Part II: For the Teacher

Curriculum Areas

Problem 1 - Measurement and Number Sense

Problem 2 - Number Sense and Pattern/Algebra

Problem 3 - Probability and Number Sense

Problem 4 - Data Management and Algebra and Number Sense

Problem 5 - Geometry

Problem 6 - Problem Solving and Geometry

Hints and Suggestions:

Problem 1 a)

Hint 1 - How many 100 metre races would make 1000 metres?

Problem 1 b)

Hint 1 - How many 100 metre races would make a marathon?

Hint 2 - How many seconds are there in one minute? In one hour?

Problem 1 c)

Hint 1 - How many minutes does it take the cheetah to run 1 kilometre?

Problem 2

Suggestion: Before beginning the problem, discuss with the class whether numbers with hundreds digit 0 are to be included (e.g., 077). The solutions below have assumed they are NOT allowed.

Problem 2 a)

Hint 1 - What pairs of digits have a sum of 3?

Problem 2 b)

Hint 1 - What pairs of digits have a sum of 5?

Problem 2 c)

Hint 1 - What will be the hundreds digit of the least 'Lorna' number? Of the greatest?

Problem 2 d)

Hint 1 - How many 'Lorna' numbers have 1 as the hundreds digit? How many have 2 as the hundreds digit? How many have 3? Can you see a pattern?

Suggestion: For a more challenging version of this problem, instead of starting with the definition of a 'Lorna' number, pose this initial question:

These are 'Lorna' numbers: 202, 312 440, 523, 514, 752.
These are NOT 'Lorna' numbers: 222, 311, 443, 521, 732, 908.
Write the definition of a 'Lorna' number.

Have students procede with parts a), b), c), d) as given.

***Extension* :**

Suggestion: Ask students the same questions as suggested in the Hints for 'Lorna' numbers. Do they have the same answers for 'Dennis' numbers?

Problem 3 a)

Hint 1 - If you add an odd number and an even number, will the sum be odd or even?

Hint 2 - How many sums are possible in total from a roll of the two dice?

Problem 3 b)

Hint 1 - If you multiply a number from one die by a number from the other, what sort of number will you get for the product?

Hint 2 - How many products are possible in total from a roll of the two dice?

Suggestion: Listing the possibilities in tree form may be helpful for parts a)(ii) and b)(ii).

Problem 4 b)

Hint 1 - If the average number of figures owned is 30 per person, how many figures do the four children own in total?

Problem 4 c)

Hint 1 - If the average number of figures owned is 32 per person, how many figures would the four children have to own in total?

***Extension* :**

Hint 1 - If the average number of figures owned is 27 per person, how many figures would the four children own in total?

Hint 2 - What is the total number of figures owned by Billy-Bob and Buffy-Sue?

Problem 5

Suggestions: Discuss part a) with the class as a whole, and then let them try part b). If students have difficulty with part c) (or the Extension), have them cut out the three nets and try various possibilities with pencils (and erasers). Alternatively, blank dice and erasable markers could be used.

Extension **:** Emphasize that students should try to place the letters **M, A, T, H** appropriately without cutting out the net. Then have them do so to verify that the letters of the word **MATH** do appear, in the correct order, on the vertical sides of the resulting cube.

Problem 6

Hint 1 - How many 90° angles do the hands make between noon and 1 p.m.? Between 5 and 6 p.m.?

Hint 2 - How many 90° angles do the hands make between 2 p.m. and 4 p.m.?

Suggestions:

1. Have students create a model clock by following the directions on the problem sheet. They will need a piece of cardboard or a cork board in order to pin and manipulate the hands freely.
2. If students have access to the web, you may wish to suggest the following web reference from the National Library of Virtual Manipulatives, which contains a virtual clock manipulative: http://nlvm.usu.edu/en/nav/frames_asid_316_g_2_t_4.html?from=category_g_2_t_4.html

Solutions

Problem 1

a) Since 1000 m = 10 × 100 m, if Usain Bolt could maintain his 100 m pace for 1000 m, his time would be 10 × 9.69 = 96.9 seconds.

b) Since 42.2 km = 42,200 m = 422 × 100 m, his time for the marathon would be 422 × 9.69 = 4089.18 s; 4089.18 s ÷ 60 s/min = 68.153 min; 68.153 min ÷ 60 min/hr ≈ 1.136 hr; 0.136 hr × 60 min/hr ≈ 8 min, so his time is about 1 hour and 8 minutes.

c) The cheetah's time for the marathon would be 42.2 ÷ 120 ≈ 0.35 hours ≈ 21 minutes.

Extension **:** Since the runners are doing 400 metre races, we would expect them to run more slowly overall than someone running 100 metres. Thus we would expect their average times for 100 metres to be greater than Usain Bolt's time for 100 metres.

