Part II: For the Teacher

Curriculum Areas

Problem 1 - Number Sense

Problem 2 - Number Sense and Data Management

Problem 3 - Number Sense

Problem 4 - Geometry and Measurement

Problem 5 - Number Sense

Problem 6 - Geometry and Spatial Sense

Hints and Suggestions:

Problem 1 a)

Hint 1 - What does "divide me evenly" mean?

Hint 2 - Do you have to check divisibility by both 4 and 8? Why/or why not?

Problem 1 b)

Hint 1 - If a number has both 2 and 7 as factors, what other number must also be a factor?

Suggestion: Students may approach these problems in different ways, the simplest of which is to write down all the multiples of 4 and 8 in part a), and of 7 and 2 in b). It would perhaps be worthwhile to do a) together as a class, and discuss why the desired number need only be divisible by 8.

Problem 2 b)

Hint 1 - If a player scored 20 points in 10 games, how many points per game did she score?

Suggestion: To help students with parts b) and c), you may wish to review how to round decimal numbers to the nearest tenth.

Problem 3 a)

Hint 1 - How many vehicles were there in total, if the parking lot was $\frac{5}{8}$ full?

Problem 3 c)

Hint 1 - How many spaces are there for cars?

Problem 4 a)

Hint 1 - Use the grid on your model to see how many lengths of 2 cm occur on each of the width, height, and depth of the table.

Suggestion: Manipulatives such as Cube-A-Links could be used to make a three-dimensional model of the cheese block.

Problem 4 b)

Hint 1 - Which blocks DO have wax on them? How many are there?

Problem 5

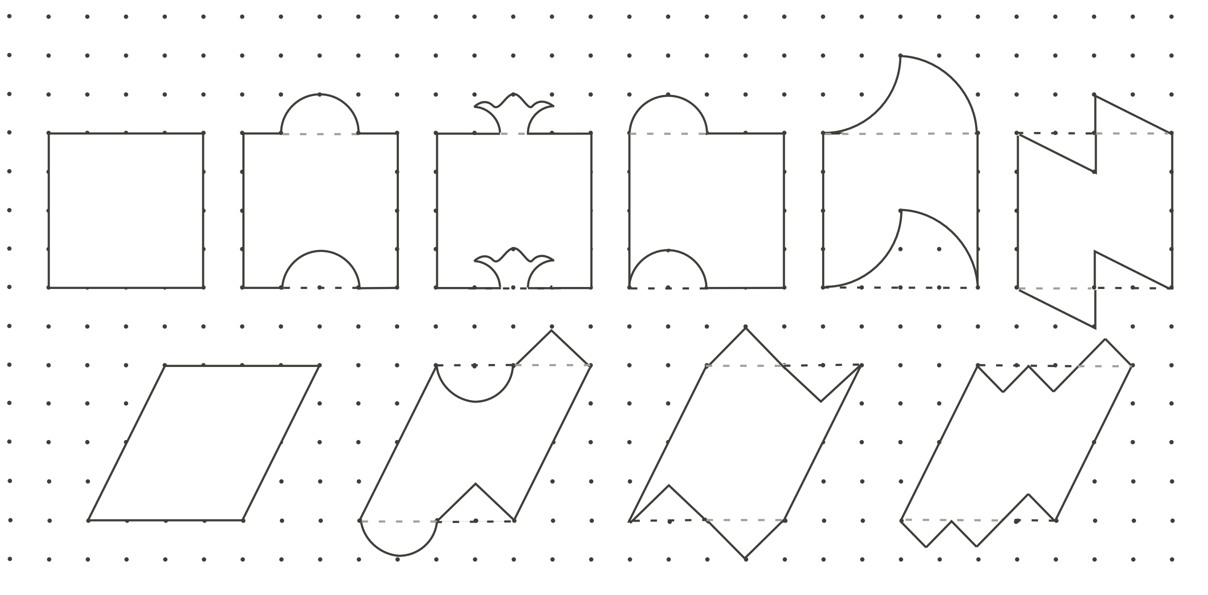
Suggestion: The list from the hundred chart, and possibly part a), could be done with the class as a whole, to introduce the more thought-provoking ideas in parts b) and c).

