Problem

a) Make a list of all the factors of 24. How many different triangles can be formed using any three of these numbers as the side lengths?

b) Write a sentence explaining why not all combinations of any three numbers from your list in a) will form a triangle.
Hints

**Hint 1** - Can you form a triangle with sides of lengths 2, 3, and 6? Why or why not?

*Suggestion:* Supply students with about 30 toothpicks each, with a single toothpick having unit length, and suggest they try to form the triangles.
Solution

a) The factors of 24 are 2, 3, 4, 6, 8 and 12. Thus the five combinations which can form triangles are \{2, 3, 4\}, \{3, 4, 6\}, \{3, 6, 8\}, \{4, 6, 8\} and \{6, 8, 12\}.

b) The key idea is that the sum of the lengths of any two sides must be greater than the third side.
For example, \{2, 4, 6\} can’t form a triangle because 2 + 4 = 6, so the sides \(\frac{2}{6}, \frac{4}{6}\) do not ‘contain’ any area. Similarly for the set \{2, 8, 12\}, 2 + 8 = 10 < 12 \(\frac{2}{12}, \frac{8}{12}\), so no triangle can be formed.
This is a famous mathematical theorem known as the ‘Triangle Inequality’: If a, b, c are the lengths of the sides of a triangle, then \(a + b > c\).