Problem 2

a) Possible Lorna numbers with hundreds digit equal to 3 are: 303, 312, 321, and 330.

b) Possible Lorna numbers with hundreds digit equal to 5 are: 505, 514, 523, 532, 541 and 550.

c) The least possible Lorna number is 101; the greatest is 990.

d) Using a chart to record all possible 'Lorna' numbers reveals a pattern:

Hundreds Digit	Possible 'Lorna' Numbers	Number of 'L' Numbers
1	101, 110	2
2	202, 220, 211	3
3	303, 312, 321, 330	4
4	404, 413, 422, 431, 440	5
...	...	...
9	909, 918, 927, 936, 945, 954, 963, 972, 981, 990	10

Thus the total number of 'Lorna' numbers is 2+3+4+5+6+7+8+9+10=54.
Clearly, for each hundreds digit H there are H+1 'Lorna' numbers H T U with T = H - U, or H = T + U, giving possible values T = 0,1,2,. . . ,H while U = H, H-1, H-2,. . . ,0. For example, for H = 7, the 8 'Lorna' numbers are 707, 716, 725, 734, 743, 752, 761, 770.

***Extension* :**

A chart recording all possible 'Dennis' numbers also reveals a pattern:

Units Digit	Possible 'Dennis' Numbers	Number of 'Dennis' Numbers
1	101	1
2	112, 202	2
3	123, 213, 303	3
4	134, 224, 314, 404	4
...	...	...
9	189, 279, 369, 459, 549, 639, 729, 819, 909	9

Clearly, for each units digit U there are exactly U 'Dennis' numbers H T U, with T = U - H, or U = H + T, giving possible values H = 1,2,3,...,U while T = U-1, U-2,...,1,0. (Note that we have not permitted H = 0, i.e., 022 is not allowed, even though T=2, U=2, H=0 gives T=U-U.) For example, if U = 7, the 7 'Dennis' numbers are 167, 257, 347, 437, 527, 617, 707.
Thus the total number of 'Dennis' numbers is 1+2+3+4+5+6+7+8+9=45, so it is NOT the same as the number of 'Lorna' numbers.

Problem 3

Die 1 and Die 2 have numbers as shown:

Die 1	1	3	5	7	9	11
Die 2	2	4	6	8	10	12

a) Hamed rolls the dice and adds the two numbers:

(i) Odd sums will occur in every case, since the sum of an even number and an odd number is always odd. Thus the probability is $\frac{36}{36} = 1$ (i.e., it is a certain event that the sum will be odd).

(ii) Here is a table showing the possible sums:

Die 2 \ Die 1	1	3	5	7	9	11
2	3	5	7	9	11	13
4	5	7	9	11	13	15
6	7	9	11	13	15	17
8	9	11	13	15	17	19
10	11	13	15	17	19	21
12	13	15	17	19	21	23

Thus the total possible number of sums less than 15 is 1 + 2 + 3 + 4 + 5 + 6 = 21, out of a total of 6 × 6 = 36 possible sums. Hence the probability of a sum less than 15 is $\frac{21}{36} = \frac{7}{12}$, a chance of 7 in 12.

Note to the Teacher: Students could also use trees to list the possible sums.

b) Hamed rolls the dice and multiplies the two numbers.

(i) Since multiplying an even number by an odd number always gives an even number, there are no possible odd products. Thus the probability is $\frac{0}{36} = 0$.

(ii) A product of 18 can be obtained in two ways, 3×6, or 2×9. Thus the probability is $\frac{2}{36} = \frac{1}{18}$.

***Extension* :**

Hamed: Odd numbers are only divisible by odd numbers. Thus no number on Die 2 can be evenly divided into by an odd number on Die 1. However, odd numbers can divide even numbers. Examining divisors from Die 1, possible divisions are:

- 1 into any of 2, 4, 6, 8, 10, or 12, giving 6 pairs;
- 3 into 6 or 12, giving 2 pairs;
- 5 into 10, giving 1 pair.

Thus there are 9 possible pairs in total, giving a probability of $\frac{9}{36} = \frac{1}{4}$ of Hamed getting a point.

Beth: Examining the table of sums for part a) (ii) above, we see that the only sums with digit sums divisible by 4 are 13 ($1 + 3 = 4$), and 17 ($1 + 7 = 8$). Since 13 occurs 6 times, and 17 occurs 4 times, there are 10 possibilities. Thus the probability of Beth getting a point is $\frac{10}{36} = \frac{5}{18}$.

Conclusion: The game is not fair since Beth has a greater chance of getting a point.

