Problem

Aunt Sybil wants to plant herbs to sell at the market. The herb garden is to be a rectangular plot measuring 5 metres by 12 metres. She wants five different herbs, each in a square patch, as follows:

- a 1 m by 1 m square of rosemary;
- a 2 m by 2 m square of sage;
- a 3 m by 3 m square of thyme;
- a 4 m by 4 m square of basil;
- a 5 m by 5 m square of parsley.

Will all of these fit in her plot? Explain your reasoning, and sketch a plan for her garden if possible, using the grid below. How many square metres does Aunt Sybil have unplanted?

 Extensions:

1.  a) If Aunt Sybil’s plot is only 5 metres by 11 metres, would her five square patches fit? Explain.
b) What if she decides the individual plots do not have to be squares, but must have the same areas as above? Can she then fit them into a $5 \times 11$ plot? Explain.
Hints

**Hint 1** - Which of the five plots should Aunt Sybil fit first into her plan? Why?

*Extension 1a)*:

**Hint 1** - If you place the $5 \times 5$ square plot at one end, will there be enough space for the $4 \times 4$ square and the $3 \times 3$ square?

*Extension 1 b)*:

**Hint 1** - What is the total area required for all five plots?

**Hint 2** - What is the area of the garden?
Solution

Yes, Aunt Sybil can fit all the desired square patches in a 5 metre by 12 metre garden plot. One way to do this is shown at right.

Extension:

1 a) If the plot is only 5 metres by 11 metres, then she cannot fit all five square patches, as there is no way to fit both the $3 \times 3$ metre and $4 \times 4$ metre patches in the $5 \times 6$ metre rectangle remaining after the $5 \times 5$ metre patch is placed.

1 b) If the patches for each herb do not have to be squares, then she needs the following areas:

- 25 square metres for parsley;
- 16 square metres for basil;
- 9 square metres for thyme;
- 4 square metres for sage;
- 1 square metre for rosemary.

This gives a total of 55 square metres, which is all the area of a $5 \times 11$ metre plot. Thus there are many ways she can fit the desired areas; two are shown below.

Suggestion: Give students the right-hand diagram below, and challenge them to explain why the diagonal patches have the correct areas.