Extension 2:

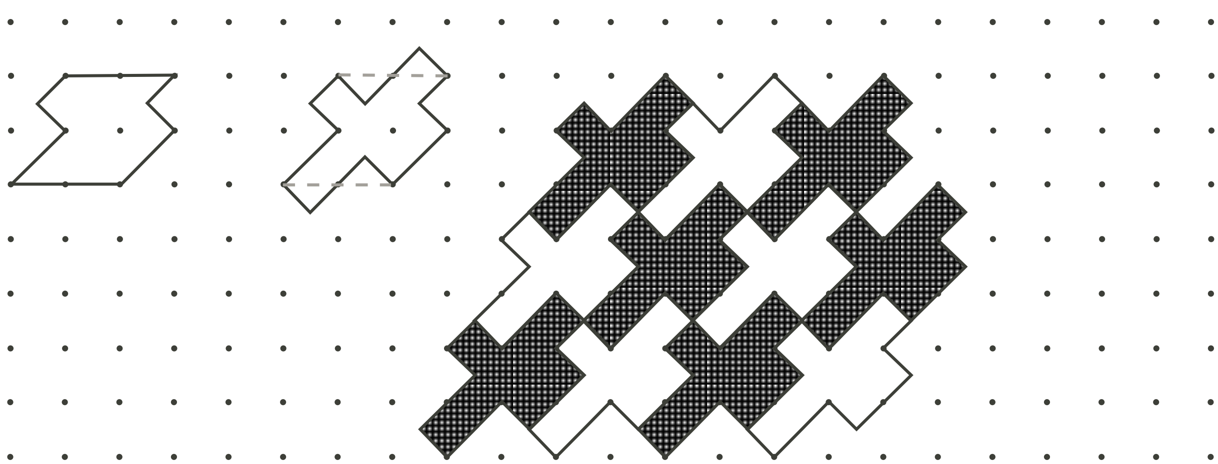
Hint 1 - What do you know about the sum of the digits of a number that is divisible by 3?

Problem 6

Suggestions:
It may be beneficial to start with tesselations formed by re-shaping only one pair of opposite sides in an identical way. Here are some sample starters:



Once you have discussed the example given in the problem, you may also wish to use the alternate shape [2] as the basis for a second example. Here is one possibility:



This is a completely open-ended activity with a strong artistic component.

Solutions

Problem 1

a) The only two-digit numbers less than 30 and divisible by both 4 and 8 are 16 and 24. Since the sum of the two digits must be greater than 6, the number must be 16.

b) The only number greater than 30 and less than 50, divisible by both 7 and 2 (i.e., by 14) is 42.

Extension:

The three-digit multiples of 11 which are also less than 200 are 110, 121, 132, 143, 154, 165, 176, 187, and 198. The one with digits having a sum of 12 is 165.

Problem 2

a) Gretzky's total for his first three years is 513 points, while Crosby's total for his first two years is 222 points. So Crosby will need 513 - 222 = 291 points in 2007-08 to equal Gretzky's record.

b) Gretzky's 1981-82 points per game equals $\frac{212}{80} = 2.65\ldots \approx 2.7$, and his goals per game equals $\frac{92}{80} = 1.15\ldots \approx 1.2$. (Note: The symbol '$\approx$' means 'approximately equals'.)

c) The percentages of points that were goals are:
for Gretzky, in 1979-80, $\frac{51}{137} \times 100\% \approx 37.2\%$;
for Crosby, in 2005-06, $\frac{39}{102} \times 100\% \approx 38.2\%$.

Extension:

Encourage students to explore the statistics more fully before making a decision about who they think is the better player (e.g, repeat b) and c) for other years, and both players). There will no doubt be an emotional element to these discussions!

Problem 3

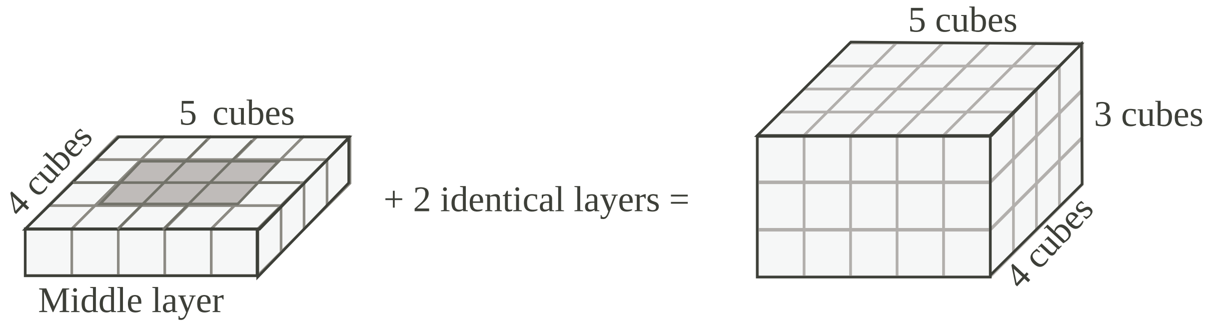
a) If the parking lot was $\frac{5}{8}$ full, then there was a total of $\frac{5}{8}$ of 800 which is equal to 500 vehicles in the lot. Since 140 of them were trucks, there were 500 - 140 = 360 cars in the lot.