Problem 4

a) Since all together they have $23 + 36 + 15 + 34 = 108$ action figures, the four people have an average of $108 \div 4 = 27$ figures each.

b) If they have an average of 30 figures each, then all together, the four people have a total of 30 $\times$ 4 = 120 action figures. Thus Buffy-Sue now has 120 - (27 + 38 + 20) = 35 figures.

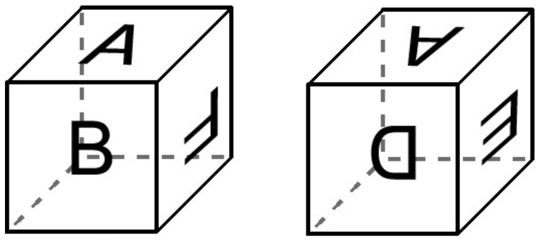
c) To have an average of 32 figures each, the four people must have a total of $32 \times 4 = 128$ action figures. There are two possible answers to this question: if you assume the previous total from part a), they need 128 - 108 = 20 more figures; however, if you assume the previous total from b), they will need 128-120 = 8 more figures.

***Extension* :**

All of the given statements could be true. The total number of figures is $27 \times 4 = 108$. Thus, for a), the friends could have, say, 50, 50, 2, and 6 figures each. A distribution of 40, 40, 19, and 9 shows b) and c) could be true.

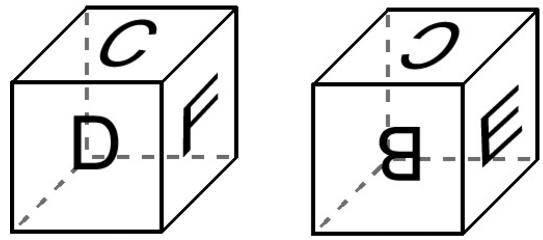
Problem 5

a) Views with the letter A at the top:



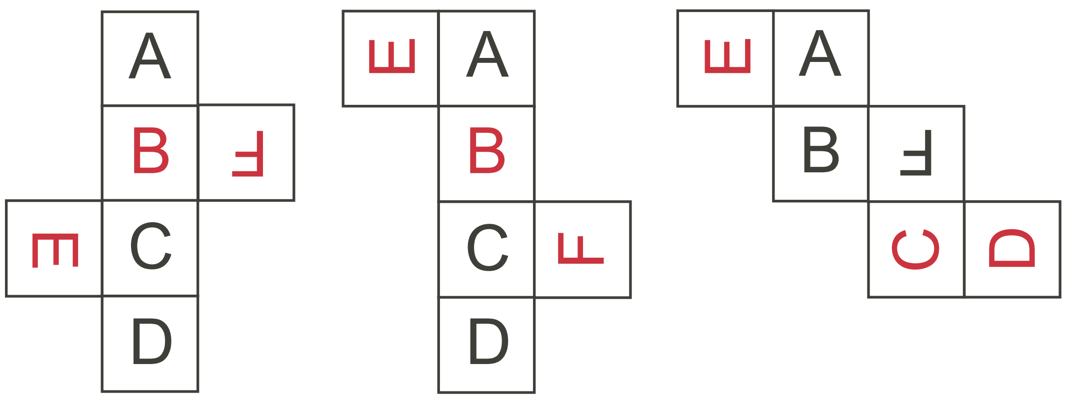
'Front view' 'Back' view

b) Views with the letter B at the top:

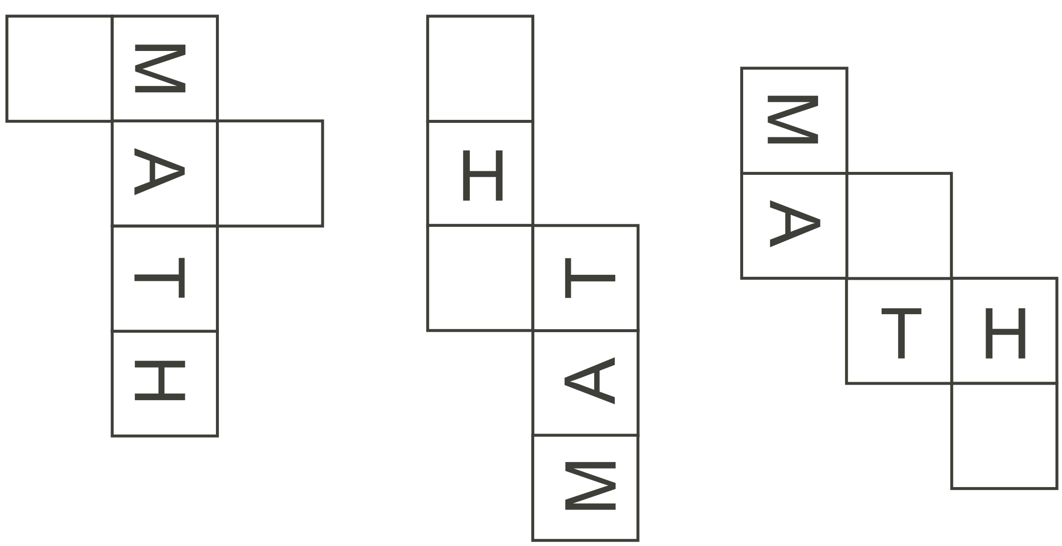


'Front' view 'Back' view

c) The completed nets are:

