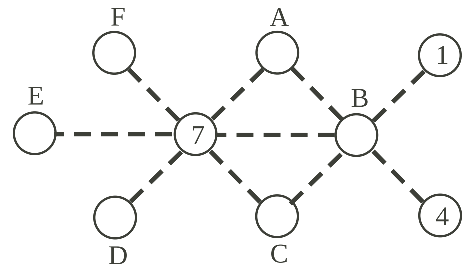
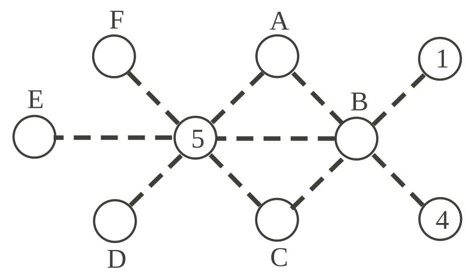
Problem

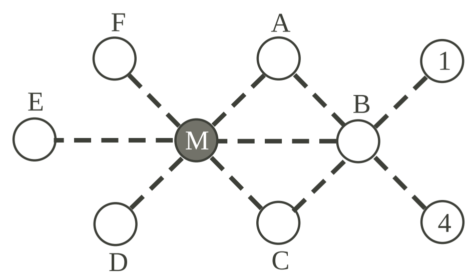


a) In the diagram at right, the sum of the three numbers on each of the five straight lines is equal to 18. Each of the numbers 1,2,3, . . . ,9 is each used once, and only once. Find the numbers which could go in the circles labelled A,B,C,D,E,F.



b) Compare your solution with those of several classmates. Did anyone find a solution different from yours?

c) Try to solve the problem with the 7 replaced by 5, as shown at right. What happens?



Extension:

1. Explore the possible solutions when the number in the shaded circle M is something other than 7 or 5. Hence show that 7 is the only possible choice for which there are solutions to this problem.

Hints

Part a)

Hint 1 - What numbers could go in circles B and C?

Hint 2 - What numbers could go in circles A and D?

Part c)

Hint 1 - What number MUST go in circle B?

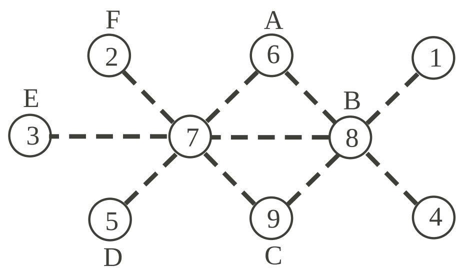
Extension:

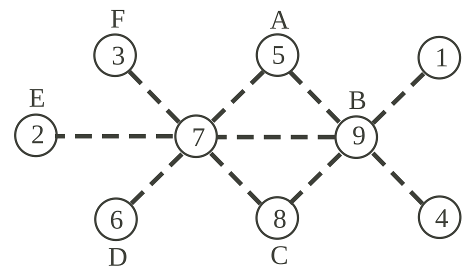
Hint 1 - Start with the one of the two known possibilities for circle B. What does that tell you about A and C?

Hint 2 - Pick a value for circle M. Does it work?

Solution

a),b) There are only two possible numbers for circle B, namely, 8 or 9. Once B is chosen, A, C, and E are fixed, which in turn determines D and F. The two possible solutions are:

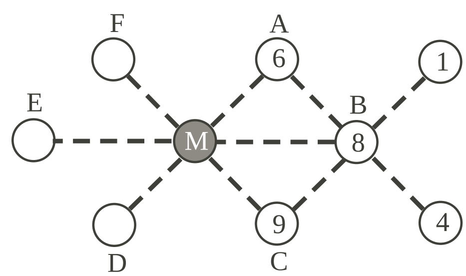




c) When the centre-left number is 5 instead of 7, we have two possible scenarios for the horizontal line of three circles. If B = 8, then 8 + 5 + E = 18 would require E = 5, which is already used. If B = 9, then 9 + 5 + E = 18 would require E = 4, which is already used. Thus no solutions are possible when 7 is replaced with 5.

Extension:

1. To show why 7 is the only possible choice, just try the possibilities one-by-one. There are still only two possible values for B, namely, 8 or 9. Here is the argument for B = 8, with initial configuration:



The four numbers available for the remaining four spaces are 2, 3, 5, and 7. So all we need to do is try each of these four for one of the spaces. Since we have already seen that M = 7 has two solutions, and M = 5 has none, we need only test M = 2 and M = 3. If we choose M = 2, then B + M + E = 18 gives E = 18 - 8 - 2 = 8, which is already used. If M = 3, then A + M + D = 18 gives D = 18 - 6 - 3 = 9, which is already used. Thus, if B = 8, the only possible choice of M is 7.

A similar argument shows that B = 9 also yields no solutions other than M = 7.