b) To solve part a), we did not need to know that one-quarter of the spaces in the lot are designated for trucks.

c) Since one-quarter of the spaces in the lot are designated for trucks, three-quarters of the spaces, or $\frac{3}{4}$ of 800 which is equal to 600 spaces were empty, a fraction of $\frac{240}{600} = \frac{2}{5}$.

Problem 4

a) In terms of two-centimetre cubes, the block's dimensions are 5 cubes wide by 3 cubes high by 4 cubes deep. Thus we can picture it as consisting of 3 layers, each with 20 cubes. Thus, in total, there are 60 such cubes in the block.



b) Since any exterior cube has wax on it, all of the cubes in the top layer and the bottom layer will have wax on them. Further, since the vertical exterior sides also have wax on them, only six cubes in the middle layer have no wax, as shown by the shaded cubes in the left diagram. Thus there are 3 x 2 = 6 cubes that are entirely interior to the block of cheese, and hence have no wax on them.

Problem 5

a),b) The table below shows the list of remaining (prime) numbers and their digit sums.

No.	2	3	5	7	11	13	17	19	23	29	31	37	41	43
Digit Sum	2	3	5	7	2	4	8	1	5	2	4	1	5	7

No.	47	53	59	61	67	71	73	79	83	89	97
Digit Sum	2	8	5	7	4	8	1	7	2	8	7

Note that:

- the factors of each of the remaining (prime) numbers are just 1 and the number itself;
- each of the numbers 2, 11, 29, 47, and 83 has digit sum 2;
- none of the numbers has digit sum 6.

c)(i) If we consider numbers 1 to 9, only number 6 has a digit sum of 6. We now consider the sum of the digits of numbers 10 to 99. For these numbers, the sum of the digits varies from 1 (for number 10) to 18 (for number 99). If this sum is equal to 1, 2, 3, 4, 5, 6, 7, 8 or 9, it is also the digit sum of the number. Therefore if, initially, the sum of the digits of the number is equal to 6, the digit sum of the number is also equal to 6. If the sum of the digits is initially equal to 10, 11, 12, 13, 14, 15, 16, 17 or 18, then only 15 gives a digit sum of 6.b Therefore if a number from 1 to 99 has a digit sum of 6, then the sum of its digits must be equal to 6 or to 15.

(ii) Numbers whose digits have sum 6 are 6, 15, 24, 33, 42, 51 and 60. Numbers whose digits have sum 15 are 69, 78, 87 and 96. Therefore, numbers 6, 15, 24, 33, 42, 51, 60, 69, 78, 87 and 96 have a digit sum of 6.

Extension:

1. Consider the sum of the digits for the numbers 101 to 199, which varies from 2 to 19. As in part c), it must be equal to 6 or to 15 to get a digit sum of 6. To find the numbers whose sum of digits is equal to 6 or to 15, we can work more rapidly if we only look at the units digit and the tens digit whose sum must be equal to 5 or to 14, since the hundreds digit is 1. These numbers are 105, 114, 123, 132, 141, 150, 159, 168, 177, 186 and 195. There are 11 such numbers.

2. We know that a number is divisible by 3 if the sum of its digits is divisible by 3. From part c), we know that numbers whose digit sum is equal to 6 are such that the sum of their digits is equal to 6 or to 15. Since these two sums are divisible by 3, the numbers whose digit sum is equal to 6 must be divisible by 3, and hence will never be prime numbers. Similarly, if the digit sum of a number is 9, then the number is divisible by 9, and hence cannot be prime.

Problem 6

This problem offers limitless variation in solutions. Two possibilities have been offered in the original problem, and in the Hints above. Since drawing the tesselation may be challenging, students may want to discuss their initial attempts with one another or the teacher. Colouring can also be quite effective here... encourage creative play! You may also wish to explore the tessellations for symmetry.