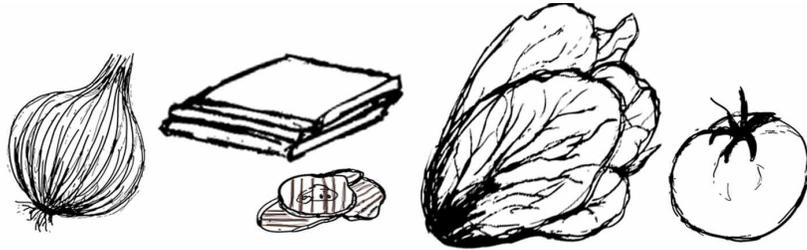


Problem

At the school picnic, the teachers are serving hamburgers to the students. Each student gets a single burger, with the choice of any three condiments from the five the teachers provided: lettuce, cheese, tomatoes, pickles, and onions.



- Ali wants cheese for sure. How many possible combinations of his other two condiments could Ali choose?
- Tanya wants tomatoes for sure. Does she have fewer, more, or the same number of choices as Ali? Explain your answer.
- Xiao knows he wants either cheese or tomatoes, but not both. How many possible combinations of condiments could Xiao choose?
- What is the total number of possible combinations of condiments students could choose for their burgers if they have no preferences and use three condiments?



Hints**Part 2a)**

Hint 1 - Ali has chosen cheese as his first condiment. If he chooses lettuce as his second condiment, what choices does he have for his third condiment? How many combinations are there with lettuce as his second condiment?

Hint 2 - For each choice of his second condiment, what choices does Ali have for his third condiment? Are all the resulting sets of three condiments different?

Part 2b)

Hint 1 - In what way is this question different from part a)? In what way is it the same?

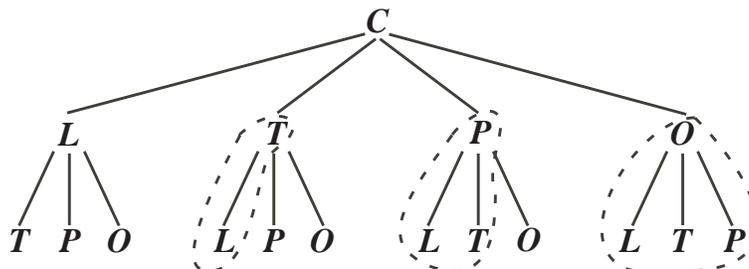
Suggestions:

1. If students find this problem too complicated, start them off with fewer choices, e.g., cheese, onions, and tomatoes only.
2. Students could use a tree diagram in parts a) and b) of this problem. But they may need a suggestion to look for overlapping combinations, since trees give permutations rather than combinations.

Solution

- a) With cheese for sure as his first choice, Ali can choose any two of lettuce, tomatoes, pickles, or onions. So he has 4 choices for his second condiment. For each of those, he has 3 possible choices for his third condiment. In tree form, with C for cheese, L for lettuce, T for tomato, P for pickle, and O for onion, a 'tree' to illustrate all these choices looks like this:

Tree Diagram



BUT... some of these choices give identical combinations! For example, CLT and CTL give the same combination, cheese, lettuce, and tomato. (The dotted ovals enclose combinations which already exist to the left in the tree.) Thus there are only 6 unique combinations for Ali to choose.

- b) If Tanya wants tomatoes for sure, she also has 4 choices for her second condiment and 3 for her third. Thus she has the same number of choices as Ali, namely 6. (This is the same problem as a), with different names!)
- c) Since Xiao wants either cheese or tomatoes, he has 2 choices for his first condiment. Using the abbreviations above, the unique combinations with either cheese or tomato but not both are: CPL, CLO, CPO, TLP, TLO, TPO, a total of 6.
- d) No matter what choice is made for the first condiment, there will always be 6 unique combinations as in a) above. Since there are 5 different possible choices for the first condiment, the total number of possible combinations is $5 \times 6 = 30$.

Comment: Part c) could be illustrated by combining five separate trees like the one in part a), but with different entries for each tree.