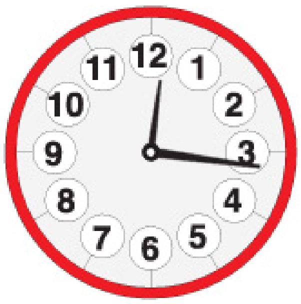


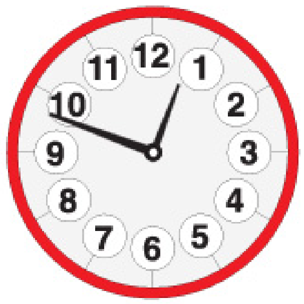
Extension **:**
Solutions will vary. Three possible solutions are shown below. Students can check their answers by constructing the cubes.

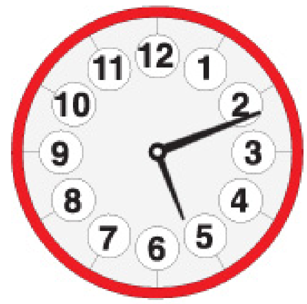


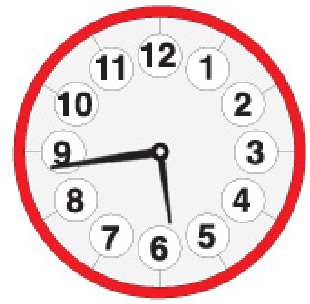
Problem 6

To see whether Sarah is right, we need to discover how many occurrences there are of 'right-angled hands' (90° angle between the two hands), and of 'straight-line hands' (180° angle between the two hands).
Right-angled hands generally occur twice in each hour; the four clock faces below reveal the two right-angled positions between noon and 1 p.m., and between 5 and 6 p.m.

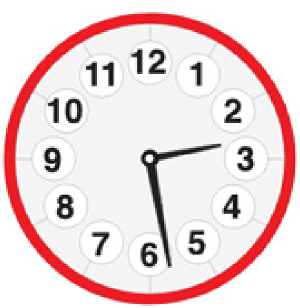

Between 2:15 and 12:20

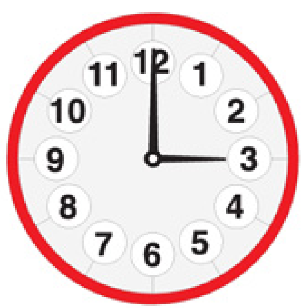

Between 12:45 and 12:50


Between 5:10 and 5:15

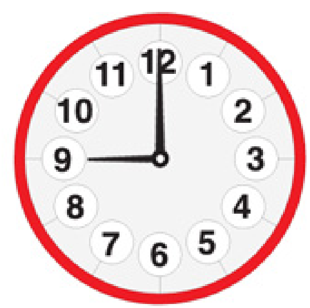

Between 5:40 and 5:45

This occurs for each one-hour interval between noon and midnight, except for two, namely, between 2 and 3 p.m., and between 8 and 9 p.m. In those two intervals, the second occurence of right-angled hands is actually ON the next hour, and hence coincides with the first occurrence for the next hour, as shown below.


Between 2:25 and 2:30


at 3:00


Between 8:25 and 8:30


at 9:00

Thus, between noon and midnight, there are 10 intervals with 2 right-angled positions, and 2 intervals with only 1, giving a total of $10 \times 2 + 2 = 22$ such positions.
Straight-line hand positions are easier to picture; they occur only once per hour in general. For example, between 12 noon and 1 p.m., the hands make 180° somewhere between 12:30 and 12:35; between 2 and 3 p.m., this occurs somewhere between 2:40 and 2:45; between 8 and 9 p.m., somwhere between 8:10 and 8:15, etc. The 'odd-ball' case here is between 5 and 7 p.m., where there is only one occurrence in 2 hours, namely at 6 p.m. Thus there are only 11 occurrences of 'straight-line' hands between noon and midnight.
We thus conclude that Sarah's allowance will be $ $1 \times 22 =$ $ 22 for right-angled hands, or $ $2 \times 11 =$ $ 22 for straight-line hands. So both choices give her the same allowance, i.e., neither gives her a larger allowance.