - If this box is similar in shape to a cereal box, what shape are the faces?  
How do you find the area of these faces?

**Hint 3** - What are possible lengths and widths for the top of the box, to make an area of 32 cm²?  
Which of these possibilities are reasonable?

**Hint 4** - Remember that the length of one side must match at least one length of the other side and of the top.
Solution

Since each edge length is a whole number, we examine the possible factors of each of the given areas, each area being the product of two lengths. The possibilities are:

<table>
<thead>
<tr>
<th>Side</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side 1</td>
<td>96 cm²</td>
</tr>
<tr>
<td>Side 2</td>
<td>48 cm²</td>
</tr>
<tr>
<td>Top</td>
<td>32 cm²</td>
</tr>
</tbody>
</table>

Now we need to select three lengths $a, b, c$ which appear in pairs among the products of factors, say, $a, b$ for side 1, $b, c$ for side 2, and $c, a$ for the top. Since the top has the fewest possibilities, it is sensible to start with those. If we select $2 \times 16$, then side 2 has to be $2 \times 24$ (or $3 \times 16$), and side 3 has to be $24 \times 16$ (or $2 \times 3$), neither of which gives $96$ cm². So the top must be $4 \times 8$; then side 2 is $4 \times 12$ (or $6 \times 8$), and side 3 is $8 \times 12$ (or $6 \times 4$), of which only $8 \times 12 = 96$. So the dimensions of the box are 4 cm by 8 cm by 12 cm, and its volume is $4 \times 8 \times 12 = 384$ cm³.