Part II: For the Teacher

Curriculum Areas

Problem 1 - Number Sense

Problem 2 - Pattern and Algebra

Problem 3 - Measurement and Geometry

Problem 4 - Logic and Number Sense

Problem 5 - Geometry

Problem 6 - Logic and Number Sense

Hints and Suggestions:

Problem 1

Hint 1 - What number is the only reasonable one for the last blank? OR Which blank in the story should be filled with 85?

Hint 2 - Who is the oldest? Which number should represent her age?

Problem 2

Hint 1 - Would a place value chart be helpful?

Extension:
How would you adapt your place value chart?

<table>
<thead>
<tr>
<th>Multiple</th>
<th>100’s digit</th>
<th>10’s digit</th>
<th>1’s digit</th>
</tr>
</thead>
<tbody>
<tr>
<td>99</td>
<td>0</td>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

Problem 3

Hint 1 - Which of the five plots should Aunt Sybil fit first into her plan? Why?

Extension 1a):
Hint 1 - If you place the 5 × 5 square plot at one end, will there be enough space for the 4 × 4 square and the 3 × 3 square?

Extension 1 b):
Hint 1 - What is the total area required for all five plots?

Hint 2 - What is the area of the garden?

Problem 4

Hint 1 - What is the ones digit for any multiple of 5?

Hint 2 - What multiples of 4 and 7 could also be multiples of 5?

Hint 3 - How can you check your answer?
Problem 5

**Hint 1** - Can your congruent pieces have shapes other than squares, rectangles, or triangles?

*Suggestion:* Provide the students with graph paper or geopaper if desired.

*Extension:*
*Suggestion:* Carefully cut out four of each shape and try to assemble them into a square. Mark the top side with an “X”, and do not ‘flip’ the pieces. Supply students with graph paper so they can sketch their solutions accurately.

Problem 6 a)

**Hint 1** - In which innings did the Jays score runs?

**Hint 2**  How many runs did the Jays score in each inning?

**Hint 3**  How many runs did the Tigers score in the third inning?

Problem 6 b)

**Hint 1** - What was the final score?

**Hint 2**  In which three innings did the Tigers score runs?

*Extension:*

**Hint 1** - Work backwards. Check your answers by making sure they fit the clues as you reread the question.

*Suggestion:* Have students trade their problems with other teams to check whether their clues work (i.e., determine a unique score chart).

**Solutions**

**Problem 1**

The blanks should be filled, in order, with 35 (Bev’s age), 2, 3 (Jennifer’s age), 8 (John’s age), 6 (Robin’s age), 33 (Mike’s age), and 85 (the sum 3 + 6 + 8 + 33 + 35).

*Note:* There are two possibilities for Robin and Jennifer: since 6 = 3 × 2, Robin could be 6 and Jennifer 2 years old, or Robin could be 6 and Jennifer 3 years old. The proper choice is determined by the sum, 85 years, since 85 = 35 + 33 + 8 + 6 + 3.

**Problem 2**

The first five multiples of 99 are 99, 198, 297, 396, 495. Thus the place value patterns are:

- the ones digit decreases by 1 with each multiple;
- the tens digit remains 9 in each multiple;
- the hundreds digit increases by 1 with each multiple.

Hence the next four multiples are 594, 693, 792, and 891.
Extensions:
1. The next multiples are 990, 1089, 1188, 1287, 1386, 1485, 1584, 1683, 1782, 1881, 1980, 2079, 2178, 2277, 2376, 2475. So, for the multiples from $11 \times 99$ to $20 \times 99$, the patterns are:
   - the ones digit decreases by 1 with each multiple, going from 9 to 0;
   - the tens digit remains 8;
   - the hundreds digit increases by 1, going from 0 to 9;
   - the thousands digit remains 1 in each multiple.

2. From $21 \times 99$ to $25 \times 99$, the patterns for the ones and hundreds digit are the same as for $11 \times 99$ to $15 \times 99$, the tens digit decreases to 7, and the thousands digit changes to 2.

3. We can conclude from our observations in 1. that with each ten multiples of 99 (i.e., each decade of multiples), the patterns for the ones and hundreds digits are the same within each decade. But the tens digit decreases by 1 for each decade (i.e., the tens digit is 9 for $1 \times 99$ to $10 \times 99$, 8 for $11 \times 99$ to $20 \times 99$, 7 for $21 \times 99$ to $30 \times 99$, etc), while the thousands digit is 0 for $1 \times 99$, to $10 \times 99$, 1 for $11 \times 99$ to $20 \times 99$, 2 for $21 \times 99$ to $30 \times 99$, etc). Hence $89 \times 99$ would be 8811.

4. The sum of the digits in each product is 18, due to the fact that the sum of the thousands digit and the tens digit is always 9, and the sum of the hundreds digit and the ones digit is always 9.

Problem 3

Yes, Aunt Sybil can fit all the desired square patches in a 5 metre by 12 metre garden plot. One way to do this is shown at right.

Extension:

1 a) If the plot is only 5 metres by 11 metres, then she cannot fit all five square patches, as there is no way to fit both the $3 \times 3$ metre and $4 \times 4$ metre patches in the $5 \times 6$ metre rectangle remaining after the $5 \times 5$ metre patch is placed.

1 b) If the patches for each herb do not have to be squares, then she needs the following areas:
   - 25 square metres for parsley;
   - 16 square metres for basil;
   - 9 square metres for thyme;
   - 4 square metres for sage;
   - 1 square metre for rosemary.

This gives a total of 55 square metres, which is all the area of a $5 \times 11$ metre plot. Thus there are many ways she can fit the desired areas; two are shown below.

Suggestion: Give students the right-hand diagram below, and challenge them to explain why the diagonal patches have the correct areas.
Problem 4

If Lloyd has $n$ marbles, then $(n + 1)$ must be a multiple of 4, and of 5, and of 7. The least multiple of these is 140. Thus Lloyd has 139 marbles.

Problem 5

Six ways to divide a square into four congruent pieces are shown below. Once students discover either of the last two, they will realize there are countless ways to do this subdivision, using symmetrical divisions of the square with lines intersecting at the centre.

*Extension:* Templates 1, 2, 3, 4, 5, and 7 can each be used to form an $8 \times 8$ square.
Problem 6

a) Inning

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<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
</tr>
</tbody>
</table>

b) There are three possible solutions:

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<th>4</th>
<th>5</th>
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<th>Total</th>
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<tbody>
<tr>
<td>Tigers</td>
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<td>0</td>
<td>2</td>
<td>2</td>
<td>x</td>
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</tbody>
</table>

**Extension:**

*Note to the Teacher:* Students may have difficulty creating relevant clues which do not give away the answer. You may wish to do this as a class activity so that these issues arise naturally and can be discussed. One possibility would be to present them with a completed score chart, and then work backwards to create an appropriate set of clues.